

The Geometry Measuring Machine (GEMM) Project at NIST

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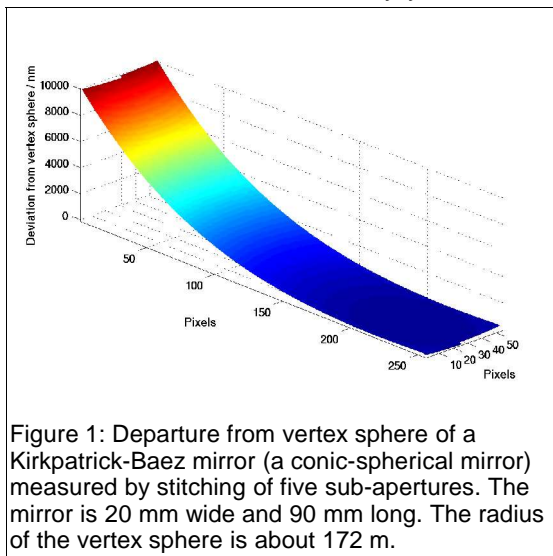
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1. INTRODUCTION

Optics with aspheric surfaces are used in modern optical systems which must perform well, be light-weight, and low-cost. The metrology of aspheric surfaces, however, often poses a formidable problem, especially for precision optics. When very precise aspheric surfaces are tested with interferometers, refractive or diffractive “null-optics” are often used to engineer test wavefronts that match the test part surface. This becomes necessary because the dynamic range of interferometers is generally insufficient to measure aspheres with large departures from the vertex sphere. The use of null-optics however not only adds cost but also creates additional metrology problems and increases measurement uncertainty. Perhaps the most problematic aspect of aspheric surface testing using interferometers and null-lenses is the loss of universality in the metrology tool. For example, spherical Fizeau interferometers, equipped with transmission spheres, are widely used to measure form errors of spherical optical surfaces. Within certain limitations, one transmission sphere can be used to test spherical surfaces over a wide range of radii. However, an interferometer equipped with a null-lens can only be used to test the one surface for which the null-lens was designed. A new surface geometry will generally require a new null-lens.

For the measurements and calibrations which NIST is called upon to make, it is desirable to have methods for interferometric aspheric surface metrology that do not require null optics. The cost of a null lens would usually be prohibitive and the time needed to fabricate the null lens unacceptable. Apart from the relatively simple double-pass tests, two methods are known for asphere metrology without null-lenses. One method that has been used for many years is sub-aperture interferometry (see [1,2]). If the departure of an asphere from a sphere is too large to be measured with an interferometer, the surface is measured in several overlapping parts, or sub-apertures, that are within the dynamic range of the interferometer. Subsequently, the sub-aperture measurements are combined, or “stitched” together, to give a form error map for the entire surface of the part. An example of a “stitched” sub-aperture measurement is shown in Figure 1. It shows the measurement of a conic-spherical mirror which is part of the Kirkpatrick-Baez optics of a synchrotron radiation beamline. The measurement was made with the “eXtremely accurate CALIBration Interferometer” at NIST. The mirror has a rectangular shape with a length of 90 mm and a width of 20 mm. In the short direction it is spherical with a radius of about



172m and elliptical in the long direction. The large vertex radius of 172 m made it possible to measure the form error in five sub-apertures against a flat reference mirror. Figure 1 shows sub-aperture stitching interferometry measurements made with low uncertainty. The disadvantage is that the measurements can be very time consuming and the data analysis is cumbersome.

Recently, a group at the Physikalisch-Technische Bundesanstalt (PTB) in Germany proposed a new method for aspheric surface metrology: the Large Aperture Curvature Sensor (LACS) [3,4]. LACS measures a one-dimensional profile of an aspheric surface. The local curvature of the surface along the direction of the profile is measured at many locations along the profile and the profile is then reconstructed from the curvature measurements. A small interferometer serves as the curvature sensor. The advantage of the LACS concept is that null-optics are not needed. A LACS can, in principle, measure profiles of surfaces with arbitrary shape although the mechanical system that moves the curvature sensor across the surface may impose some practical limitations. The LACS method is also called a “reference free” surface metrology method because a specific reference artefact, such as a transmission sphere, is not required. Once the curvature sensor is calibrated it can be used to measure any profile.

The “Geometry Measuring Machine” (GEMM) at NIST will be similar to the LACS at the PTB. Unlike LACS it will be able to measure two-dimensional surfaces. The goal of the GEMM project is to construct an instrument that can sense the local geometry at any point of an arbitrary surface and then reconstruct the surface from the local geometry information. The work volume of the prototype machine is about the size of an apple which will allow parts with diameters up to about 100 mm to be measured. GEMM will be used to evaluate the feasibility of the geometry sensing concept and to determine the measurement uncertainty that can be achieved.

2. MECHANICAL DESIGN

The design of the GEMM prototype is shown in Figure 2. Like the LACS instrument the GEMM uses a compact phase-measuring interferometer to measure the local geometry, or curvature, at many points of a measurement grid on the part surface under test. It should be possible to use either a Shack-Hartmann sensor or an interferometer as the local geometry sensor. We chose the interferometer primarily because a suitable compact interferometer was commercially available. The main disadvantage of the phase-measuring interferometer is that it is relatively slow which is a minor concern in a prototype instrument. The test beam of the interferometer is collimated and by changing the objective lenses the test beam diameter can be 2 mm, 5 mm, or 10 mm.

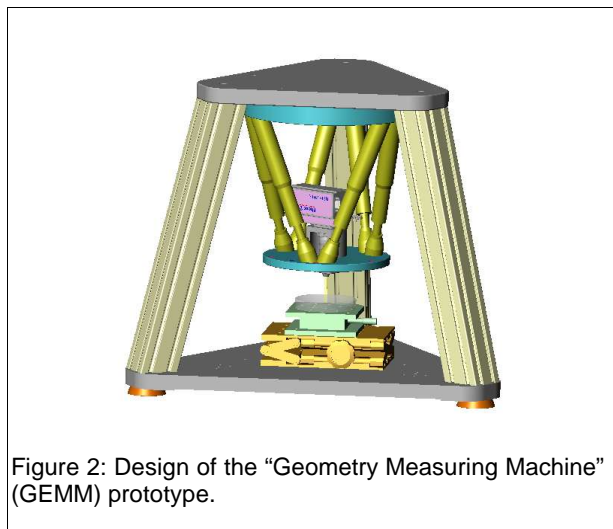


Figure 2: Design of the “Geometry Measuring Machine” (GEMM) prototype.

During a surface measurement the interferometer must be moved from one grid point to the next and it must be possible to adjust the tilt of the interferometer so that the test beam is normal to the surface. An actuator system with at least 5 degrees-of-freedom (DOF) is required. In GEMM, a Stewart platform, or Hexapod robot, is used to move the interferometer. The Stewart platform is a stiff, stable motion system that has 6 DOF and achieves good positioning accuracy. The Stewart platform used in GEMM can position the interferometer with a resolution of 1 μ m in a volume of about 80 mm x 80 mm x 25 mm. Three sturdy aluminum legs connect the Stewart platform actuator and the interferometer to the machine base. The part under test is mounted on a simple linear 3-axis stage that is used to position the part during initial alignment and to bring the part surface onto the object plane of the interferometer's imaging system. A computer system controls both the interferometer and the motion system. GEMM can be used for both one-dimensional profile measurements and complete surface measurement.

3. PROFILES

It can be shown [5] that the shape of a (torsion free) one-dimensional profile P in space is uniquely determined by its curvature k , or the rate of change of the tangent vector, and this is an intrinsic property of the curve. Therefore we can reconstruct the shape of a profile $P(x)$ from measurements of its curvature $k(x)$. This is the concept that underlies the LACS. Schulz [3] and Elster *et al.* [4] have shown that the reconstruction is indeed possible with an uncertainty that makes the method an attractive candidate for precision optical metrology. In cartesian coordinates the relation between a one-dimensional profile $P(x)$ and its curvature $k(x)$ is given by the following non-linear differential equation [5]:

$$\frac{d^2 P(x)}{dx^2} - k(x) \left[1 + \left(\frac{dP(x)}{dx} \right)^2 \right]^{\frac{3}{2}} = 0 \quad (1)$$

When an optical surface is to be measured using this method, the metrologist will decide on a profile across the surface, usually a diameter for a rotationally symmetric part, and then measure the curvatures $k(x_i)$ at a set of locations x_i . Several methods can then be used to solve Equation 1 for the profile $P(x)$ some of which are discussed in [4]. As part of the construction of the GEMM we are evaluating algorithms for the solution of Equation 1. An example is shown in Figure 3 which shows the reconstruction of a simulated measurement of an ellipsoidal mirror with a conic constant $K = -0.1$ and a vertex radius of 200 mm. The curvature of the surface along the profile was determined by fitting a circle to local surface data. Curvature data were thus generated on a grid with 0.1 mm spacing between the grid points. The maximum error seen in the reconstruction is 5 nm. We expect that the error can be reduced with a better reconstruction algorithm.

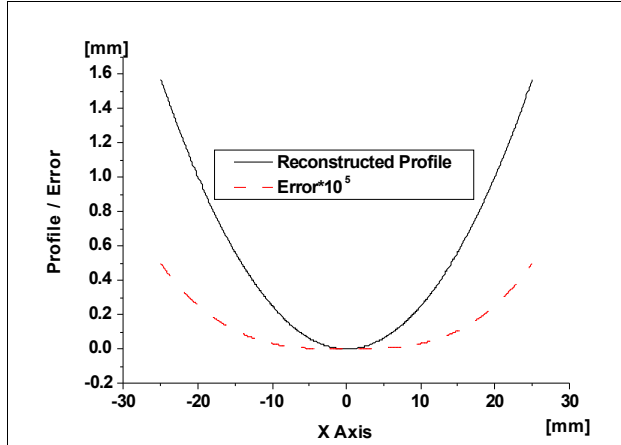


Figure 3: Simulated reconstruction of a surface profile. The surface is an ellipsoid with a conic constant $K = -0.1$.

4. SURFACES

As mentioned above, the goal of the GEMM project is to develop methods for measuring the geometry of a surface in two dimensions, not merely a one-dimensional profile. Unlike the profile, a surface $S(u,v)$ is not simply determined by its curvatures. (u,v denotes a pair of coordinates, for example cartesian coordinates x,y or polar coordinates r,θ .) In the early 19th century one of the great accomplishments of C. F. Gauß [6] was to show that a surface is uniquely determined, up to its orientation in space, by the coefficients of the first and second fundamental forms I and II :

$$I = E du^2 + 2F du dv + G dv^2, \quad (2)$$

where

$$E = \partial_u S \cdot \partial_u S; F = \partial_u S \cdot \partial_v S; G = \partial_v S \cdot \partial_v S, \quad (3)$$

and

$$II = L du^2 + 2M du dv + N dv^2 \quad (4)$$

where

$$\mathbf{L} = \mathbf{n} \cdot \partial_u \partial_u \mathbf{S} ; \mathbf{M} = \mathbf{n} \cdot \partial_u \partial_v \mathbf{S} ; \mathbf{N} = \mathbf{n} \cdot \partial_v \partial_v \mathbf{S} . \quad (5)$$

\mathbf{n} is the unit normal vector field on the surface and we use the abbreviation $\partial_x \equiv \partial / \partial x$. The coefficient functions of the first and second fundamental forms E, F, G , and L, M, N , can be determined experimentally for a given surface just as the curvature for a one-dimensional profile. In analogy to Equation 1, a partial differential equation, the Beltrami equation [7], allows reconstruction of the surface :

$$\frac{1}{\sqrt{EG - F^2}} \left[\frac{\partial}{\partial_u} \left(\frac{G \partial_u \mathbf{S} - F \partial_v \mathbf{S}}{\sqrt{EG - F^2}} \right) + \frac{\partial}{\partial_v} \left(\frac{E \partial_v \mathbf{S} - F \partial_u \mathbf{S}}{\sqrt{EG - F^2}} \right) \right] = 2H \mathbf{n} . \quad (6)$$

H is the mean curvature which depends on coefficients of the curvature form Equation 4. The surface measurement procedure is analogous to the measurement procedure in the one-dimensional case. After a choice of coordinates (u, v) , for example cartesian coordinates, six functions must be measured instead of just one in the one-dimensional case. Equation 6 is then solved for the surface S .

6. REFERENCES

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