THEORETICAL ANALYSIS OF ROTATIONAL ACCURACY FOR THRUST HYDROSTATIC BEARINGS

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INTRODUCTION

Hydrostatic spindles have extremely high rotational accuracy in both the radial and axial directions. It is well known from multiple measurements that rotational accuracy for hydrostatic bearings is usually many times higher than the manufacturing accuracy of the individual bearings components. This effect is called “errors averaging”; it is the result of averaging that occurs due to the oil layer that surrounds the rotating shaft.

In [1] and [2], the theoretical rotational accuracy for hydrostatic journal bearings is analyzed, and the averaging mechanism is explained. However, the averaging process differs significantly between journal and thrust bearings. In this paper the averaging mechanism for a hydrostatic thrust bearing is theoretically analyzed and explained.

MODEL AND ASSUMPTIONS

To simplify the equations used, a simple type of hydrostatic thrust bearing will be analyzed (Fig 1). Oil from the hydraulic power unit is supplied to the pocket through a constant inlet restrictor, R_0. Oil leaves the pocket across two annular lands located on the thrust surface. A large pocket of constant pressure acts on the rear of the shaft and preloads the thrust pocket. This thrust bearing design is widely used, especially in high speed applications.

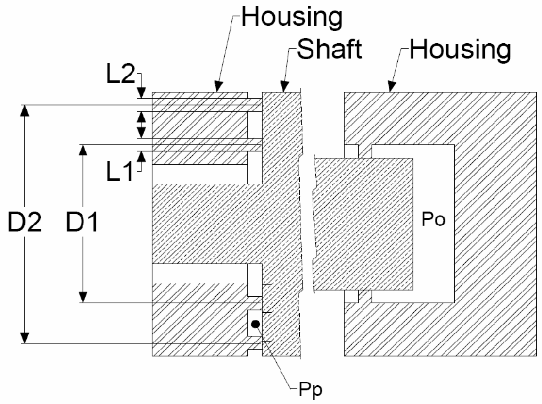

FIGURE 1. A simple hydrostatic thrust bearing.

To make the analysis easier only the simplest type of manufacturing error will be taken into consideration. Both thrust surfaces, the one on the non-rotating housing and the one on the rotating shaft, are flat but not square to the rotational axis (Fig. 2): the angle between thrust surface of the housing and the rotational axis will be called α, and the angle between the shaft’s thrust surface and the rotational axis will be called β.

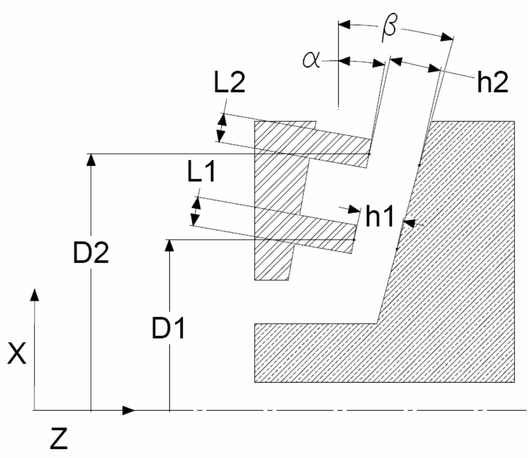

FIGURE 2. Thrust pocket detail.

The goal of this investigation is to find how the shaft axial coordinate, $z(\phi)$, varies as the shaft rotates, ϕ.

Bearing Equations

The quasi static behavior of this system is completely defined by two equations; the flow balance equation,

$$\frac{p_s - p_p}{R_0} = p_p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \tag{1}$$

and the force balance equation,

$$p_p S_p = p_0 S_0 \tag{2}$$

where p_s is supply pressure; p_p is pressure in the thrust pocket; p_0 is pressure in the preloading chamber; R_1 and R_2 are the hydraulic resistances for the inner and outer annular lands respectively; S_p and S_0 are the areas of the thrust pocket and preloading chamber respectively.

The outlet restrictors R_1 and R_2 are described by the following formulas:

$$\frac{1}{R_1}=\frac{D_1}{24\,\mu\,L_1}\int_0^{2\pi} h_1^3(\varphi,\phi)\,d\varphi$$

$$\frac{1}{R_2}=\frac{D_2}{24\,\mu\,L_2}\int_0^{2\pi} h_2^3(\varphi,\phi)\,d\varphi \tag{3}$$

where L_1 and L_2 are the width of the inner and outer annular lands respectively; D_1 and D_2 are the effective diameters for inner and outer lands respectively (Fig. 1); $h_1(\varphi,\phi)$ and $h_2(\varphi,\phi)$ are the gaps at the inner and outer restrictors' lands respectively (Fig. 2); φ is the angular coordinate in non-rotating coordinates system related to the housing; and ϕ is the shaft angle of rotation relative to the housing.

The non-rotating coordinates system will be chosen so that the z-axis is along the axis of rotation, and the plane X-Z,(Y=0) crosses the point where deviation of the housing's thrust surface is maximal.

The initial shaft's angular position *(ϕ=0)* is chosen when maximal deviation of the shaft's thrust surface coincides with maximal deviation of the housing's thrust surface.

Using these definitions, the formulas for the gaps at the inner and outer lands can be written as:

$$\begin{aligned} &h_1(\varphi,\phi)=\\ &z(\phi)-\frac{1}{2}D_1\sin\alpha\,\cos\varphi+\frac{1}{2}D_1\sin\beta\,\cos(\varphi-\phi)\\ &\equiv z(\phi)-u_1\cos\varphi+v_1\cos(\varphi-\phi) \end{aligned} \tag{4}$$

$$\begin{aligned} &h_2(\varphi,\phi)=\\ &z(\phi)-\frac{1}{2}D_2\sin\alpha\,\cos\varphi+\frac{1}{2}D_2\sin\beta\,\cos(\varphi-\phi)\\ &\equiv z(\phi)-u_2\cos\varphi+v_2\cos(\varphi-\phi) \end{aligned} \tag{5}$$

where,

$$u_1=\frac{1}{2}D_1\sin\alpha\;;\quad v_1=\frac{1}{2}D_1\sin\beta\;;\quad u_2=\frac{1}{2}D_2\sin\alpha\;;$$

$$v_2=\frac{1}{2}D_2\sin\beta \tag{6}$$

Parameters u_1, v_1, u_2, and v_2 all have very simple metrological meanings: u_1 and u_2 are half of the total run-out (T.I.R), measured on the housing at the middle of the inner and outer annular lands respectively. v_1 and v_2 are measured on the shaft and are half of the total run-out at the inner and outer lands respectively.

By substituting (4) and (5) into (3), one can get a new set of equations for the outlet restrictors R_1 and R_2:

$$\frac{1}{R_1}=\frac{D_1}{24\mu L_1}\left(2\pi z^3(\phi)+3\pi z(\phi)\left(u_1^2+v_1^2-2\,u_1 v_1\cos\phi\right)\right) \tag{7}$$

$$\frac{1}{R_2}=\frac{D_2}{24\mu L_2}\left(2\pi z^3(\phi)+3\pi z(\phi)\left(u_2^2+v_2^2-2\,u_2 v_2\cos\phi\right)\right) \tag{8}$$

Substituting (7) and (8) into (1) and (2) and combining the equations by eliminating the pocket pressure, p_p, the following equation for $z(\phi)$ can be obtained:

$$\begin{aligned} &\frac{D_1}{24\,\mu\,L_1}\left(2\pi\,z^3(\phi)+3\pi\,z(\phi)\left(u_1^2+v_1^2-2\,u_1\,v_1\cos\phi\right)\right)+\\ &\frac{D_2}{24\,\mu\,L_2}\left(2\pi\,z^3(\phi)+3\pi\,z(\phi)\left(u_2^2+v_2^2-2\,u_2\,v_2\cos\phi\right)\right)\\ &=\frac{1}{R_0}\left(\frac{p_s\,S_p}{p_0\,S_0}-1\right) \end{aligned} \tag{9}$$

To simplify equation (9), as well as the following calculations, the parameter z_0 will be introduced; z_0 is the axial gap when u_1, v_1, u_2, and v_2 all equal zero (which is the ideal thrust bearing, where all surfaces are flat and parallel); z_0 satisfies equation (9) when parameters u_1, v_1, u_2, and v_2 are all zero.

$$\frac{\pi}{12\,\mu}z_0^3\left(\frac{D_1}{L_1}+\frac{D_2}{L_2}\right)=\frac{1}{R_0}\left(\frac{p_s\,S_p}{p_0\,S_0}-1\right) \tag{10}$$

Setting the left sides of (9) and (10) equal to each other and dividing both sides of the resulting equation by z_0^3, one can get the following equation for the dimensionless variable: $\bar{z}(\phi)$,

$$\bar{z}^3(\phi)\left(\frac{D_1}{L_1}+\frac{D_2}{L_2}\right)+\frac{3}{2}\bar{z}(\phi)\left[\begin{array}{l}\frac{D_1}{L_1}\left(\bar{u}_1^2+\bar{v}_1^2-2\bar{u}_1\,\bar{v}_1\cos\phi\right)+\\ \frac{D_2}{L_2}\left(\bar{u}_2^2+\bar{v}_2^2-2\bar{u}_2\,\bar{v}_2\cos\phi\right)\end{array}\right]$$

$$=\frac{D_1}{L_1}+\frac{D_2}{L_2} \tag{11}$$

The dimensionless parameters are defined as follows:

$$\bar{z}(\phi)=\frac{z(\phi)}{z_0};\qquad \bar{u}_1=\frac{u_1}{z_0};\qquad \bar{u}_2=\frac{u_2}{z_0};\qquad \bar{v}_1=\frac{v_1}{z_0};$$

$$\bar{v}_2=\frac{v_2}{z_0} \tag{12}$$

The dimensionless function, $\bar{z}(\phi)$, can be written as a combination of a nominal dimensionless gap and a dimensionless spindle run-out term, ε.

$$\bar{z}(\phi)=1+\varepsilon\left(\phi,\bar{u}_1,\bar{u}_2,\bar{v}_1,\bar{v}_2\right) \tag{13}$$

By substituting (13) into (11) and leaving only the first order terms for ε, and the first and second order terms for $\bar{u}_1$, $\bar{v}_1$, $\bar{u}_2$, and $\bar{v}_2$, the function ε can be approximated as:

$$\varepsilon\left(\phi,\,\bar{u}_1,\,\bar{u}_2,\bar{v}_1,\bar{v}_2\right)=\left(\frac{D_1}{L_1}\bar{u}_1\,\bar{v}_1+\frac{D_2}{L_2}\bar{u}_2\,\bar{v}_2\right)\cos\phi$$
$$-\frac{1}{2}\left[\frac{D_1}{L_1}\left(\bar{u}_1^2+\bar{v}_1^2\right)+\frac{D_2}{L_2}\left(\bar{u}_2^2+\bar{v}_2^2\right)\right] \tag{14}$$

The second term in equation (14) does not change when the shaft is rotating; the first term in (14) describes the shaft's axial motion during rotation.

SUMMARY OF RESULTS

The difference between maximal and minimal values for ε, $\Delta\varepsilon$, is the spindle's axial run-out:

$$\Delta\varepsilon=2\left(\frac{D_1}{L_1}\bar{u}_1\,\bar{v}_1+\frac{D_2}{L_2}\bar{u}_2\,\bar{v}_2\right) \tag{15}$$

To obtain the actual run-out of the spindle, both sides of (15) need to be multiplied by the nominal gap, z_0. As long as the nominal gap is significantly larger than the run-out in the parts, the axial run-out of the spindle will be very small compared to the run-out of the individual parts.

REFERENCES

[1] L. Kashchenevsky. PhD Thesis, 1981

[2] Leonid Kashchenevsky, Byron R. Knapp. Predicting the Rotational Accuracy of Hydrostatic Spindles. ASPE 2001 summer topical meeting Volume 24, pages 52 – 55.