DESIGN ANALYSIS OF RECESS-TYPE HYDROSTATIC SLIDE SYSTEMS

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ABSTRACT
This paper aims to develop the design theory and analysis tool for hydrostatic slide system in precision machine tool applications. More specifically, to develop a design and analysis platform for one specific configuration (i.e. recess-type) of hydrostatic slide system applied in precision lathe machines. Such system should realize the objectives of high quality, ultra precision and high reliability.

1. INTRODUCTION
The design theory and analysis tool development of the hydrostatic slide system will be based on considering a single bearing pad, which constitutes then the basic, modular element to be used to construct any desired table/slide configuration. Once the characteristics of one pad are modelled/ analyzed satisfactory in function of the design parameters, the obtained results can be used in other dedicated design environments to simulation and/or optimise the behaviour of any integrated system of interest, which comprises a number of different pads.

Three main tasks will be investigated in this research, namely:
(1) Definition of one suitable pad configuration as subject of investigation: The pad configuration chosen to be analyzed in this study is the recess-type configuration. It is important to restrict the analysis to a single configuration, otherwise we risk making the analysis and the results too difficult to handle. This single pad is, however, sufficiently versatile being described by a set of parameters, i.e., length, width, land width, recess depth, feed-hole/orifice/valve type, etc. In this way different pad sizes and arrangements can be studied.

(2) Description of the working principle and basic characteristics of the pad:
The basic characteristics of the bearing pad will include the properties of load, stiffness, damping, friction and heat generation, etc. These are functions of the design parameters: pad geometry, gap height, supply pressure and sliding speed. This task describes qualitatively how the performance characteristics are influenced by the design parameters.

(3) Development of an analysis tool:
On the basis of the design theory, an analysis software tool will be developed to calculate the basic performance characteristics of the bearing pad.

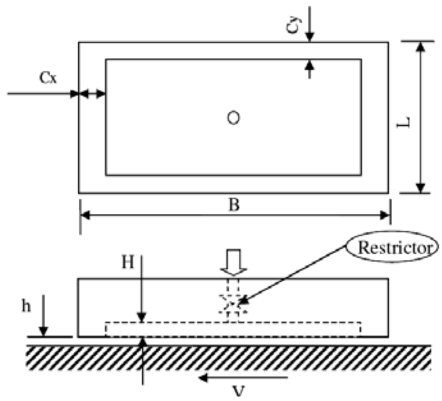


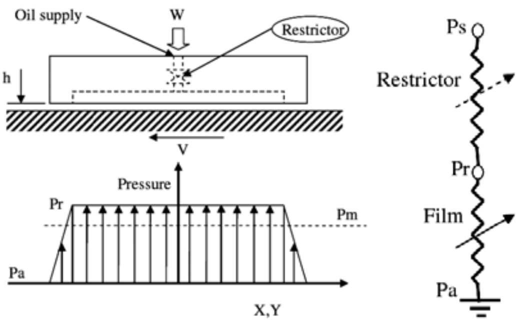
FIGURE 1. Hydrostatic bearing pad configuration

2. DESIGN ANALYSIS OF HYDROSTATIC SLIDE SYSTEM

2.1 Pad Configuration
In this study one pad configuration, shown as Figure 1, is considered. The pad is rectangular and has a central recess (or pocket), which has usually a depth, **H**, that is much larger than the lubricant film thickness, **h**, at the lands. In the middle of the recess, there is a feed hole for

supplying the lubricating oil. This later is usually supplied from a source at a fixed supply pressure $\mathbf{p_s}$, through a restrictor (or a compensator), which causes the pressure to drop to $\mathbf{p_r}$ at the entrance of the recess. The recess is surrounded by flat (finely machined) lands, which may generally have different widths ($\mathbf{C_x}$ and $\mathbf{C_y}$) in the two different bearing directions. The bearing maintains a small gap, **h**, above those lands, depending on the load, supply and restrictor conditions.

The pad may be machined directly on the slideway or manufactured as a separate, modular unit with suitable means to secure it to the slideway. Aspects of manufacture, assembly and piping/oil supply are outside the scope of this study.



(a) (b)

FIGURE 2. (a)Pressure distribution in the pad; (b) Electrical analogy

2.2 Working Principle

Referring to Figure 2, the oil is fed from the supply point at $\mathbf{p_s}$. Before entering the bearing recess, it passes through a restrictor (or a compensator), which may be a capillary tube, an orifice or a special-purpose flow valve and may be placed inside the pad or outside it. The oil reaches the recess at a (reduced) pressure $\mathbf{p_r}$. Since the recess is deep in comparison with the film over the lands, the recess pressure will generally be constant at that value. Finally, the pressure drops approximately linearly over the lands to reach atmospheric pressure at exit (Figure 2a). The average value of the pressure under the bearing pad is $\mathbf{p_m}$. The bearing supports a load, W (= $\mathbf{p_m}$ x pad area), made up usually of a (fixed) pre-load and a (variable) pay-load. This load is clearly proportional to the recess pressure, for a given pad geometry. The fact that the bearing should have a stiffness, i.e. that it would not collapse when the load is increased, is thanks to the presence of the restrictor/compensator. Considering the analogy of Figure 2b, we can see that when the gap reduces (owing to an increase in load), the film resistance to the flow increases. If the restrictor's resistance is a non-decreasing function of the flow (e.g. constant as in the case of a capillary), this would lead to an increase in the recess pressure, which compensates the increase in the load. Thus, besides the geometry of the lands, the characteristics of the restrictor plays an important role in determining the stiffness characteristics of the bearing.

Finally, the sliding speed, **V**, may have two effects on the behaviour of the bearing; namely, (i) warming up the oil, by shearing and thus altering its viscosity, and (ii) causing an uneven distribution of the pressure over the pad. The first aspect could lead to variation in the gap height; the second would lead to a tilting moment on the bearing.

Design parameters:

(1) Pad geometry

- Length and width, **L** and **B**. [m]
- Land widths, $\mathbf{C_x}$ and $\mathbf{C_y}$. [m]
- Nominal (operating) gap height, h. [m]

(2) Supply conditions

- Supply pressure, $\mathbf{P_s}$. [Pa]
- Restrictor/compensator characteristics, which can be specified generally as: $\mathbf{Q} = f(P_s , P_r)$
- Sliding speed, **V**. [m/s]
- (Generally also) the thermal characteristics of pad/slideway, which are however outside the scope of this study. They will be shown to be of minor significance in normal operating conditions.

Basic performance characteristics:

- Load capacity, $\mathbf{W} = \mathbf{p_m}$ x total pad area. [N]
- Stiffness, $\mathbf{k}$ =-dW/dh. [N/m]
- Damping, $\mathbf{c}$ = -dW/dh' (h'=dh/dt). [N/m/s]
- Flow rate of oil, **Q**. [m^3/s]
- Pumping power, $\mathbf{P_p}$. [Watt]
- (Viscous) friction force, $\mathbf{F_f}$. [N]
- Friction power loss, $\mathbf{P_f}$.[Watt]

How the performance characteristics are influenced by design parameters?

The designer is interested (i) in the way that changes in the design affect the performance at the usual (specified) operating conditions, and (ii) in the way that the performance changes with the operating conditions.

Regarding point (i), it may be generally said that, at usual operating conditions, a hydrostatic pad should be designed so as to satisfy, as far as possible, the following conditions:

I. The ultimate load capacity, which is reached as the clearance is reduced to zero, should be as high as possible;
II. The stiffness (at nominal gap) should be as high as possible;
III. The flow should be as low as possible;

Generally, the influence of the supply pressure, the gap height and the restrictor characteristics on the performance can be deduced relatively easily from appropriate formulas. However, the influence of the geometrical pad form and that of the outflow resistance is much less obvious. It may be shown, e.g., that for a constant thrust load (and stiffness at a fixed gap), variation of the supply pressure and land geometry will yield a point of minimum flow. In other words, it is important to have an effective design tool as well as an optimisation methodology in order to arrive at an effective design. The design task is summarised in Figure 3, where the green block represents the essential pad performance.

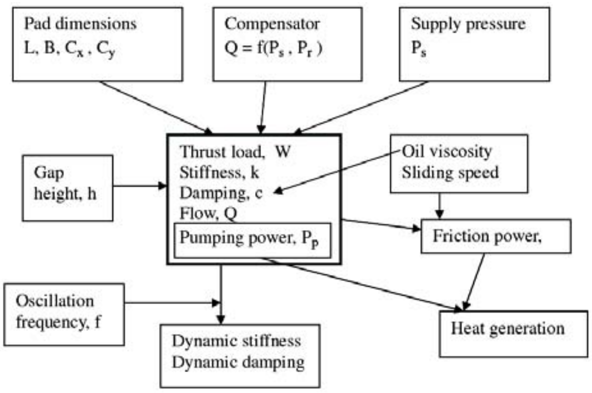


***FIGURE 3.** The interrelation between the principal design parameters and the operating characteristics of an individual pad*

2.3 Design Theory and Basic Relationships

The following notions and relationships[1,2,3,4,5] are needed as a starting point for building up the calculation scheme.

(1) Equivalent bearing area:
Referring to Figure. 2a, the load capacity of the pad is given by:

W = $\mathbf{p_m}$ x pad area, or, equivalent pad area $\mathbf{A_v}$.

W = $\mathbf{p_r}$ x $\mathbf{A_v}$,……………………………………(1)

In Figure1, $\mathbf{A_v}$ can be shown as:

$$A_v \approx BL - BC - LC \quad \text{………………………(2)}$$

(2) Flow resistance:
Considering the analogy of Figure 2b, we define the flow resistance, **R**, by the relation:

$$Q = \frac{\Delta p}{R}, \quad \text{………………………………………(3)}$$

where Q is the volumetric flow of the oil and Δp is the pressure drop over the flow element.

We need to determine the outflow resistance and the inflow resistance.

(1) **Outflow resistance**, R_o, is the viscous flow resistance over the peripheral land of the bearing. It is given approximately by:

$$R_o \approx \frac{6\eta}{h^3(L/C + B/C - 3)}, \quad \text{…………………(4)}$$

where, $\boldsymbol{\eta}$ is the oil viscosity.

(2) **Inflow resistance**, R_i, depends on the type of restrictor used:

(a) Capillary:

$$R_i = \frac{128 l \eta}{\pi d_c^{\,4}}, \quad \text{……………………………(5)}$$

where ***l***= length of capillary tube and $\mathbf{d_c}$ is its internal diameter.

(b) Orifice:

$$R_i = \frac{8\rho}{\pi^2 d_o^{\,4} C_o^{\,2}} Q, \quad \text{…………………(6)}$$

where:
$\boldsymbol{\rho}$ = density of the oil, d_o = orifice diameter, $\mathbf{C_d}$ = discharge coefficient.

Note that, unlike the capillary case, orifice flow is proportional to the square root of the pressure drop. That is why the resistance depends of the value of the flow.

(3) $\mathbf{p_r}$ as function of $\mathbf{p_s}$:
Applying the flow analogy of Figure 2b, we have:

$$p_r = p_s \frac{R_o}{R_o + R_i} = \frac{p_s}{1 + \frac{R_i}{R_o}} \quad \text{…………………(8)}$$

This enables us to calculate the load capacity as:

$$W = p_r A_v = \frac{p_s A_v}{1+\dfrac{R_i}{R_o}} \quad \text{.............................(9)}$$

It is expedient to define the value of the resistance ratio, R_o/R_i at nominal, or design gap, $\mathbf{h = h_d}$ and use this as design parameter to calculate the bearing characteristics. Thus, we define:

$$\xi = \frac{R_i}{R_o}, \quad for \quad h = h_d \quad \text{........................(10)}$$

Obviously there will be an optimum value for this parameter depending on which type of restrictor used.

2.4 Calculation of Basic Performance Characteristics

First we give the characteristics that depend on the compensation type (load and stiffness). After that, we give the characteristics, which are independent of restriction type (flow rate, pumping power, friction force and damping).

Load capacity, and stiffness

(1) Capillary compensation:

The load is given by:

$$W = \frac{p_s A_v}{1+\xi\left(\dfrac{h}{h_d}\right)^3}, \quad W_d = \frac{p_s A_v}{1+\xi} \quad \text{.........(11)}$$

Where $\mathbf{W_d}$ is the load at nominal (or design) gap.

The stiffness is given by:

$$k = 3\frac{W_d}{h_d}\frac{\xi(1+\xi)\left(\dfrac{h}{h_d}\right)^2}{\left(1+\xi\dfrac{h^3}{h_d{}^3}\right)^2} \quad \text{.....................(12)}$$

The stiffness at nominal (or design) gap is then:

$$k_d = 3\frac{W_d}{h_d}\frac{\xi}{(1+\xi)} \quad \text{.............................(13)}$$

It can be shown that the maximum stiffness (at nominal gap) is obtained, for capillary compensated bearing, when:

$$\xi = 1. \quad \text{..(14)}$$

This means that:

$$W_{optimal} = \frac{p_s A_v}{2} \quad \text{................................(15)}$$

and

$$k_{optimal} = \frac{3}{2}\frac{W_d}{h_d}. \quad \text{................................(16)}$$

(2) Orifice compensation:

The resistance ratio can be expressed as:

$$\frac{R_i}{R_o} = \frac{1}{2}\left(\sqrt{1+4\xi(\xi+1)(h/h_d)^6}-1\right) \quad \text{......(17)}$$

The load can then be calculated from the general formula (9):

$$W = \frac{p_s A_v}{1+\dfrac{R_i}{R_o}} \quad \text{......................................(18)}$$

The stiffness is given by:

$$k = 6\frac{W_d}{h_d}\frac{\xi(1+\xi)^2}{\left(1+2\dfrac{R_i}{R_o}\right)\left(1+\dfrac{R_i}{R_o}\right)^2}\left(\frac{h}{h_d}\right)^5 \quad (19)$$

The stiffness at nominal (or design) gap is then:

$$k_d = 6\frac{W_d}{h_d}\frac{\xi}{(1+2\xi)} \quad \text{.............................(20)}$$

It can be shown that the maximum stiffness (at nominal gap) is obtained, for an orifice compensated bearing, when:

$$\xi = \sqrt{\frac{1}{2}}. \quad \text{...(21)}$$

This means that:

$$W_{optimal} = \frac{p_s A_v}{1+\sqrt{1/2}} \approx 0.585 p_s A_v \quad \text{...........(22)}$$

and

$$k_{optimal} = \frac{6\sqrt{1/2}}{1+2\sqrt{1/2}} \approx 1.757\frac{W_d}{h_d} \quad \text{............(23)}$$

Common Bearing Characteristics

(1)Flow rate of oil, Q is calculated from:

$$Q = \frac{p_r}{R_o} = \frac{p_s - p_r}{R_i} = \frac{p_s}{R_i + R_o} \quad \text{..............(24)}$$

(2)Pumping power, P_p, is given by:

$$P_p = p_s Q \quad \text{..(25)}$$

(3)Damping, c = -dW/dh' (h'=dh/dt).

$$c \approx \frac{2(L+B-2C)C^3}{h^3} \quad \text{..........................(26)}$$

(4)(Viscous) **friction force**, $\mathbf{F_f}$, is given by:

$$F_f = 2C(L+B-2C)\eta\frac{V}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

N.B. The **friction power**, $\mathbf{P_f}$, = $\mathbf{F_f\,V}$.

2.5 Development of An Analysis Tool

A software code using *Matlab*® is developed to calculate the performance characteristics of the bearing pad. The programme code developed here is written in a generic form, which is both user-friendly and flexible, where the user can possibly add other aspects to the code on the one hand, and integrate it into a more elaborate design environment, on the other.

3. DESIGN EXAMPLE

An example of one pad with different compensation methods is considered in this paper. The pad has dimensions **L**=75 mm, **B**= 125 mm, land width, $\mathbf{C_x}$= $\mathbf{C_y}$= 10 mm, and pocket depth of 0.5 mm. Supply gauge pressure is 100 bar, ambient pressure is 1 bar. The feed hole is located at the center of the pad. The oil viscosity is 10 mPa (or 0.01 Ns/m^2). The performance is examined over a gap height variation 20 to 40 micrometers, at intervals of 5 micrometers. It is assumed that the operating gap is 30 micrometers; this means that for capillary and orifice restrictors, the stiffness will have a maximum value around 30 micron gap.

Two compensation cases are considered separately as follows.

(1) Capillary

We here choose pocket gauge pressure, at nominal gap, **Po** equal to half of the gauge supply pressure for maximum stiffness. Table 1 depicts the results of bearing characteristics with respect to different gap heights. Note that both of the friction power and the hydrodynamic power are null owing to the absence of sliding. Figure 5 is the graphical outputs of the main bearing characteristics.

(2) Orifice

We set nominal **Po** = 0.5858* **Ps** for maximum stiffness at nominal gap. Table 2 shows the results of bearing performance characteristics with respect to different gap heights, and Figure 6 are the graphical outputs of bearing characteristics.

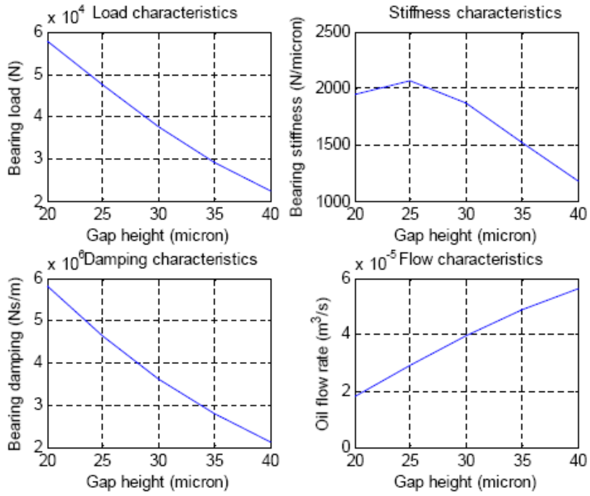


FIGURE 5. The basic characteristics of capillary: Load, stiffness, damping and flow are plotted in function of the gap height.

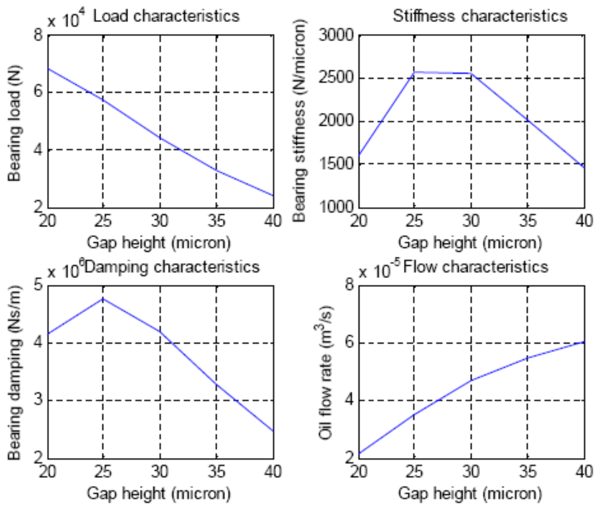


FIGURE 6. The basic characteristics of orifice: Load, stiffness, damping and flow are plotted in function of the gap height.

4. DISCUSSIONS

Comparing Figures 5 and 6, it can be concluded that the orifice yields better performance than the capillary, as expected from theoretical analysis. In order to show the effect of sliding, we consider here the same pad as above, with capillary compensation, but with a sliding speed of 5m/s, which is an extreme value for hydrostatic slideways (but not for rotating systems). Table 3 shows the results of bearing characteristics with respect to different gap heights while considering sliding speed. We notice here non-zero values of the friction power and the hydrodynamic power. However, they are much smaller than the pumping power. If the speed is reduced to 2 m/s or less, those values will be an order of magnitude smaller than the pumping power. In other words, they become negligible.

5. CONCLUSION

A software tool has been developed for the calculation of the characteristics of hydrostatic bearing pads. This software code is able to determine all the relevant performance characteristics of the bearing, given any set of design parameters.

6. ACKNOWLEDGEMENTS

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TABLE 1. Bearing characteristics with respect to different gap heights for capillary restrictor.

Gap (μm)	Load (N)	Stiffness (N/μm)	Damping (Ns/m)	Flow Rate (cm^3/s)	$P_{Pumping}$ (Watt)	$P_{Friction}$ (Watt)	$P_{Hydrodynamic}$ (Watt)
20	57907	1950.9	5.80×10^6	18.102	179.21	0	0
25	47698	2067.3	4.64×10^6	29.119	288.28	0	0
30	37770	1865.6	3.63×10^6	39.836	394.38	0	0
35	29272	1524.1	2.79×10^6	49.013	485.23	0	0
40	22531	1178.5	2.14×10^6	56.296	557.33	0	0

TABLE 2. Bearing characteristics with respect to different gap heights for orifice restrictor.

Gap (μm)	Load (N)	Stiffness (N/μm)	Damping (Ns/m)	Flow Rate (cm^3/s)	$P_{Pumping}$ (Watt)	$P_{Friction}$ (Watt)	$P_{Hydrodynamic}$ (Watt)
20	68315	1599.1	4.16×10^6	21.356	211.43	0	0
25	57555	2568.1	4.77×10^6	35.137	347.85	0	0
30	44373	2556.5	4.21×10^6	46.801	463.33	0	0
35	32855	2018.6	3.28×10^6	55.013	544.63	0	0
40	24197	1461.5	2.46×10^6	60.459	598.54	0	0

TABLE 3. Bearing characteristics with sliding speed for orifice restrictor.

Gap (μm)	Load (N)	Stiffness (N/μm)	Damping (Ns/m)	Flow Rate (cm^3/s)	$P_{Pumping}$ (Watt)	$P_{Friction}$ (Watt)	$P_{Hydrodynamic}$ (Watt)
20	57907	1950.9	5.80×10^6	18.102	179.21	48.692	14.658
25	47698	2067.3	4.64×10^6	29.119	288.28	39.476	14.229
30	37770	1865.6	3.63×10^6	39.836	394.38	33.323	13.813
35	29272	1524.1	2.79×10^6	49.013	485.23	28.922	13.410
40	22531	1178.5	2.14×10^6	56.296	557.33	25.615	13.020