# A Framework for Uncertainty Evaluation of GPS Standard-chain

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**Abstract** The uncertainty is used as an economic tool to enable optimum allocation of resource amongst specification, manufacturing and verification. For a given GPS standard-chain, the key question is to calculate the compliance uncertainty. The determination of compliance uncertainty is a complex question because specification uncertainty and measurement uncertainty arise from many causes and propagate through the GPS standard-chain. A framework for compliance uncertainty of GPS standard-chain is proposed in this article, which is based on the modeling of GPS standard-chain. According to ISO 17450–2, a GPS process should be either in default state or in special state. The biggest difference between the two states is that whether the specification operator is accordant with the verification operator. Aiming at the two states, the flow for the computation of compliance uncertainty is given respectively. It enables us to generate compliance uncertainty on the verification of GPS standard-chain, which makes the acceptance or refusal of feature characteristic can be conducted in a quantitative way, so the veracity of verification could be improved.

*Key words*: GPS, uncertainty, standard-chain

## 1 Introduction

Since the establishment in 1996, ISO/TC213 has been working to develop the geometrical product specification (GPS) system, which links the whole course of geometrical products from specification, manufacturing to verification [1]. The “uncertainty” is used as an economic tool to enable optimum allocation of resource amongst specification, manufacturing and verification. In the GPS system, correlation uncertainty, specification uncertainty, compliance uncertainty and total uncertainty are defined besides measurement uncertainty [2]. The relationship of various uncertainties is shown in fig. 1.

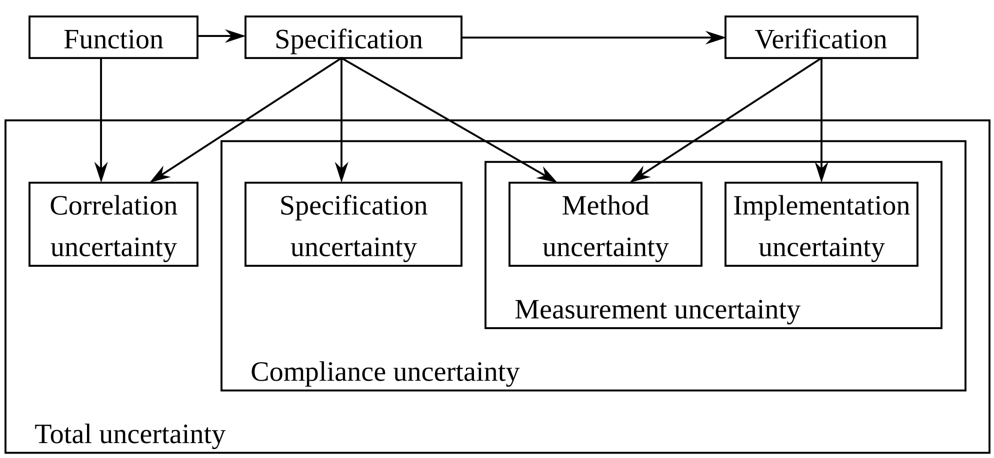


Fig. 1 Relationship of various uncertainties

Correlation uncertainty is usually not related to one single GPS specification. Commonly it takes a number of single GPS specifications to simulate a function. Therefore, for a given single GPS specification, the most important question is to determine compliance uncertainty of the corresponding GPS standard-chain, which is used to prove conformance or non-conformance of feature characteristic. Compliance uncertainty is the sum of specification uncertainty and measurement uncertainty, with which it can be proven that a workpiece complies with all possible interpretation of a specification. The computation method of measurement uncertainty is given in GUM [3] and ISO 14253-2 [4]. However, there is no specific method to calculate the compliance uncertainty until now. In order to solve this urgent question, a framework for the compliance uncertainty evaluation of GPS standard-chain is presented in this article.

## 2 Modeling of GPS standard-chain

A GPS standard-chain consists a set of standards related to a given GPS specification. The standards are collected in a few groups – "links" of the chain. There are 7 chain links associated with different geometrical characteristics of features [5]. It is clear that each GPS standard-chain is related to the complete process of specification, manufacturing and verification. Specification is the primary variation-control tool in design and verification is the primary tool to assess conformance to specification. A manufacturing process is selected to realize the design in a cost effective manner and to check the geometrical characteristic accuracy. In a GPS standard-chain, geometrical feature occurs in three disciplines: nominal feature, specification feature and verification feature, which correspond to nominal surface model, skin model and sampled surface model respectively.

### 2.1 Nominal surface model

Nominal surface model is the surface model of ideal geometry defined by the technical product documentation. The integral feature part of a nominal surface model is nominal feature *NF*, which is an ideal geometric entity.

### 2.2 Skin model

Skin model is the surface model of non-perfect geometry. The integral feature part of a skin model is specification feature *SF*, which is a non-ideal geometric entity. The skin model and *SF* are accordant with specification operator, which is an ordered set of such specification operations as partition, extraction, filtration, association, collection, construction and evaluation. The specification operator can be characterized by a function $F_{SP}$, so the input of $F_{SP}$ is *SF* and the output is the specification of characteristic $T_{SP}$. Accordingly, every operation of the specification operator can be looked on as a sub-function of $F_{SP}$.

### 2.3 Sampled surface model

Sampled surface model is approximation of the discrete surface model obtained by sampling of the workpiece with measuring instruments. The integral feature part of a sampled surface model is verification feature *VF*, which is a non-ideal geometric entity. The sampled surface model and *VF* are accordant with verification operator, which is an ordered set of such verification operations as partition, extraction, filtration, association, collection, construction and evaluation. The verification operator can be characterized by a function $F_{VP}$, so the input of $F_{VP}$ is *VF* and the output is the evaluation of characteristic $S_{VP}$. Accordingly, every operation of the verification operator can be looked on as a sub-function of $F_{VP}$.

### 2.4 Comparison for conformance

The measurement uncertainty causes "grey-zones" around the specification limits given on a drawing. According to the decision rules given in ISO 14253-1 [6], only measured values in the conformance zone can prove conformance, and only measured values in the non-conformance zone can prove non-conformance. If measured values are inside the "grey-zones", it is neither possible for the customer to reject the workpiece, nor for the supplier to accept the workpiece. Obviously, the decision rules are improper because the specification uncertainty is not taken into account, which are existent actually in GPS specifications. Furthermore, sometimes the specification uncertainty evaluated for specifications given on existing product documentation, is much larger than the measurement uncertainty. In this instance, the measurements performed with small measurement uncertainty are too expensive and of no use, if the specification uncertainty is not reduced drastically. Therefore, the decision rules should be based on compliance uncertainty for a GPS standard-chain, which takes into account specification uncertainty as well as measurement uncertainty.

Summarizing above-mentioned, the model of GPS standard-chain can be shown as in figure 2.

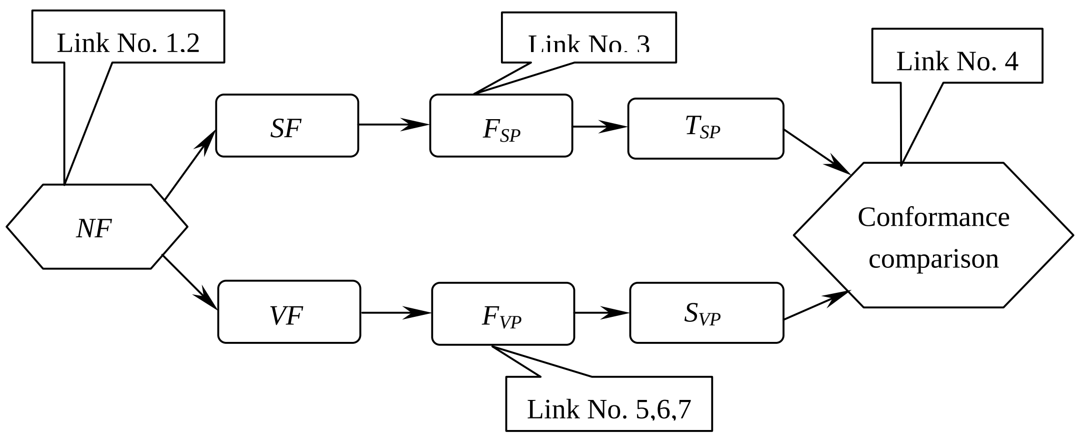


Fig. 2 Model of GPS standard-chain

## 3 Compliance uncertainty of GPS standard-chain

It can be seen clearly that the key question is to calculate the compliance uncertainty for a given GPS standard-chain. The determination of compliance uncertainty is a complex question because specification uncertainty and measurement uncertainty arise from many causes and propagate through the GPS standard-chain. In the past, the measurement uncertainty was regarded mainly but the specification uncertainty was ignored. Although the GPS system gives the definition of specification uncertainty, the ISO standard for specification uncertainty is not existent until now. According to ISO 17450–2, a GPS process should be either in default state or special state. The biggest difference between the two states is that whether the specification operator is accordant with the verification operator. Aiming at the two states, the flow for the computation of compliance uncertainty is proposed respectively as follows.

### 3.1 Default state

The specification operator is a default operator, and the verification operator is accordant with the specification operator completely. In this state, the method uncertainty equals to zero, so the compliance uncertainty should be calculated by the propagation of the implementation uncertainty though the specification operator or verification operator, in which the specification uncertainty is inherent, as shown in fig. 3.

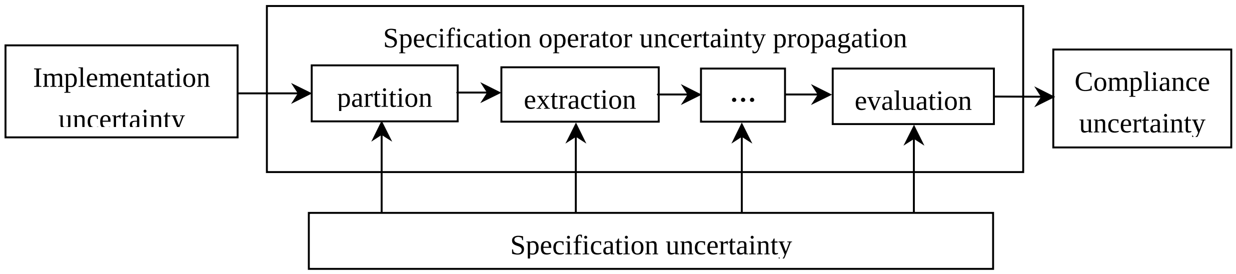


Fig. 3 Calculation flow for compliance uncertainty in default state

### 3.2 Special state

The specification operator is a special operator, and/or the verification operator is not accordant with the specification operator. In this state, the method uncertainty doesn't equal to zero commonly, so the compliance uncertainty should be obtained by the propagation of the implementation uncertainty though the verification operator, in which both the specification uncertainty and method uncertainty are inherent, as shown in fig. 4.

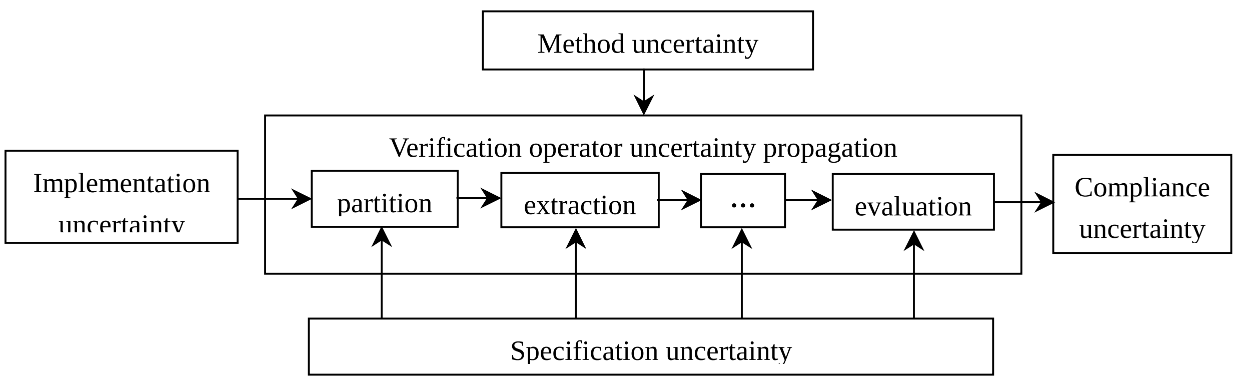


Fig. 4 Calculation flow for compliance uncertainty in special state

## 4 A case study

In this section, a case study for the GPS specification of $R_y$ 16.0μm is given in the default state.

### 4.1 Modeling of GPS standard-chain corresponding to *Ry* 16.0μm

According to ISO 17450-2, the default specification operator of $R_y$ 16.0μm indicates that:

- partition from the skin model of a non-ideal surface;
- partition of non-ideal lines from this non-ideal surface;
- extraction using the evaluation length given in ISO 4288;
- filtration using Gaussian filter with cut-off wavelength determined by the rules in ISO 4288 and the corresponding stylus tip radius and sample spacing shall be used;
- evaluation of $R_y$ values as defined in ISO 4287 and 4288.

Because it is in the default state, the verification operator is accordant with the specification operator completely, which consists of partition, partition, extraction, filtration, evaluation orderly, too.
The specification of characteristic is measured by the stylus instrument with sampling interval $\Delta_x$ = 5μm, sampling length $l_r$ = 2.5mm, evaluation length $l_n$ = 5mm and cut-off wavelength $\lambda c$ = 2.5mm. After sampling data are filtered, the result of $R_y$ can be numerated, which equals to 15.0μm.

## 4.2 Compliance uncertainty of the GPS standard-chain

According to the flow chat in default state, the calculation of compliance uncertainty can be divided into three steps: the first step, to determine the implementation uncertainty; the second step, to determine the uncertainty after the filtration operation and the last step, to determine the uncertainty after evaluation operation, which is the final result of compliance uncertainty.

The first step: Usually, it is supposed that the sampling points are totally uncorrelated and the uncertainty of sampling points is equal, so the implementation uncertainty $u_{im}$ is the uncertainty of single sampling point, which is equated to be 0.648μm.

The second step: The Gaussian filter according to ISO 11562 [7] is a linear profile filter of a continuous weighting function $s(x)$ defined by the equation

$$s(x)=\frac{1}{\alpha\lambda_c}\exp\left[-\pi\left(\frac{x}{\alpha\lambda_c}\right)^2\right] \tag{1}$$

According to the uncertainty propagation rule of Gaussian filter [8], the uncertainty $u_{fi}$ after the filtration operation can be expressed as

$$u_{fi}=\sqrt{1-\frac{\Delta_x}{a\lambda_c}\left(2-\frac{1}{\sqrt{2}}\right)}u_{im}=0.646\mu m \tag{2}$$

The last step: The filtered data are correlated, although the input data are totally uncorrected. The reason is that each filtered value, due to the convolution process, is calculated from several input values in the neighborhood of this particular value. The compliance uncertainty after evaluation operation can be calculated by the uncertainty propagation formula

$$u_{co}=\sqrt{u_{fi}^{\ 2}+u_{fi}^{\ 2}-2\rho_{i,j}u_{fi}^{\ 2}}=0.9\mu m \tag{3}$$

## 4.3 The result of conformance comparison

In conformity to the decision rules given in 2.4, the result of conformance comparison is shown in fig. 5. It is can be seen that the measured value ($R_y$ = 15.0μm) is inside the conformance zone, so the measured value is conformable to the measurand.

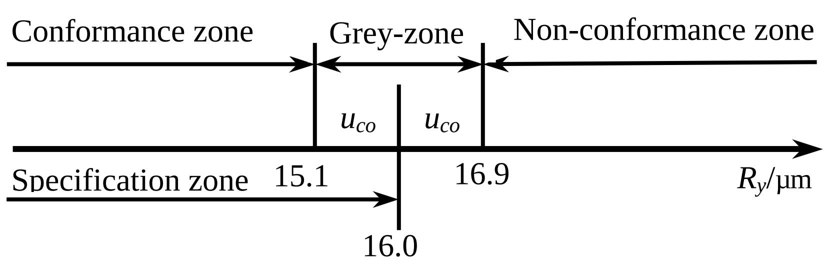


Fig. 5 Result of conformance comparison

## 5 Conclusions

A framework for uncertainty evaluation of GPS standard-chain is proposed in this article, which is based on the modeling of GPS standard-chain. It enables us to generate compliance uncertainty on the verification of GPS standard-chain, which makes the acceptance or refusal of feature characteristic can be conducted in a quantitative way, so the veracity of verification could be improved. Furthermore, it can be seen clearly that the calculation process of compliance uncertainty is easier in the default state than the special state because the specification operator is accordant with the verification operator. So the “default “ should be used as much as possible in a GPS standard-chain, which embodies the simplification principle advocated by the GPS system.


## Acknowledgements

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