

Evaluation of the geometrical uncertainty of helix deviation measurements using the Monte Carlo simulation

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Introduction

In the evaluation of uncertainty, the usual approach is to write down a model equation and apply the law of propagation of uncertainty to it. However, when the measurand is a complex derived quantity, the model equation occasionally involves an inverse function that cannot be solved analytically. It is not possible to directly apply the law of propagation of uncertainty to such a situation.

In this study, the uncertainty of a quantity involving an inverse function that cannot be solved analytically is evaluated by using the Monte Carlo simulation. Prior to this, we constructed a program that calculates the uncertainties of the profile deviation of gear measurement¹. We pick up the helix deviation measurement of the gear in this study.

Principle and model equation of helix deviation measurement

The principle of helix deviation measurement is shown in Fig. 1.

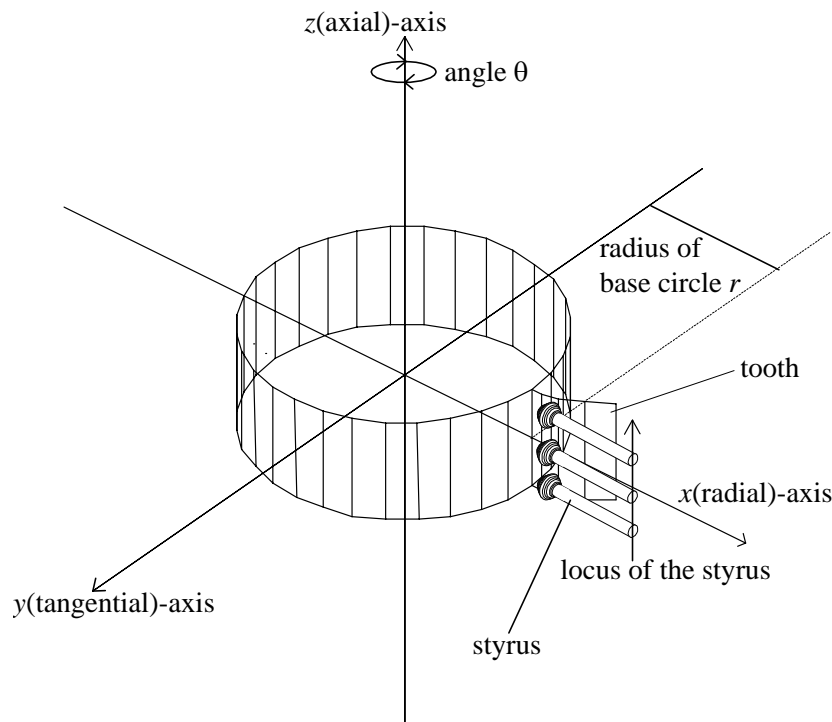


Fig. 1: Principle of helix deviation measurement

When the helix deviation of the gear is measured, the y -position of the stylus may be obtained. Subsequently, we calculate the model equation of the y -position of the stylus.

Position of the gear tooth:

The ideal position of the gear tooth (x_0, y_0, z_0) is expressed by the following equations:

$$\begin{aligned}
x_0 &= r(\cos a + a \sin a) \\
y_0 &= r(\sin a - a \cos a) \\
-W_0 &\leq z_0 \leq W_0
\end{aligned}$$

Here, r is the radius of the base circle, a is the parameter of the involute function, and W_0 is the width of the gear tooth. The Gear measurement machine has an eccentricity and a origin setting error. Let the sum of those errors be (x_{ecc}, y_{ecc}) , and the position of the gear tooth that includes the eccentricity and origin setting error be (x_1, y_1, z_1) .

$$\begin{aligned}
x_1 &= x_0 + x_{ecc} \\
y_1 &= y_0 + y_{ecc} \\
z_1 &= z_0
\end{aligned}$$

The spindle of the gear inclines and the gear rotates. Let the inclination of the gear spindle be ϕ , the angle of rotation be θ_1 , and the position of the gear tooth after inclination and rotation be (x_2, y_2, z_2) .

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = rot(\phi, \theta_1) \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

Here, $rot(\phi, \theta_1)$ represents the rotation matrix, and θ_1 is the sum of the identical value of rotation and the error of rotation. (x_2, y_2, z_2) represents the actual position of the gear tooth.

Position of the stylus:

Let the identical x -position and z -position of the stylus be X_0 and Z_0 , respectively.

$$\begin{aligned}
X_0 &= P_x \\
Z_0 &= P_z
\end{aligned}$$

P_x is the fixed measurement point. Identically, P_x is a static value; and P_z is the displacement in the direction of measurement; therefore, the value of P_z varies.

Let the x -position and z -position of the stylus that includes the geometrical errors be X_1 and Z_1 , respectively.

$$\begin{aligned}
X_1 &= X_0 \cos(X_{slopeV}) \cos(Y_{slopeH} - X_{slopeH}) + Z_0 \sin(Z_{slopeX}) \\
Z_1 &= Z_0 \cos(Z_{slopeX}) \cos(Z_{slopeY})
\end{aligned}$$

Here, X_{slopeV} is the inclination of the liner gage against the reference plane, and X_{slopeH} is the inclination of the linear gage for the reference plane. The other constants obey this law.

Let the x -position and z -position of the stylus that includes random errors be X_2 and Z_2 , respectively.

$$\begin{aligned}
X_2 &= X_1 + P_{x_rand} + P_{z_yaw} \\
Z_2 &= Z_1 + P_{z_rand}
\end{aligned}$$

Here, P_{x_rand} and P_{z_rand} are the measurement errors of the x - and z -direction, respectively. P_{z_yaw} represents the yawing of the z -axis. Accordingly, the x - and z -position of the stylus can be calculated.

Calculation of y-position of the stylus:

Simultaneous equations will be obtained if X_2 and Z_2 are substituted for x_2 and z_2 , which express the position of the gear. The y-position of the stylus is calculated by solving the simultaneous equations. These simultaneous equations are solved uniquely. However, since they include the inverse involute function, they cannot be solved analytically. Therefore, we construct a virtual machine simulating the gear measurement machine, and we solve the simultaneous equations using the simulation. Subsequently, we can use the Monte Carlo method to obtain the distribution of the measurement data. This flowchart is shown in Fig. 2.

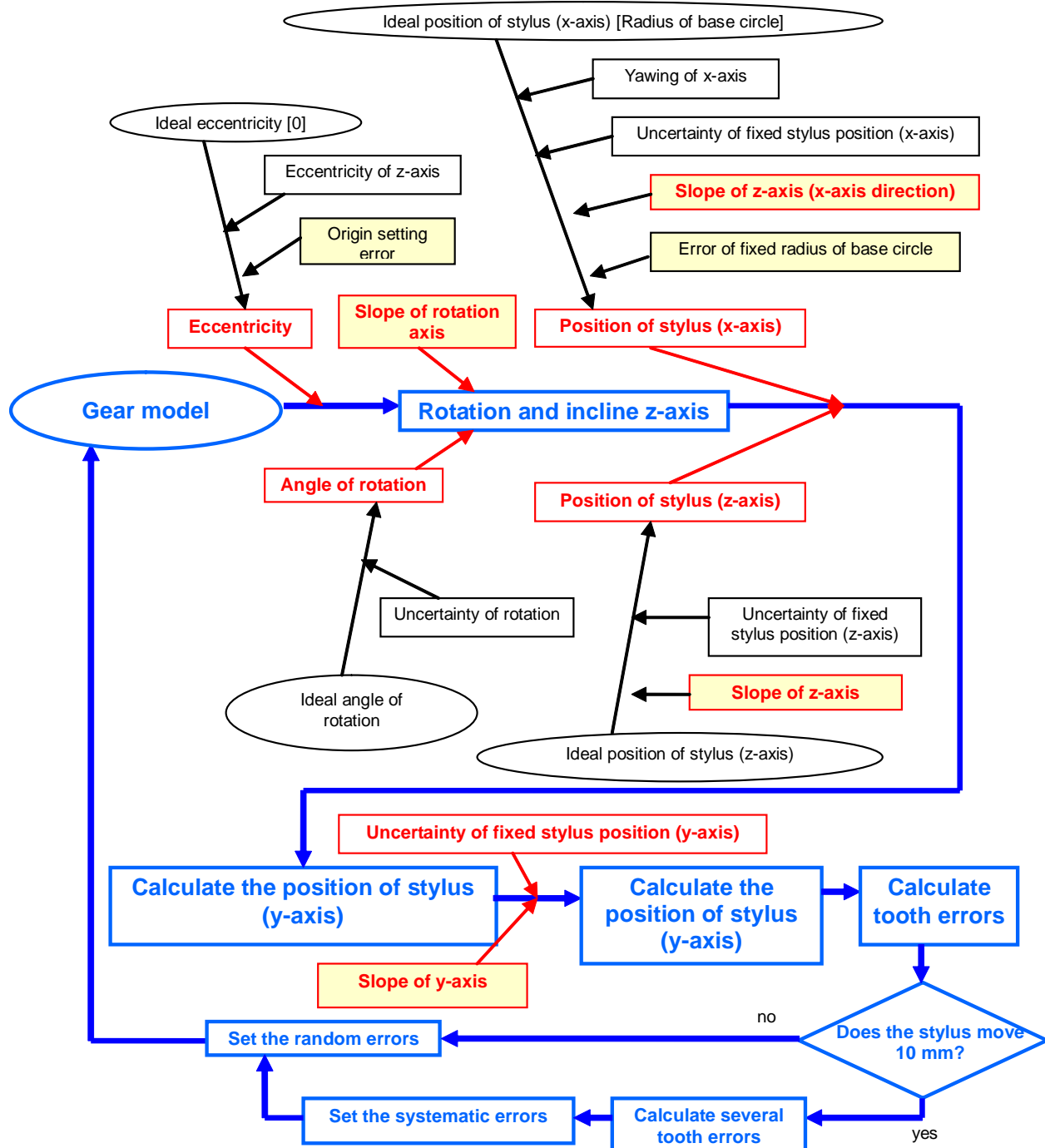


Fig. 2: Flowchart of the simulation

About the simulation and results

The factors of the measurement uncertainty taken into consideration included: eccentricity of the rotation axis, origin setting error, slope of the rotation axis, measurement uncertainty of the rotary encoder, measurement uncertainty of the scales, pitching of the z (axial)-axes, and slopes of x (radial)-, y (tangential)- and z (axial)-axes.

The preceding individual uncertainties were measured at the University of Electro-Communications and Osaka Seimitsu Kikai Co., Ltd²⁾. The results of the measurements are listed in Table 1, and the simulation conditions are listed in Table 2.

The individual uncertainties listed in Table 1 are limited values except for the length measurement uncertainty (y -axis direction). The distributions of individual uncertainties, except for the length measurement uncertainty (y -axis direction), are assumed to be rectangular, and the distribution of the length measurement uncertainty (y -axis direction) is assumed to be normal.

Table 1: Individual uncertainties

Source of uncertainties	Value
Eccentricity (x -axis direction)	0.51 μm
Eccentricity (y -axis direction)	0.51 μm
Origin setting error (x -axis direction)	0.32 μm
Origin setting error (y -axis direction)	0.12 μm
Length measurement uncertainty (x -axis direction)	0.3 μm
Length measurement uncertainty (y -axis direction)	0.0296 μm
Length measurement uncertainty (z -axis direction)	0.3 μm
Pitching of z -axis	0.15 μm
Angle of x -axis for the reference plane	0.001 $\mu\text{m}/100\text{ mm}$
Angle of x -axis against the reference plane	0.001 $\mu\text{m}/100\text{ mm}$
Angle of y -axis for the reference plane	0.000245 rad
Angle of y -axis against the reference plane	0.000118 rad
Angle of z -axis for x -axis	0.0005 $\mu\text{m}/150\text{ mm}$
Angle of z -axis for y -axis	0.0005 $\mu\text{m}/150\text{ mm}$
Uncertainty of an angle of rotation	0.00000242 rad
Slope of rotation axel (x -axis direction)	0.0000060 rad
Slope of rotation axel (y -axis direction)	0.0000154 rad

Table 2: Simulation conditions

Number of partitions of measurement diameter	100 points/10 mm
Radius of base circle	49.4 mm
Model of tooth	300,000 points per line, 0.1 μm pitch in the z -axis direction
Repetitions	20,000

The helix deviations evaluated by us correspond to the total helix deviation, the helix form deviation, and the helix slope deviation, which were defined by ISO 1328-1: 1995 “Cylindrical gears—ISO system of accuracy—Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth.”

The histogram of total helix deviation is shown in Fig. 3, the histogram of helix form deviation is shown in Fig. 4, the histogram of helix slope deviation is shown in Fig. 5, and one of the results which was obtained from 20,000 simulations is shown in Fig. 6.

Consequently, the uncertainty of the total helix deviation measurement was $0.47\ \mu\text{m}$, the uncertainty of the helix form deviation measurement was $0.35\ \mu\text{m}$, and the uncertainty of the helix slope deviation measurement was $0.22\ \mu\text{m}$, where the uncertainties are expressed in terms of expanded uncertainties corresponding to the 95% level of confidence.

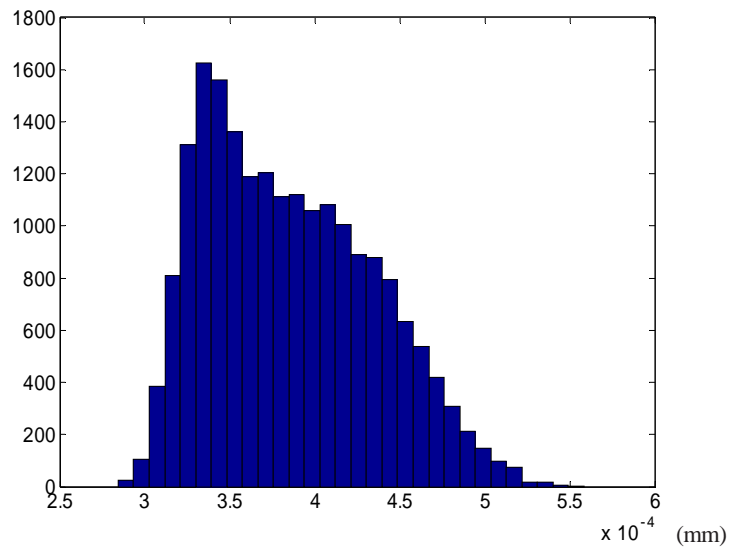


Fig. 3: Histogram of total helix deviation

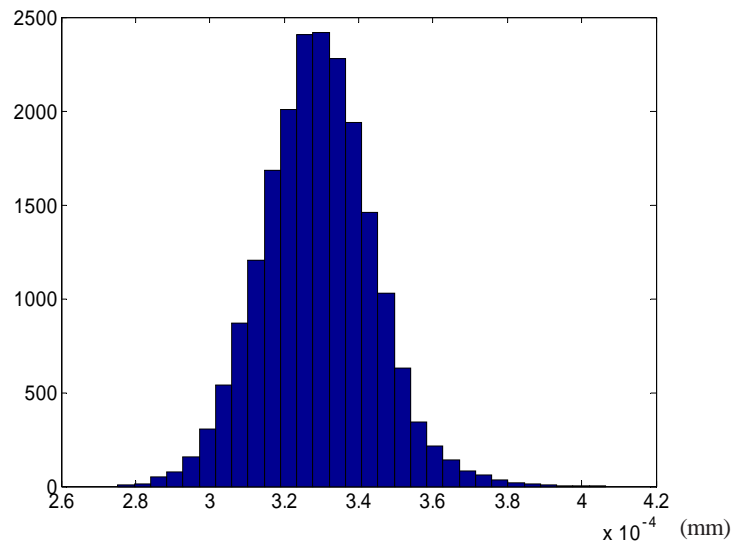


Fig. 4: Histogram of helix form deviation

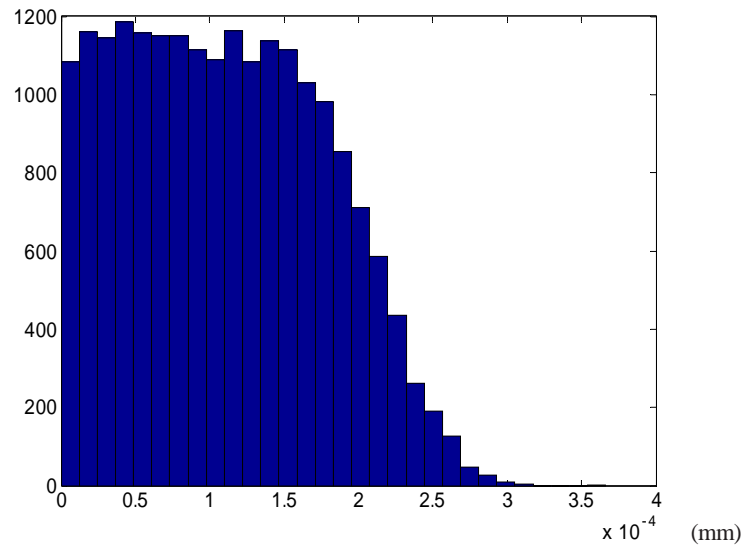


Fig. 5: Histogram of helix slope deviation

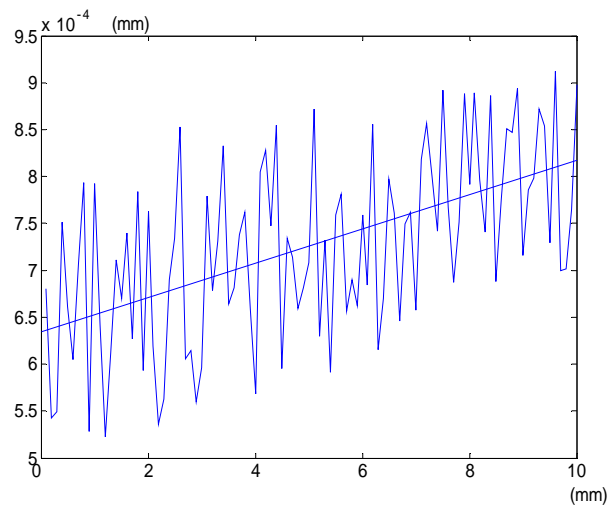


Fig. 6: Example of result of simulation

Conclusions

This study estimates the expanded uncertainty of the measurement of the gear helix profile whose model equations include the inverse involute function. However, this model does not include whole uncertainties; this point must be improved upon. In future, we will structure a simulation for helical gear and pitch deviations.

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Bibliography

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- 2) Committee for gear measurement accuracy. Performance evaluation and estimation uncertainty of a high precision gear measurement machine. Report of WG2 of committee for gear measurement accuracy (2002) (in Japanese).