IN-SITU CALIBRATION OF A REDUNDANT MEASUREMENT SYSTEM FOR MANIPULATOR POSITIONING

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1. Introduction

Achieving high positioning accuracy in a precision manipulator usually relies on calibration of its metrology system, i.e. the determination of reproducible geometry errors as represented by parameters in a metrology model, which is then used for position control.

The issue of metrology calibration in the presence of external sensors, which can provide reliable information on the true spatial position of the manipulator's end-effector, is well understood. A problem arises, however, in the absence of such external sensors, when the manipulator's internal sensors, all of them encumbered with unknown geometry errors, provide the entire position information.

Many high-performance manipulators feature a redundant metrology system. In such a system more spatial measurements are available than the number of degrees of freedom. This paper shows how that redundancy can be utilised to calibrate the manipulator in situ, i.e. without the use of external measurement systems.

2. Coordinate Transformations for Redundant Metrology

Our method requires introducing the inverse and forward coordinate transformations for the measurement system in a formal manner. Firstly, the **inverse coordinate transformation** maps the 6 DoF spatial position **p** of the end-effector into the sensor readings **q**:

$$\mathbf{q} = \mathbf{f}\left(\mathbf{p}, \mathbf{g}\right) \tag{1}$$

Such a transformation essentially constitutes the metrology model and is uniquely defined. The unknown geometry errors **g** are included as parameters in (1), either through analytical modelling of the manipulator geometry or through arbitrary expressions, such as polynomials. Both linear and non-linear treatment of the parameters is possible. It is the purpose of calibration to determine the geometry parameters **g**.

Secondly, the **forward coordinate transformation** maps the measurement values **q** into the 6 DoF spatial position **p** of the end-effector:

$$\mathbf{p} = \mathbf{f}^{+}\left(\mathbf{q}, \mathbf{g}\right) \tag{2}$$

This is the transformation which allows us to control the manipulator, provided we know the geometry parameters **g**. It is not uniquely defined due to redundancy and has to be based on an arbitrary decision that its result always satisfy some of the measurements, and minimise the differences over the remaining measurements in the least-square sense. In particular, the simplest assumption would be a uniform least-square fit over all the sensors.

For consistency, we must demand that transforming a given 6 DoF position into sensors values and then transforming the latter back to a 6 DoF position result in the original position. The converse is not generally true, though, due to the fact that **q** is redundant and does not have to be consistent with our model. Formally:

$$\begin{aligned} \mathbf{f}^{+}\left(\mathbf{f}\left(\mathbf{p}, \mathbf{g}\right), \mathbf{g}\right) &= \mathbf{p} \\ \mathbf{f}\left(\mathbf{f}^{+}\left(\mathbf{q}, \mathbf{g}\right), \mathbf{g}\right) &\neq \mathbf{q} \end{aligned} \tag{3}$$

In this sense the forward transformation is not a full inverse of the inverse transformation, but a “pseudo-inverse”, by analogy with linear algebra.

3. Computation of the Forward Transformation

Computing the inverse transformation does not pose any problems, however the forward transformation is not available in the closed form and has to be computed with the use of numerical methods. If, as previously suggested, we aim at the best fit to our redundant measurements, we seek the minimum of the fit error E (here, we omit the parameters **g** for brevity):

$$\begin{aligned} E = \left\|\mathbf{f}\left(\mathbf{p}\right) - \mathbf{q}\right\|_2 &= \left(\mathbf{f}\left(\mathbf{p}\right) - \mathbf{q}\right)^T \left(\mathbf{f}\left(\mathbf{p}\right) - \mathbf{q}\right) \\ \frac{\partial E}{\partial \mathbf{p}} &= 2\left(\frac{\partial \mathbf{f}}{\partial \mathbf{p}}\right)^T \left(\mathbf{f}\left(\mathbf{p}\right) - \mathbf{q}\right) = \mathbf{0} \end{aligned} \tag{4}$$

Denoting the Jacobian of the inverse transformation by **A**:

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{p}} \tag{5}$$

we can write the equation, whose unique solution is the stage position **p**, as follows:

$$\mathbf{F}(\mathbf{p}) = \mathbf{A}^T\left(\mathbf{f}(\mathbf{p}) - \mathbf{q}\right) = \mathbf{0} \tag{6}$$

A Newton iteration follows automatically:

$$\begin{aligned} \mathbf{p}_{n+1} &= \mathbf{p}_n - \left(\frac{\partial \mathbf{F}}{\partial \mathbf{p}}\right)^{-1} \mathbf{F}(\mathbf{p}_n) = \\ &= \mathbf{p}_n - \left(\mathbf{A}^T\mathbf{A}\right)^{-1} \mathbf{A}^T\left(\mathbf{f}(\mathbf{p}_n) - \mathbf{q}\right) = \\ &= \mathbf{p}_n - \mathbf{A}^+\left(\mathbf{f}(\mathbf{p}_n) - \mathbf{q}\right) \end{aligned} \tag{7}$$

Denoting the matrix which appears in front of the error term, in this case a pseudo-inverse of **A**, by **G**:

$$\mathbf{G} = \mathbf{A}^+ \tag{8}$$

we observe that, at the limit of the Newton iteration, **G** becomes the Jacobian of the forward transformation:

$$\begin{gathered} \mathbf{p}_n - \mathbf{p}_{n+1} = \mathbf{G}\left(\mathbf{f}(\mathbf{p}) - \mathbf{q}\right) = \mathbf{G}\left(\mathbf{f}\left(\mathbf{f}^+(\mathbf{q})\right) - \mathbf{q}\right) = \mathbf{0} \\ \mathbf{G}\frac{\partial \mathbf{f}}{\partial \mathbf{p}}\frac{\partial \mathbf{f}^+}{\partial \mathbf{q}} - \mathbf{G} = \mathbf{0} \quad \Rightarrow \quad \mathbf{G}\mathbf{A}\frac{\partial \mathbf{f}^+}{\partial \mathbf{q}} = \mathbf{G} \quad \Rightarrow \quad \frac{\partial \mathbf{f}^+}{\partial \mathbf{q}} = \mathbf{G} \end{gathered} \tag{9}$$

If our arbitrary choice of the forward transformation is different than the uniform least-square fit, the form of the matrix **G** will be more complex. In fact any matrix **G** which satisfies

$$\mathbf{G}\mathbf{A} = \mathbf{I} \tag{10}$$

is admissible as the Jacobian of a valid forward transformation. One can convince oneself that this is a necessary condition by differentiating the first line of (3) over **p**.

4. Calibration Equation

Once a manipulator moves through its workspace, the only reliable information at our disposal is the actual sensor readings, which we will denote by $\mathbf{q}_1$. The manipulator, in reality, corresponds to a certain vector $\mathbf{g}_1$ of geometry parameters, which is unknown to us. Instead, we use another vector $\mathbf{g}_0$ of uncalibrated parameters, e.g. their nominal values. Let us apply the composition of the inverse and forward coordinate transformation to the actual sensor readings, assuming in both operations the incorrect parametrs $\mathbf{g}_0$:

$$\mathbf{q}_0 = \mathbf{f}\left(\mathbf{f}^+(\mathbf{q}_1, \mathbf{g}_0), \mathbf{g}_0\right) \neq \mathbf{q}_1 \tag{11}$$

We shall call the resulting $\mathbf{q}_0$ **anticipated sensor readings**. If we had the knowledge of the correct parameters $\mathbf{g}_1$ instead of $\mathbf{g}_0$, the anticipated sensor readings would be identical with the actual ones. This suggests that the difference between the actual and anticipated readings is related to the mismatch between the real manipulator ($\mathbf{g}_0$) and the model ($\mathbf{g}_1$). We will now proceed to approximate that difference with the linear term of the Taylor series:

$$\begin{aligned} \Delta\mathbf{q} &= \mathbf{q}_1 - \mathbf{q}_0 = \mathbf{q}_1 - \mathbf{f}\left(\mathbf{f}^+\left(\mathbf{q}_1, \mathbf{g}_0\right), \mathbf{g}_0\right) = \mathbf{h}\left(\mathbf{g}_0\right) \\ \Delta\mathbf{q} &\approx \frac{\partial\mathbf{h}}{\partial\mathbf{g}}\left(\mathbf{g}_0 - \mathbf{g}_1\right) = \left(\frac{\partial\mathbf{f}}{\partial\mathbf{p}}\frac{\partial\mathbf{f}^+}{\partial\mathbf{g}} + \frac{\partial\mathbf{f}}{\partial\mathbf{g}}\right)\left(\mathbf{g}_1 - \mathbf{g}_0\right) \end{aligned} \tag{12}$$

Both the partial derivatives $\partial\mathbf{f}/\partial\mathbf{p}$ and $\partial\mathbf{f}/\partial\mathbf{g}$ appearing in (12) are available; it is only $\partial\mathbf{f}^+/\partial\mathbf{g}$ which requires a separate derivation. Differentiating the first line of (3) over $\mathbf{g}$ gives:

$$\frac{\partial\mathbf{f}^+}{\partial\mathbf{q}}\frac{\partial\mathbf{f}}{\partial\mathbf{g}} + \frac{\partial\mathbf{f}^+}{\partial\mathbf{g}} = \mathbf{0} \quad \Rightarrow \quad \frac{\partial\mathbf{f}^+}{\partial\mathbf{g}} = -\frac{\partial\mathbf{f}^+}{\partial\mathbf{q}}\frac{\partial\mathbf{f}}{\partial\mathbf{g}} \tag{13}$$

Substituting (13) to (12) yields the sought expression for $\Delta\mathbf{q}$:

$$\Delta\mathbf{q} = \left(-\frac{\partial\mathbf{f}}{\partial\mathbf{p}}\frac{\partial\mathbf{f}^+}{\partial\mathbf{q}}\frac{\partial\mathbf{f}}{\partial\mathbf{g}} + \frac{\partial\mathbf{f}}{\partial\mathbf{g}}\right)\left(\mathbf{g}_1 - \mathbf{g}_0\right) = \left(\mathbf{I} - \mathbf{AG}\right)\frac{\partial\mathbf{f}}{\partial\mathbf{g}}\left(\mathbf{g}_1 - \mathbf{g}_0\right) \tag{14}$$

We will now introduce the following symbols for frequently occurring expressions:

$$\mathbf{L} = \frac{\partial\mathbf{f}}{\partial\mathbf{g}}, \quad \mathbf{H} = \mathbf{I} - \mathbf{AG}, \quad \Delta\mathbf{g} = \mathbf{g}_1 - \mathbf{g}_0 \tag{15}$$

Please note that $\mathbf{L}$ can be thought of as a **sensitivity matrix**. With the new nomenclature, our calibration equation (14) acquires its final form:

$$\Delta\mathbf{q} = \mathbf{H}\,\mathbf{L}\,\Delta\mathbf{g} \tag{16}$$

As the manipulator performs a sequence of movements, all the measurement values $\mathbf{q}_1$ are registered and anticipated sensor readings $\mathbf{q}_0$ are computed for each point, according to (11). Consequently, each point contributes equations of the type (16). Each of those equations relates the geometry errors to the observed discrepancies between true and anticipated sensor readings. Calibration is accomplished though a least-squares fit to the system of all the available equations.

Since, in general, both $\mathbf{H}$ and $\mathbf{L}$ are functions of $\mathbf{g}$, the entire calibration fit has to be iterated a number of times until the fit residue decreases below a desired level. Furthermore, both $\mathbf{H}$ and $\mathbf{L}$ are functions of $\mathbf{p}$, for which, in every iteration, we substitute the result of our increasingly more accurate forward transformation.

5. Redundancy Matrix

It is interesting to note that the key matrix in our calibration formalism (16), **H**, which we also call **redundancy matrix**, has a rank equal to the number of redundant sensors in the system, since it is a null space of both **A** and **G**:

$$\begin{aligned}\mathbf{HA} &= (\mathbf{I}-\mathbf{AG})\mathbf{A} = \mathbf{A}-\mathbf{A}(\mathbf{GA}) = \mathbf{A}-\mathbf{AI} = \mathbf{0}\\ \mathbf{GH} &= \mathbf{G}(\mathbf{I}-\mathbf{AG}) = \mathbf{G}-(\mathbf{GA})\mathbf{G} = \mathbf{G}-\mathbf{IG} = \mathbf{0}\end{aligned} \tag{17}$$

Another interesting observation arises from differentiating the second line of (3) over **q**:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{p}}\frac{\partial \mathbf{f}^{+}}{\partial \mathbf{q}} = \mathbf{AG} \neq \mathbf{I} \quad \Rightarrow \quad \mathbf{H} = \mathbf{I}-\mathbf{AG} \neq \mathbf{0} \tag{18}$$

In general, **H** is a non-zero matrix; it degenerates to a zero matrix only for a non-redundant system.

6. Simulation Example

We have demonstrated the operation of our method in a Mathematica simulation on a simple example involving a 6 DoF motion stage, whose position is measured by 8 displacement sensors interrogating 3 metrology surfaces on the stage (Figure 1). The stage has the following ranges of motion:

X, Y, Z: ±125 mm
Rx, Ry, Rz: ±0.1 rad

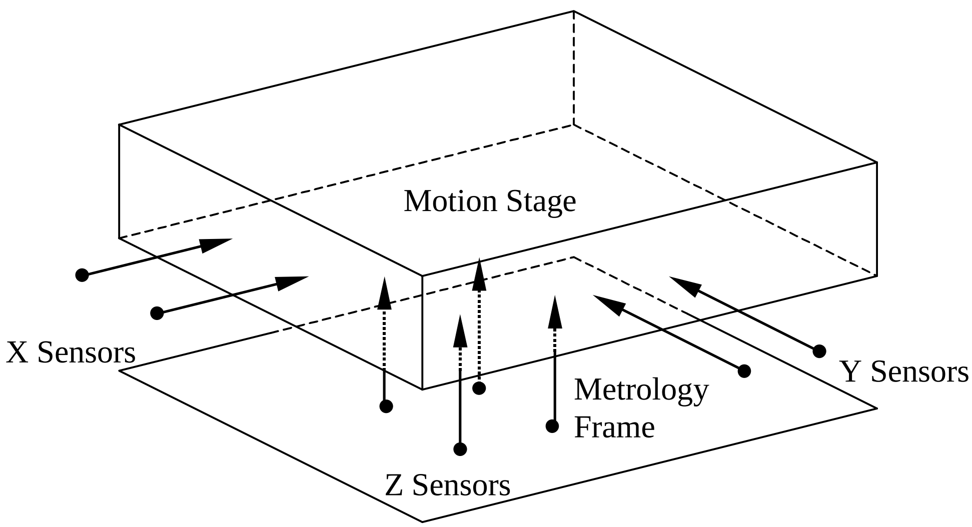

Figure 1 Schematic layout of the idealised motion stage chosen for simulating our calibration method. All the metrology surfaces of the motion stage, as well as all the sensor locations are encumbered with errors, which we are able to calibrate with our method.

The sensors are idealised as providing absolute distances between certain points stationary in the reference frame and the metrology surfaces, which are idealised as planes associated with the moving stage. There are 2 sensors approximating the X coordinate of the stage, 2 for the Y coordinate and 4 for the Z coordinate. This arrangement gives us 8 – 6 = 2 redundant measurements. The inclination of metrology surfaces and sensor locations in 3 dimensions are initially unknown. Thus the system includes a total of 8 x 3 + 3 = 27 geometry parameters. The tolerance which we assume for simulating the translational errors amounts to 1 mm, the tolerance for angular errors is 0.01 rad. The starting iteration assumes a nominal layout. In the course of the simulation, the stage is brought to $6^3 = 729$ different positions representing all possible combinations of the extreme negative, zero and the extreme positive value of each of the 6 DoF.

The simulation shows that all the geometry parameters can be calibrated in-situ, utilising only the internal displacement sensors. The entire process requires 5 iterations, while the condition numbers of the calibration matrix are on the order of 10^8. The calibration matrix has 12 zero singular values, which correspond to the arbitrary locations of coordinate frames used for defining the stage geometry. Those frame locations can be constrained artificially with additional equations or left to the matrix pseudo-inversion, which will constrain them naturally.

When controlled with the parameters obtained from calibration, the manipulator is capable of accurate movements up to the constant spatial offset at both coordinate frames used for defining positions.

7. Summary

Our paper presents a methodology of calibrating the geometry of a redundant position measurement system in situ. We introduce the notions of inverse and forward coordinate transformation associated with the degrees of freedom we seek to control and the sensor readings at our disposal. We subsequently propose to apply a composition of both the transformations to sensor readings obtained at various manipulator poses and prove that the result of that composition is algebraically related to the mismatch between the manipulator and its model. Crucially, we introduce the notion of the redundancy matrix **H** governing the manner in which the mismatch between reality and our model becomes visible to our sensors. In conclusion, as a proof of principle, we simulate an idealised 6 DoF motion stage equipped with 8 displacement sensors and succeed in calibrating its complete geometry up to constant spatial offsets. We believe that the methodology we present is applicable to a wide range of manipulators and can be extended theoretically in many directions.