Evaluation of CMM Tolerance Calculation Software

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Abstract

Coordinate measuring machine (CMM) software has long used least-squares fit calculations to construct resolved geometry representations of measured features. The National Institute of Standards and Technology (NIST) and other national testing bodies have been evaluating the performance of least-squares algorithms for many years, long before the ASME B89.4.10-2000 and the ISO 10360-6:2001 standards for evaluating CMM software were available. Least-squares analysis is a very useful tool when dealing with measurement uncertainty or the statistical control of a manufacturing process. The problem, however, is that the ASME Y14.5M-1994 and the ISO 1101 standards on dimensioning and tolerancing never specify the use of a least-squares fit.

Tolerances of form such as straightness of a line, flatness of a plane, circularity of a circle, and cylindricity of a cylinder require a Chebyshev (minimum zone) fit. A feature of size such as a circle, cylinder, sphere, or parallel plane requires a mating envelope (maximum inscribed or minimum circumscribed) fit for tolerances of location and orientation and when used as a datum feature. If CMM software gives inaccurate results for non-least-squares resolved geometry calculations, then the tolerance calculation based on the resolved geometry is also inaccurate.

A formal evaluation of the performance of non-least-squares fits is not currently available from the national testing bodies. This problem is especially critical because the well-tested algorithms used to solve least-squares fits, such as the Levenberg-Marquardt method, cannot be used on non-least-squares fits because the derivatives of the objective functions are no longer continuous. General multivariable minimization algorithms such as the downhill Simplex method sometimes give poor results for non-least-squares fits on seemingly simple data sets. Non-least-squares fits are especially difficult because there may be more than one stable solution – the global solution can only be determined after finding all possible solutions.

CMM software is tested using reference data sets that have a known solution for a specified non-least-squares fit. This paper examines the basic geometry that controls the stability of each non-least-squares fit. Once these simple geometric rules are understood, reference data sets are easy to construct.

1 Introduction

The purpose of this paper is to show how easy it is to generate reference data sets for lines, planes, circles, and spheres using simple geometric rules, that give known solutions to non-least-squares fits. By generating the reference data sets in a standard format, such as the Dimensional Measuring Interface Standard (DMIS 4.0), the CMM software can be immediately tested and evaluated by running an offline simulation. If the CMM software results don't agree with the known solution, it is a simple calculation to verify which solution is best and whether the reference data set generator or the CMM software is flawed. If desired, the reference data set can be verified using a brute force calculation that tries all possible combinations of the minimum number of points of contact.

By understanding the stability criteria of each non-least-squares fit, the minimum number of points of contact can be established for a given feature with a known size, orientation, and location. Additional measure points can be randomly shifted, since they should have no influence on the solution. If desired, the random shifts can be sorted to produce a specific form error, such as a 3-lobed circle.

Generating reference data sets for non-least-squares cylinders requires more complicated mathematics. Reference data sets are not required for cones and tori because a geometrical tolerance cannot be applied to the resolved geometry of a cone or a torus, and the standards don't define conicity and toricity form tolerances. The mathematical description of the circularity tolerance of a sphere doesn't exactly match the Chebyshev sphere (sphericity), but the reference data set generation is included because they are so similar. Reference data sets for maximum inscribed and minimum circumscribed parallel planes are required because slots and tabs are subject to orientation and position tolerances and may be used as datum features, requiring a mating envelope calculation.

Besides testing the accuracy of the CMM software's non-least-squares fits, another consideration is whether or not the algorithm can find the global solution when multiple, local solutions exist. For most non-least-squares fits, a single, unique solution can be guaranteed by following some simple measurement rules. If these measurement rules are acceptable, then reference data sets with multiple solutions aren't required. Otherwise, reference data sets with multiple solutions should be specifically created. For each non-least-squares fit, simple methods are described to eliminate multiple solutions or to force them to exist.

2 Chebyshev Fit

The Chebyshev fit squeezes two features of perfect form together as closely as possible, while containing the measured points of the actual feature. The Chebyshev fit is used to calculate form tolerances because it minimizes the deviation of the measured feature from a feature of perfect form. For lines and planes, the two features of perfect form must remain parallel. For circles and spheres, the two features of perfect form must remain concentric, but the diameter may change.

The Chebyshev fit is stable when the force squeezing the two features together can no longer change the location and/or orientation of the two features of perfect form. For most features, a stable Chebyshev fit can be guaranteed to be unique simply by measuring the feature with evenly spaced points. The maximum number of measure points is determined by the form error relative to the size, or length, of the feature. The exception is the Chebyshev plane, which may commonly have two, stable solutions when measured in a grid pattern. For the other features, a reference data set with two independent, stable solutions can be constructed by forcing a point to be measured twice.

2.1 Chebyshev Line

A Chebyshev line fit squeezes two parallel lines about the measure points of an actual line until it contacts three points – two points on one line and one point on the other line, forming a triangle as shown in Fig. (1). If the two angles of the triangle on the side that contacts two points, θ_1 and θ_2, are both less than 90°, then the Chebyshev line fit is stable to contraction. If one of these two angles is greater than or equal to 90°, then the two parallel lines can come closer together by rotating, losing contact with one of the points, but eventually contacting another point.

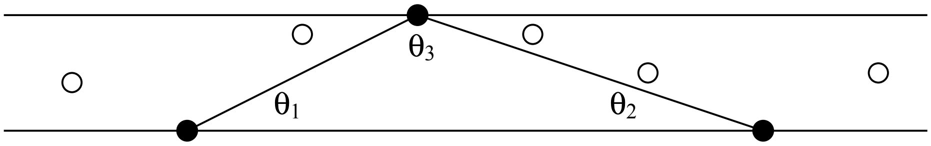

Fig. (1)

The Chebyshev line fit is unique if the angle of the triangle on the side with one point, θ_3, is greater than or equal to 90°. Uniqueness can be guaranteed by restricting the number of evenly distributed measure points on the line to:

$$N < 4\left(\frac{L}{d}\right)^2 \quad \text{Eq. (1)}$$

where N is the number of points, L is the length of the line, and d is the distance between the Chebyshev lines. For a straightness form error that is 10% of the length of the line, this allows 399 measure points, a condition that is easy to meet.

To construct a Chebyshev line reference data set that has a unique solution, start with a perfect, equally spaced line with N points, where N satisfies Eq. (1). Pick three random points on the line. Move the middle point a distance $d/2$ away from the line and move the other two points a distance $-d/2$ away from the line. For each of the other N-3 points, pick a random number, $-d/2 < r < d/2$, and move the point a distance r away from the line. These random deviations may be sorted, if desired, to produce a specific type of form error. The Chebyshev line solution is identical to the nominal line definition, with a separation distance of d.

A second solution for a Chebyshev line fit is extremely rare, requiring measurements to be repeated at the same location, and for the measurement uncertainty error at that point to dominate the form error of the line. To construct a Chebyshev line reference data set that has two solutions, start with a perfect, equally spaced line with N-1 points. Select a random point, n, and add an Nth point at that location. Move the nth point a distance $d/2$ away from the line and move the Nth point a distance $-d/2$ away from the line. For the other N-2 points, pick a random number, $-d/2 < r < d/2$, and move the point a distance r away from the line. Both Chebyshev line solutions contact points n and N, but the third contact point of each Chebyshev line solution has to be determined from the other measure points. The separation distances of the two solutions are different, but both will be slightly less than d.

A reference data set with two stable Chebyshev line solutions is shown in Fig. (2). The separation between the two points measured at the same location dominates the form error of the line. The Chebyshev line fit can pivot about these two points in two different directions, each leading to an independent, stable solution.

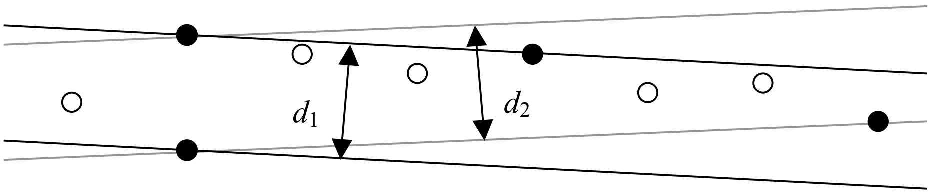

Fig. (2)

2.2 Chebyshev Plane

A Chebyshev plane fit squeezes two parallel planes about the measure points of an actual plane until it contacts four points – either three points on one side and one point on the other side (3-1) or two points on each side (2-2). Examples of these points of contact are shown in Fig. (3), where lines connect points that are on the same side. The 3-1 Chebyshev plane fit is stable to contraction if the single point is inside the triangle formed by the other three points. The 2-2 Chebyshev plane fit is stable to contraction if the two line segments cross each other. If these conditions aren't met, then the two parallel planes can come closer together by rotating, losing contact with one of the points, but eventually contacting another point.

Fig. (3)

The 3-1 Chebyshev plane fit is unique if the single point is at least a critical distance, ε, away from all three faces of the triangle. The 2-2 Chebyshev plane fit is unique if all four of the points are at least ε away from the intersection of the two line segments. The critical distance, ε, is a function of the form error of the plane and the length and width of the plane. The exact nature of this relationship is unimportant, since it is common for a point to be exactly in line with another pair of points, and zero is always less than ε. A second, stable solution for a Chebyshev plane fit is common, requiring only that three of the contact points be in line with each other. If a plane measured in a 3x3 grid pattern has a random form error, there is a 38% chance of two independent, stable Chebyshev plane fits existing.

To construct a Chebyshev plane reference data set that has a unique solution, start with a perfect, equally spaced rectangle with M points in each row and N points in each column. Pick four random points on the plane such that no three of the points are in line with each other. If the four points form a triangle that contains the fourth point, then this is a 3-1 Chebyshev plane. Otherwise, the four points form a 2-2 Chebyshev plane. The points contacting one side of the Chebyshev plane are moved a distance $d/2$ away from the plane and the other points are moved a distance $-d/2$ away from the plane. For each of the other MN-4 points, pick a random number, $-d/2 < r < d/2$, and move the point a distance r away from the line. These random deviations may be sorted, if desired, to produce a specific type of form error. The Chebyshev plane solution is identical to the nominal plane definition, with a separation distance of d.

There are a few measurement patterns that guarantee a unique Chebyshev plane fit, but they will almost never be used by a typical CMM operator. The square, 4-point pattern shown in Fig. (4) guarantees a unique solution because no line can be drawn that is within ε of touching 3 points. This can be extended to a circular pattern of measurement points, but this does a poor job of representing the interior of the plane. The 8-point measurement pattern shown in Fig. (4) adds an extra point in the middle of each triangle of the 4-point pattern, guaranteeing a unique solution. It becomes difficult to add more points to the pattern without forming three points in a line, especially while trying to maintain an even spacing between the points.

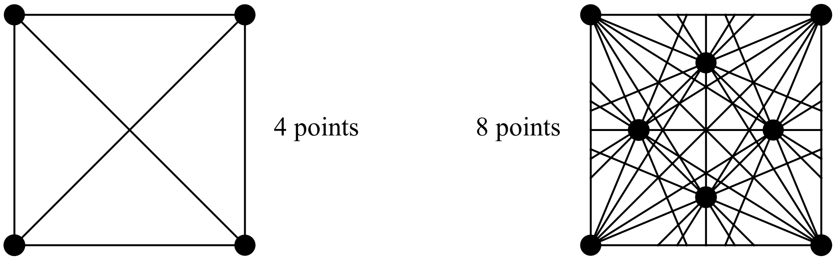

Fig. (4)

A second solution for a Chebyshev plane is common, since most planes are measured in a grid pattern, and 3-point lines can be found in each row, each column, and most diagonals. To construct a Chebyshev plane reference data set that has two solutions, start with a perfect, equally spaced rectangle with M points in each row and N points in each column. Select three random points that are in line with each other. Move the middle point a distance $d/2$ away from the plane and move the other two points a distance $-d/2$ away from the plane. For the other $MN-3$ points, pick a random number, $-d/2 < r < d/2$, and move the point a distance r away from the plane. Both Chebyshev plane solutions contact these three points, but the fourth contact point of each Chebyshev plane solution has to be determined from the other measure points. The separation distances of the two solutions are different, but both will be slightly less than d.

A reference data set with two stable Chebyshev plane solutions can be visualized by assuming that Fig. (1) is an end-on look at a plane, where the three points of contact are in line with each other. The vertical separation between the three points dominates the form error of the plane. The Chebyshev plane fit can pivot about this triangle in two different directions, each leading to an independent, stable solution.

2.3 Chebyshev Circle

2.4 Chebyshev Sphere

3 Minimum Circumscribed Fit

The minimum circumscribed fit squeezes a feature of perfect form as small as possible, while containing the measured points of the actual feature. The minimum circumscribed fit is used to calculate the size, orientation tolerance and position tolerance of an outer feature, such as a shaft or a tab, because it measures the smallest mating part that the feature fits inside. The minimum circumscribed fit is stable when the squeezing force can no longer change the location and/or orientation of the feature of perfect form. The minimum circumscribed fit should not be used on partial arcs of a circle or on partial hemispheres of a sphere because the answer is not very representative of the surface of the feature.

Except for parallel plane features, the minimum circumscribed fit is guaranteed to be unique. The minimum circumscribed parallel plane is not described here because the behavior is identical to the Chebyshev plane. Two independent, stable solutions are common with the minimum circumscribed parallel plane.

3.1 Minimum Circumscribed Circle

3.2 Minimum Circumscribed Sphere

4 Maximum Inscribed Fit

The maximum inscribed fit expands a feature of perfect form as large as possible, while inside the measured points of the actual feature. The maximum inscribed fit is used to calculate the size, orientation tolerance and position tolerance of an inner feature, such as a hole or a slot, because it measures the largest mating part that fits inside the feature. The maximum inscribed fit is stable when the expansion force can no longer change the location and/or orientation of the feature of perfect form. The maximum inscribed fit should not be used on partial arcs of a circle or on partial hemispheres of a sphere, because the feature expands to infinity.

Without restricting the placement of the measurement points, the maximum inscribed fit of a feature cannot be guaranteed, and it will be common for two independent, stable solutions to exist. The maximum inscribed parallel plane is not described here because the behavior is identical to the Chebyshev plane.

4.1 Maximum Inscribed Circle

4.2 Maximum Inscribed Sphere

5 Conclusions

Reference data sets can be easily generated using simple, geometric rules for Chebyshev fits of lines, planes, circles, and spheres, and for minimum circumscribed and maximum inscribed fits of circles, spheres, and parallel planes. For non-least-squares fits of a cylinder, more complicated mathematics is required to generate reference data sets. For non-least-squares fits where a second, stable solution is possible, the mechanisms that create a second solution can be explained using simple geometric concepts.

The minimum circumscribed fit of a circle or sphere is always guaranteed to be unique. The Chebyshev fit of a line, circle, or sphere rarely has a second, stable solution. The Chebyshev fit of a line, circle, or sphere can be guaranteed to be unique simply by requiring the measure points to be evenly spaced and not allowing a huge number of measure points.

The maximum inscribed fit of a circle or sphere, the Chebyshev fit of a plane, and the non-least-squares fit of a parallel plane commonly have a second, stable solution. By carefully controlling the placement of the measurement points, the uniqueness of these fits can be guaranteed. The maximum inscribed fit of a circle can be guaranteed to be unique if the circle is measured with an odd number (not too large) of evenly spaced measure points. For the maximum inscribed fit of a sphere and the non-least-squares fit of a parallel plane, the measurement patterns that guarantee uniqueness are more specific.

Testing CMM software using reference data sets that have a single, unique solution is sufficient for minimum circumscribed fits of circles and spheres, and for Chebyshev fits of lines, circles, and spheres. For maximum inscribed fits of circles and spheres, Chebyshev fits of planes, and the non-least-squares fits of parallel planes, however, CMM software should be tested with reference data sets that have a single, unique solution and with reference data sets that have two independent, stable solutions. Because the CMM operator may use a measurement pattern that does not guarantee a unique solution, the CMM software should able to find the global solution in these cases.