AN IMPROVED B-SPLINE APPROACH FOR THE SURFACES RECONSTRUCTION FROM DATA MEASURED BY CMM.

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Abstract

The aim of this paper is the reconstruction of a surface starting from a cloud of points. In particular we tackle with a special kind of surfaces: kinematic surfaces. A coordinate measuring machine (CMM) measures the coordinates of the curve points. Contact probing devices, as CMM, provide an accurate, sensitive and reproducible indication, but there are still uncertainties associated with them. In this work measurement uncertainty and process errors are not estimated a priori but are considered as problem variables. The starting point is a set of measured data and to approximate this data we will use a λ-spline which is an integral B-spline depending on a parameter λ; the value of this parameter affects the control points. From the computational point of view the construction of the λ-spline reduces to the computation of a linear combination of a classical B-spline and an integral B-spline. The weights of this linear combination are (1-λ) and λ hence a suitable choice of λ is crucial to obtain a good approximation. An automatic optimization algorithm has been implemented to find the optimum value of λ to get the best approximating surface.

The algorithm is deeply discussed and some examples are presented to prove the effectiveness of the proposed method.

Keywords: kinematics surfaces, reverse engineering, quasi-interpolating spline, linear algebraic transformation

1. Introduction

In the traditional product development cycle, engineers transform a concept into a 3D model and, then, into real parts. In the iterative development process, the existing design is often modified on the shop floor due to manufacturing limitation or to obtain optimal product performance. Usually, such modifications are not reflected in the CAD model, so to update the existing geometry it is necessary to reconstruct the modified model. This process of reconstruction of a virtual model from an object is known as reverse engineering. Typically, it starts measuring a physical model or a prototype in order to acquire its geometric information in form of a three-dimensional set of points. These points are then subdivided into regions (segmentation); each of them represents a single geometric feature that can be mathematically represented by various surfaces. Finally, the surfaces are reconstructed and combined into a complete geometric model [1]. Digitization of an object can be achieved either by contact probing (e.g. touch trigger sensor on a CMM) or non-contact sensing techniques (e.g. laser beam). The latter are used for scanning 3-D dense measurement data in a short time, but the scanning result is not guaranteed due to some practical reason (e.g. reflective attributes and sharp areas of the surface, extraneous vibration of the system). Contact probing devices, as CMM, provide an accurate, repeatable and reproducible indication, but there are still uncertainties associated with them. Therefore, the measure has to be expressed by a value and by an uncertainty estimate [2, 3]. A lot of work has been done to classify, determine [4] and reduce [5] those uncertainty sources. These uncertainty sources affect the subsequent shape design or shape evaluation. Therefore a correct and reliable method able to reconstruct the surface taking into account of these sources is needed. In the fields of computer graphics and computer aided design the problem of reconstruction of surface starting from a cloud of points is a well know problem and present different approach that differ from the application fields and from the starting data structure. In the surface reconstruction starting from contours the three-dimensional structure reconstruction start from stacks of two-dimensional contours. Although this problem has received a good deal of attention there remain several limitations with current methods. Perhaps foremost among these is the difficulty of automatically dealing with branching structures. Another field of interest is the surface reconstruction starting from data points produced by 3D laser range scanning system (3 D scanning), these scanning system produce large collections of points on surfaces of objects but they are unorganized, hence among them doesn't exist any topological relations. Some implemented method use a number of surface to approximate the real surface, but this number increase as the shape complexity increase. [6] [7] [8]. Some recent work addresses the problem of reconstructing smooth surfaces of arbitrary topological type using algebraic surfaces [9] [10]. However these smooth surface representations are not commonly supported within current modelling systems. The general class of non-uniform rational B-splines is considered by many the de facto CAD standard. So there has been considerable work on fitting B-splines surface to 3d points. However, most methods either assume that the surface has simple topological type as sphere plane [11] [12] [13] or require user intervention in setting up patch network [14] [15] [16] [17].

In this work, we deal with a particular class of surfaces: kinematics surfaces[18]. In this class we find all the surfaces that are generated by moving a curve along a path. So, if we consider a subgroups of motions (uniform helical, rotational or translation), a 2D curve and we apply the motion to the curve points we can obtain a cylindrical surfaces or a cylinder if the motion is a translation, a surface of revolution if the motion is a rotation about an axis and a helical surface otherwise. Therefore, to reconstruct these surfaces we need to reconstruct the path and the curve profile. In this work we deal with the curve profile reconstruction problem. A particular class of spline will be adopted (λ-spline) to reconstruct the curve used to generate the surface. This class of spline is an integral spline depending on a parameter λ. The value of this parameter, which is a real number, affects all the control points and hence the global curve shape. Namely, λ directly influences the shape of the corresponding spline curve. From a computational point of view the construction of the λ-spline reduces to the computation of a linear combination of a classical B-spline and an integral B-spline. The weights of this linear combination are (1-λ) and λ, therefore a suitable choice of λ is crucial to obtain a fair shape. Some examples will illustrate the method.

2. Quasi-interpolating (q.i.) spline, integral q.i. spline and integral q.i. λ-spline.

Here we will briefly recall the definition of quasi-interpolating (q.i.) spline, integral q.i spline and finally we will introduce the integral q.i. λ-spline.

It is well known that given a set of control point $\boldsymbol{P}_0, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_m$ the equation for the q.i. spline $(\boldsymbol{S}_m\boldsymbol{P})$ can be written as follows:

$$(\boldsymbol{S_m P})(t)=\sum_{i=0}^{m}\boldsymbol{P}_i \; B_i^k(t) \quad 0\le t\le 1$$

where $B_i^k(t)$ are the B-spline basis functions associated with the nodes $t=(t_i)_{i=-k}^{m}$ $0=t_{-k}=\ldots=t_0<t_1<\ldots<t_n<t_{n+1}=\ldots=t_{m+1}=1$; moreover the following recurrence formula holds

$$B_i^k=\frac{t-t_{i-k}}{t_{i-1}-t_{i-k}}B_i^{k-1}(t)+\frac{t_i-t}{t_i-t_{i-k+1}}B_{i+1}^{k-1}(t).$$

On the other hand the integral q.i. spline $(\boldsymbol{T_m P})$ can be seen as a q.i. spline over a suitable modified set of control points (see [17])

$$(\boldsymbol{T_m P}) = \sum_{i=0}^{m}\boldsymbol{MP}_i \; B_i^k(t) \quad 0\le t\le 1,$$

where

$$\boldsymbol{M}=\begin{bmatrix}\beta_0 & \gamma_0 & & & 0\\ \alpha_1 & \beta_1 & \gamma_1 & & \\ & \alpha_2 & \beta_2 & \ldots & \\ & & \ldots & \ldots & \gamma_{m-1}\\ 0 & & & \alpha_m & \beta_m\end{bmatrix} \quad \begin{cases}\alpha_0=0,\alpha_i=\dfrac{(\delta_i^l)^2}{2\Delta_{i-1}^k\Delta_i^{k+1}}, i=1,\ldots,m;\\ \gamma_i=\dfrac{(\delta_i^r)^2}{2\Delta_i^k\Delta_i^{k+1}}, i=1,\ldots,m-1, \gamma_m=0;\\ \beta_i=1-\alpha_i-\gamma_i, i=1,\ldots' m\end{cases}$$

with $\Delta_i^k=\xi_{i+1}^k-\xi_i^k$, $\delta_i^r=\xi_{i+1}^{k+1}-\xi_i^k$, $\delta_i^l=\xi_i^k-\xi_i^{k+1}$, $\xi_i^{k+1}<\xi_i^k<\xi_{i+1}^{k+1}$ e $\xi_i^k=\dfrac{t_{i-k+1}+\ldots+t_i}{k}$.

Finally the equation for the q.i. λ-spline can be written as follows

$$(\boldsymbol{T_m^\lambda P})=(1-\lambda)(\boldsymbol{S_m P})+\lambda(\boldsymbol{T_m P}) \qquad (1)$$

where $\boldsymbol{P}$ is the set of control point derived from measured data. In the previous formula the contributions of the q.i. spline and of the integral q.i. spline is clearly evident.

The linear combination (1) shares the same properties of the $(\boldsymbol{S_m P})$ and $(\boldsymbol{T_m P})$ q.i. splines. As for the value of parameter λ it will be chosen in order to minimize a suitable quadratic functional.

3. Kinematics surfaces

A kinematics surface can be created by moving a two dimensional entity along a path.
Given the basis curve $(\boldsymbol{T_m^\lambda P})$, and a one-parameter subgroup $\boldsymbol{EM(t)}$ of Euclidean motion [18] (uniform helical, rotational or translation) a kinematics surfaces is obtained by:

$$\boldsymbol{KS}(\boldsymbol{T_m^\lambda P})=\boldsymbol{EM(t)}(\boldsymbol{T_m^\lambda P}) \qquad (2)$$

4. Algorithm

Starting from a given motion path our kinematics surface reconstruction algorithm consist of three fundamentals steps, as show in Fig. 1.

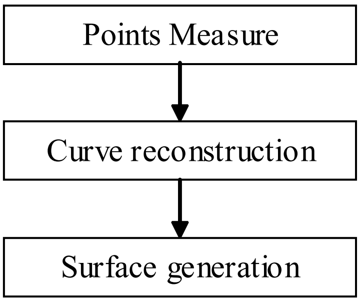

Figure 1 - Surface reconstruction algorithm.

First, a set of measured points has been detected by CMM. This set of points is considered to be located in a plane. Then, the curve profile is reconstructed using the measured points as control points of the λ-spline and the shape parameter (λ) is automatically optimized.

Finally, the kinematics surface is generated moving the reconstructed 2D profile along the relative path.

5. The quadratic functional and the minimization algorithm

Usually, in order to minimize the difference between the real part model and the reconstructed one, it must be take into account that measured points are not deterministic due to manufacturing inaccuracy and measurement uncertainty. Normally, this sources of errors are estimated a priori. In this work a similar result is obtained minimizing the following quadratic functional:

$$F(\lambda) = \sum_{i=1}^{N} \delta_i^2 (\boldsymbol{P}_i, \boldsymbol{T}_m^{\lambda} \boldsymbol{P}_i) \qquad (2)$$

where δ_i is the distance between the control points and the corresponding points on the q.i. λ-spline.

To minimize (2) the method of Nelder and Mead has been used. This method is based on the following idea: for the minimization of a function of *n* variables this method starting from an initial guess $\mathbf{x}_0$ defines a simplex with *n+1* vertices $\mathbf{x}_k=\mathbf{x}_0+h_k\mathbf{e}_k$, $k=1,\dots,n$. During an iteration, the objective function is evaluated at each vertex of the simplex, and the vertex with the lowest value ($\mathbf{x}_{\mathrm{low}}$), the vertex with the highest value ($\mathbf{x}_{\mathrm{hi}}$) and the vertex with the next-to-highest value ($\mathbf{x}_{\mathrm{nexthi}}$) are determined. Vertex $\mathbf{x}_{\mathrm{hi}}$ is reflected through the centroid $\mathbf{x}_c$ of the remaining vertices to find a new vertex $\mathbf{x}_{\mathrm{refl}}$ and the objective function is evaluated in the vertex $\mathbf{x}_{\mathrm{refl}}$. Next a new simplex is constructed using only three possible transformations: *reflections* with respect to the centroids, *dilations* and *contractions* (for a detailed description of the algorithm see [19]). The stopping criterion requires that the standard deviation of the values $f(\mathbf{x}_0),\dots,f(\mathbf{x}_n)$ is less than a fixed tolerance ε.

6. Numerical results

In order to prove the effectiveness of the proposed approach, a preliminary testing phase has been conduced. Three test cases have been considered (Table 1): one is a profile of shoes sole produced by Finproject S.p.A., one is the profile of a mechanical part manufactured by a CNC lathe, installed in the Politecnico di Milano, Dipartimento di Meccanica laboratory, and its shape is a typically revolving geometry, the last profile is taken from a marble tile with a shape that is a typical extrusion geometry. For the measurements a Zeiss Prismo 5 MPS HTG coordinate measuring machine, equipped with VAST scanning probe and Calypso as control software, has been used.

First for each test cases a set of n_i points has been acquired, in particular for the marble tile three profile taken at different height has been analysed.

As for the optimization algorithm the following values for the parameters have been used: $\varepsilon=10^{-4}$ and as starting point $\mathbf{x}_0 = 0$.

The following table presents the results obtained.

Test case	# measured points	λ	# of iterations
1	33	-0.443	22
2	203	1.7612	26
3a	338	0.0448	16
3b	338	-1.1738	25
3c	338	0.0188	13

Table 1: obtained experimental results

From these preliminary results we can draw the following main conclusion:

- Few iterations are necessary to converge;
- The third case shows that even for very similar data sets the optimal value of lambda can greatly vary, hence an heuristic procedure could be excessively time consuming;

7. Conclusion

A kinematics surface is generated by moving a 2D profile along a motion path. Therefore to reconstruct this class of surface a path and a profile have to be reconstructed. In this paper we tackle the problem of the curve profile reconstruction.

To solve the problem a particular class of spline (λ-spline) depending on a shape parameter has been considered. In order to minimize the difference between the reconstructed model and the original shape an optimization algorithm has been applied to find the optimal value of the shape parameter.

Acknowledgements

This work was carried out with the funding of the Italian M.I.U.R. (Ministry of Italian University and Research) and CNR (National Research Council of Italy).

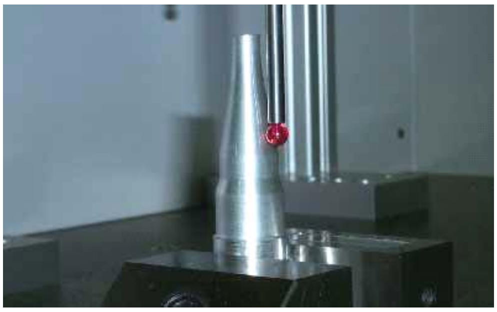

Figure 2. Marble tile.

Figure 3. Mechanical part.

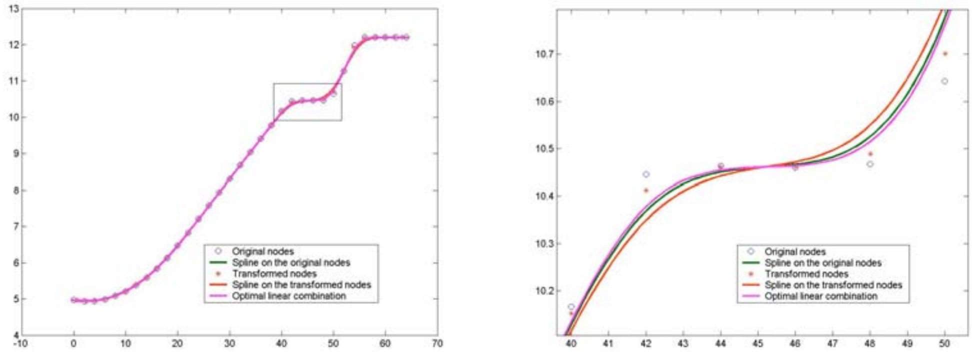

Figure 4. Case 1. Section of a rotation surface: whole curve (left) and zoom (right).

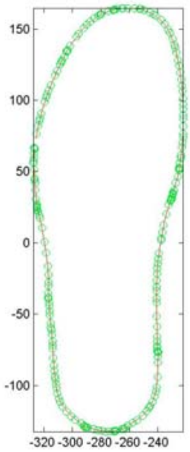

Figure 5. Case 2. Shoe sole profile: control points and reconstructed curve.

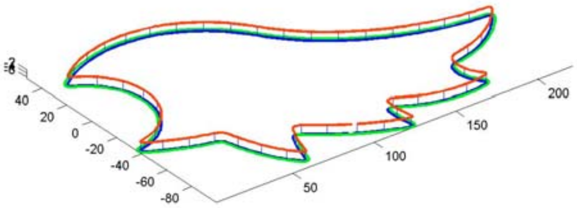

Figure 6. Case 3. Marble tile: reconstructed curves and whole surface.

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