Manufacturing Signatures and CMM Sampling Strategies

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1. Introduction

The “task specific uncertainty” in coordinate metrology is the measurement uncertainty that results, computed according to the ISO Guide to the Expression of Uncertainty in Measurement (GUM), when a specific feature is measured using a specific inspection plan. Measurements commonly are of size, form, or position and involve features dimensioned and toleranced in accordance to international and national standards. The use of coordinate measuring machines (CMMs) and other discrete point sampling devices in coordinate metrology raises questions regarding the proper interpretation of data. The methods divergence problem in coordinate metrology is a well-known phenomenon when dealing with discrete measurement data. The problem may be divided into two categories:

- different data analysis algorithms give different inspection results when using the same set of measurement data;
- different sampling schemes produce different inspection results for the same part, even when the same data analysis algorithm is used.

In addition to these fundamental issues, economic considerations argue for more efficient and reliable measurements. This has led to a search for sampling strategies and data analysis methods that maximize the information available from discrete points samples of limited size.

In this paper we tackle the problem of the choice of an appropriate sampling strategy starting from the consideration that the characteristics of a surface are directly attributable to the manufacturing methods. In fact, different type of error sources in the manufacturing system leave different signatures on the part, and the geometric errors on the part are the results of the combined effect of all these error sources. However, even if these manufacturing signatures play an important role in the task specific uncertainty estimation, they typically are not taken into account. The paper will be particularly focused on form error assessment.

In previous works the machining process analysis has been used to describe the error that affects the estimate of a circular substitute geometry in the frequency domain [1,2]. In this situation the radius of the fitted substitute geometry may be approximated by the amplitude of the harmonic of order zero, while the position of the center can be approximated by the value of the first harmonic (absolute value and phases) [3]. A further attempt to use the profile knowledge, admitted by means of a pre-sampling, to locate the sampling points is shown in [4].

This work deals with the effect of the machine-tool errors on the profiles of machined parts. A cutting profile model is derived from a machining error model [5]. It allows to foresee the fingerprint left by turning process on the machined part, i.e. every process generates surfaces with similar morphology. This model is used to choose the number of measurement points on the machined profile in order to evaluate its roundness error.

Once sampled the machined profile, the roundness error is evaluated by means of an innovative approach based on the reconstruction of the machined profile in the frequency domain. The proposed method has been tested for different numbers of measurement points. The innovative approach has been compared with the widest used techniques that consider unknown the manufacturing process, i.e. least square and minimum zone methods [6], and with the standards. The least square method associates a substitute feature, i.e. an ideal circle, to the measurement points, and calculates the maximum peak-to-valley distance of the measurement points from the substitute feature. In the literature many goal functions exist to estimate the substitute feature, such as the sum of the square deviations [7], the sum of the square normal deviations [7,8], the sum of the absolute deviations [9,10] and the average deviation [9]. The minimum zone method looks for the couple of concentric circles at minimum distance that includes the whole set of measurement points. The literature present different mathematical [11] or geometry-based techniques [12] to estimate the couple of circles.

The trend of the uncertainty due to the measurement strategy, i.e. both to the sampled points and to the elaboration technique, has been drawn with the increase of the number of the sampled data points. It has been used to evaluate the most efficient algorithm in roundness evaluation of turned parts. The same method may be used for parts machined by means of different technologies, such as grinding, boring and so on.

2. Roundness form error evaluation

A machined circular profile may be seen as a periodic wave with period 2π and may be completely described by its Fourier representation:

$$r(\vartheta)=r(\vartheta+2\pi)=a_0+\sum_{i=1}^{\infty}a_i\cos(i\vartheta)+\sum_{i=1}^{\infty}b_i\sin(i\vartheta)=a_0+\sum_{i=1}^{\infty}A_i\cos(i\vartheta-\varphi_i) \tag{1}$$

where $A_i\cos(i\vartheta-\varphi_i)$ is the i^{th} harmonic component of $r(\vartheta)$. The amplitude of this component is A_i, while its phase displacement, measured in radians, is φ_i.

When equation (1) is applied to machined circular profiles, it can be observed that the amplitude of the harmonics tends to decrease as frequency increases, even if the reduction is not monotonic. This is due to the physics of the process that generates the circular surface [13] and to the finite value of the probe radius which acts as a physical low-pass filter. Hence, over a certain harmonic N it can be supposed that $a_i=b_i=0$ for $i>N$ and the machined profile can be described with good approximation by the following equation:

$$r(\vartheta)=a_0+\sum_{i=1}^{N}a_i\cos(i\vartheta)+\sum_{i=1}^{N}b_i\sin(i\vartheta)=a_0+\sum_{i=1}^{N}A_i\cos(i\vartheta-\varphi_i) \tag{2}$$

The proposed approach evaluates the coefficients $a_i, b_i (i=1...N)$ of equation (2) by a finite number n of sampled points $r_k=r(\vartheta_k)$ through the well known relationships:

$$\hat{a}_0=\frac{1}{n}\cdot\sum_{k=1}^{n}r_k \tag{3}$$

$$\hat{b}_0=0 \tag{4}$$

$$\hat{a}_i = \frac{2}{n} \cdot \sum_{k=1}^{n} r_k \cos(i\vartheta_k) \tag{5}$$

$$\hat{b}_i = \frac{2}{n} \cdot \sum_{k=1}^{n} r_k \sin(i\vartheta_k) \tag{6}$$

for $i = 1..N$.

The circular profile is estimated by means of the rebuilt harmonic spectra:

$$\hat{r}(\vartheta) = \hat{a}_0 + \sum_{i=1}^{N} \hat{a}_i \cos(i\vartheta) + \sum_{i=1}^{N} \hat{b}_i \sin(i\vartheta) \tag{7}$$

whereas the contribution of every single harmonic may be expressed using the following notation:

$$A_i = \sqrt{a_i^2 + b_i^2} \tag{8}$$

$$\hat{\varphi}_i = \arctan\left(\frac{\hat{b}_i}{\hat{a}_i}\right) \tag{9}$$

It is well known [3] that the radius of the fitted substitute geometry, obtained using the least squares method (LSM), may be approximated by the amplitude of the harmonic of order zero and that the position of the centre can be approximated by the value of the first harmonic (absolute value and phase):

$$a_0 = \hat{r}_{LSM} \tag{10}$$

$$a_1 \cos(\varphi_1) + b_1 \sin(\varphi_1) = \sqrt{a_1^2 + b_1^2} = e \tag{11}$$

$$\varphi_1 = \nu \tag{12}$$

where $\hat{r}_{LSM}$, e and ν are the best estimate of the radius and of the position of the centre, in polar coordinates, with respect to the nominal centre of the nominal circumference in LSM.

Therefore, the form error depends on the harmonics of order higher than one, i.e. from 2 to N. Applying the LSM, it may be calculated by using the remaining N-2 harmonics:

$$LSRoundness = Max\left(\sum_{i=2}^{N} a_i \cos(i\vartheta) + \sum_{i=2}^{N} b_i \sin(i\vartheta)\right) - Min\left(\sum_{i=2}^{N} a_i \cos(i\vartheta) + \sum_{i=2}^{N} b_i \sin(i\vartheta)\right) \tag{13}$$

Equation (13) may be applied, once known the spectrum of the machined profile. This spectrum is the fingerprint left by each machining process on the machined part. The proposed method to evaluate the roundness starts from the information taken by the machining process of the part. The number of harmonics constituting the spectrum is enough to implement the method.

The number of measurement points to evaluate equation (13) may be determined by means of the well known sampling theorem that is the basis of much of the modern signal-processing technology. The sampling theorem states that it is possible to completely describe a signal with N harmonics only if the sampling frequency is higher than or equal to 2N. If the signal is sampled at a lower frequency, the phenomenon of aliasing takes place. Therefore, if a circular profile can be described by N harmonics (as supposed in equation (2)), the number of sampled point n should be higher than 2N. Estimated profile (7) obtained with $n > 2N$ is identical and is the best representation of the real profile. When the number of sampling points is less than 2N, the problem of aliasing occurs and the sampled spectrum

differs from the real one. In fact, all the harmonics $i > n/2$ of real spectrum are spuriously reflected inside range $\left[0, n/2\right]$ of the sampled spectrum.
However, the standards require to evaluate the form error through the minimum zone method, that means to look for the couple of concentric circles at minimum distance that includes the whole set of measurement points. Therefore, starting from the 2N points it is possible to reconstruct the machined profile and, then, use this profile to evaluate the minimum zone, reducing the uncertainty.

3. Application example to lathe machining

The proposed approach has been applied to a longitudinal turning operation whose data have been found in the literature [5]. Different specimens of a cylindrical steel bar of about 25.4 mm diameter and 50.8 mm length have been machined. The obtained bar section profiles has been measured by means of a rotondimeter and statistically described in the frequency domain by means of 50 harmonics.

The actual profile of the machined bar, in the frequency domain, has been generated by considering the statistical distributions of the amplitudes and the phases of the 50 harmonics shown in [5]:

$$r(\vartheta) = 25400 + \sum_{i=1}^{50} A_i \cos(i\vartheta - \varphi_i) \tag{14}$$

where A_i and φ_i follow a beta and a uniform distributions respectively:

$$A_i \sim B(\alpha_i, \beta_i) \tag{15}$$

$$\varphi_i \approx U(0, 2\pi) \tag{16}$$

for each i from 1 to 50 and $r(\vartheta)$ is expressed in micron. In particular 6 actual profiles of the machined bar have been generated. The first considers the average value of the distributions as amplitude of each harmonic. The further 5 cases have been simulated by randomly extracting the amplitude and the phase values of each harmonic.

The roundness error ε of the actual profile has been evaluated by sampling 10800 measuring points (i.e. 30 points every degree) on the profile and applying the minimum zone method.

Two different sampling strategy have been proved and compared. The first uses a number of measuring points equal to twice the number of harmonics (51 harmonics from 0 to 50); the second follows the standard ISO 12181 that states to measure 7 points for each undulation per revolution (UPR). The 102 and 360 points have been uniformly sampled along the generated circular profile. 10 replications have been carried out for each of the two sets of measuring points by randomly choosing the starting point. Therefore, we have generated 120 sets of measuring points to evaluate the roundness of the machined profile.

The coefficients of equation (14) have been estimated by means of (3)-(6) formulas and, then, the roundness value has been evaluated by using the reconstructed profile. The reconstructed profile was sampled by 2160 points and the minimum zone methods applied. The obtained results have been compared with those obtained applying the least square and minimum zone techniques directly to the samples of 102 and 360 points. We used Zeiss Calypso® software as reference software.

4. Results discussion

The percentage difference between the roundness estimate given by applying the proposed approach to each of the 10 sets of 102 measuring points and the actual value of roundness, i.e. the roundness value obtained by applying the proposed approach to the set of 10800 measuring points, has been calculated. The same difference has been evaluated, once applied to the same sets of measuring points the least squares and the minimum zone methods. We have obtained 6 distributions of percentage estimate error for each of the three applied techniques (proposed, least squares and minimum zone approaches). All the 18 distributions follow a normal probability distribution whose mean and standard deviation are shown in Table 1. The mean and the standard deviation are respectively the bias and the natural dispersion of the estimate with respect to the actual value. We can observe that the proposed method gives values of natural dispersion of the percentage error very lower than those due to both least squares and minimum zone. Moreover, the bias ranges between –3% and +3% for the proposed method, between –0.5% and 7% for least squares and between –5% and –11.4% for minimum zone.

	Least squares		Minimum zone		New approach	
Case study	μ	σ	μ	σ	μ	σ
1	4.08	5.65	-8.84	6.19	2.62	1.03
2	-0.49	6.81	-8.49	5.85	1.92	1.18
3	7.23	3.1	-8.58	2.41	-1.09	1.27
4	1.62	8.47	-11.16	4.66	-2.86	1.14
5	0.69	5.14	-11.36	6.11	1.37	0.80
6	-0.52	4.76	-4.87	5.23	0.83	0.77

Table 1. Mean and standard deviations of the distributions related to 102 points' sets

Then, it has been evaluated how the percentage estimate error improve by increasing measuring points for both the least squares and the minimum zone. The same analysis has not considered needed for the proposed approach, since the results obtained for 102 points are largely satisfactory. The 12 distributions follow a normal probability distribution, whose means and standard deviation are shown in Table 2. In particular the percentage estimate error mean and standard deviation due to least squares method range between 4% and 15% and between 1.3% and 2.4%, while those of minimum zone vary between –3.2% and 1% and between 0.9% and 2%. The least squares gives values of bias that are very higher than those of the proposed approach for each of the six considered profiles. The minimum zone achieves bias values similar to those of the proposed approach, but higher dispersion values.

	Least squares		Minimum zone	
Case study	μ	σ	μ	σ
1	13.99	1.84	0.28	1.3
2	9.95	2.36	0.94	1.49
3	14.99	1.28	-1.66	0.89
4	12.15	1.92	-3.23	1.22
5	11.76	1.84	0.86	1.61
6	4.20	2.13	-0.3	2.03

Table 2. Mean and standard deviations of the distributions related to 360 points' sets

5. Conclusions

The present paper draws the trend of uncertainty due to the measurement strategy, i.e. both to the sampled points and to the elaboration technique, with the increase of the number of the sampled data points. Both the bias and the natural dispersion of the estimate with respect to the actual value of roundness have been considered. The uncertainty trend has been used to evaluate the most efficient algorithm in roundness evaluation of turned parts. The same method may be used for parts machined by means of different technologies, such as grinding, boring and so on. This is matter of further study.

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