

Six Degree of Freedom Precision Measurement System

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Abstract

Assessing the performance of industrial robots and machine tools has been a time consuming task and involves expensive metrology devices with 6 degrees of freedom. To reduce the cost and time, a device is being developed and this paper introduces the device and the mathematical models. The device based on the principles of Stewart platform, measures both the translational errors and angular errors with six linear encoders. This measurement system including hardware and software, has the features of compact, easy to setup, easy to use, limited testing time, and low cost. The measurement is independent of setup accuracy. The device will be useful for evaluating machine tool performance, evaluating the performances of industrial robots, evaluating the motion errors of other machine systems, calibrating machine tools and industrial robots, and calibrating any other multi-axis motion systems as a six degree of freedom ruler.

Keywords: robot accuracy, performance evaluation, six degree of freedom, pose measurement, robot calibration.

1. Introduction

New robotic applications such as grinding and machining have made evaluating the performance of industrial robots more and more important. These new developments also revive the interest in robotic performance evaluation. Given the mechanical configurations of industrial robots and the popular six degrees of freedom, industrial robots have to be evaluated with metrology device or system of 3 or more degrees of freedom. Evaluation methods and equipment are needed to measure the spatial pose of robot efficiently with low cost.

Several methods are available for characterizing robot performance in accordance with ISO 9283 "Manipulating Industrial Robots Performance Criteria and Related Test Methods". Eight major performance measuring methods and techniques are introduced in the technical report ISO TR 13309 including the accurate, easy-to-use but costly laser tracking technique. The pros and cons of existing multi-degrees of freedom measuring systems, including laser tracker, straight edges, multi-probes at certain check points, image and scanning techniques etc, are well documented [Lau and Hocken, 1984; Van Brussel, 1990;

Jiang et al, 1988]. Pose measurement of robotic end effector has been the focus [Ziegert and Datseries, 1990; Zhu and Cui, 2001, 2003].

2. Mathematic Model for the Six Degree of Freedom Measurement System

Over the past two decades, six-degree-of-freedom parallel manipulators have received a great deal of attention because their attractive characteristics such as high rigidity, high local dexterity, low inertia effect and compact size. There exist various configurations of the Stewart platforms and variation of the platforms and connecting joints. Figure 1 shows a prototype 6 DOF measuring device. It was design to ease setup and reduce measurement time. It's setup, calibration, and measurement are based on the mathematic models described below.



Figure 1 6-DOF Measuring Device [Zhu and Cui, 2001, patent pending]

The forward kinematics of the 6-dof parallel manipulators is still a challenging task because it generally can not be expressed in an explicit form (closed-form). The main object is to quickly obtain the forward kinematics solution for practical measuring purpose.

Based on previous research work, three main methods are developed: the polynomial-based, the numerical-iterative and the extra-sensor approaches. The first approach, the polynomial-based method, reduces the resulting constraint equations into a high-order polynomial by an elimination method. Though it has been widely used, the polynomial-based approach may require extremely complicated formulation procedures and has been known to be much slower than numerical methods based on numerical iteration such as the Newton-Raphson method.

The second and third methods, generally have been used to obtain the forward kinematic solution quickly. The numerical-iterative approach with fast computation may be more effective than the polynomial-based one. The Newton-Raphson method among several iterative methods has been wisely employed due to its property of convergence. The Newton-Raphson method needs the derivative of the Jacobian matrix, which has a great influence on the convergence of numerical method.

The following introduces a method to find an analytical expression for the Jacobian matrix of a parallel manipulator.

2.1 Jacobian Matrix and Error Analysis

In forward kinematics and error analysis, derivation of the Jacobian matrix should be introduced after all the D-H models are established for the 6-dof kinematic problem.

In fact, the error sensitivity is formulated through $\frac{\partial x}{\partial g_i}$,

$\frac{\partial y}{\partial g_i}$, $\frac{\partial z}{\partial g_i}$ where x, y, z represent the position of the

traveling plate and ∂g_i is the error source for each component. So the following equations can be obtained:

$$dx = \sum_1^N \frac{\partial x}{\partial l_i} dg_i, \quad dy = \sum_1^N \frac{\partial y}{\partial l_i} dg_i, \quad dz = \sum_1^N \frac{\partial z}{\partial l_i} dg_i \quad (1)$$

The error model is actually a 6-DOF model since all error sources have been considered. It includes both the position variables X, Y, Z and also rotational angles α, β, γ .

From the six kinematic chains, the equations established based on D-H models are

$$\begin{aligned} f_1 &= f_1(x, y, z, \alpha, \beta, \gamma, g) = 0 \\ f_2 &= f_2(x, y, z, \alpha, \beta, \gamma, g) = 0 \\ &\dots\dots\dots \\ f_6 &= f_6(x, y, z, \alpha, \beta, \gamma, g) = 0 \end{aligned} \quad (2)$$

Differentiating all the equations against all the variables $x, y, z, \alpha, \beta, \gamma$ and g , where g is a vector including all geometric parameters. It can be written in matrix form as

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial \beta} & \frac{\partial f_1}{\partial \gamma} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial \alpha} & \frac{\partial f_2}{\partial \beta} & \frac{\partial f_2}{\partial \gamma} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial \alpha} & \frac{\partial f_3}{\partial \beta} & \frac{\partial f_3}{\partial \gamma} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial \alpha} & \frac{\partial f_4}{\partial \beta} & \frac{\partial f_4}{\partial \gamma} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial \alpha} & \frac{\partial f_5}{\partial \beta} & \frac{\partial f_5}{\partial \gamma} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial \alpha} & \frac{\partial f_6}{\partial \beta} & \frac{\partial f_6}{\partial \gamma} \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \\ dz \\ d\alpha \\ d\beta \\ d\gamma \end{bmatrix} = \begin{bmatrix} \sum_j \frac{-\partial f_1}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_2}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_3}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_4}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_5}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_6}{\partial g_j} dg_j \end{bmatrix} \quad (3)$$

In a compact form, it becomes

$$J_1 dX = dG \quad (4)$$

Where

$$dG = \begin{bmatrix} \sum_j \frac{-\partial f_1}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_2}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_3}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_4}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_5}{\partial g_j} dg_j \\ \sum_j \frac{-\partial f_6}{\partial g_j} dg_j \end{bmatrix} =$$

$$- \begin{bmatrix} \frac{\partial f_1}{\partial g_1} & \frac{\partial f_1}{\partial g_2} & \dots & \frac{\partial f_1}{\partial g_N} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_6}{\partial g_1} & \frac{\partial f_6}{\partial g_2} & \dots & \frac{\partial f_6}{\partial g_N} \end{bmatrix}_{6 \times N} \bullet \begin{bmatrix} dg_1 \\ dg_2 \\ \cdot \\ dg_N \end{bmatrix}_{N \times 1} \quad (5)$$

From Eq. (5) above, we have

$$dG = J_2 dg \quad (6)$$

Substitute Eq.(6) into Eq.(4) to obtain

$$J_1 dX = J_2 dg$$

Finally $dX = (J_1^{-1} J_2) dg$

The Jacobian matrix is obtained as $J_1^{-1} \cdot J_2$

$$J = J_1^{-1} \cdot J_2 = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial \beta} & \frac{\partial f_1}{\partial \gamma} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial \alpha} & \frac{\partial f_2}{\partial \beta} & \frac{\partial f_2}{\partial \gamma} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial \alpha} & \frac{\partial f_3}{\partial \beta} & \frac{\partial f_3}{\partial \gamma} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial \alpha} & \frac{\partial f_4}{\partial \beta} & \frac{\partial f_4}{\partial \gamma} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial \alpha} & \frac{\partial f_5}{\partial \beta} & \frac{\partial f_5}{\partial \gamma} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial \alpha} & \frac{\partial f_6}{\partial \beta} & \frac{\partial f_6}{\partial \gamma} \end{bmatrix} \bullet \begin{bmatrix} \frac{\partial f_1}{\partial g_1} & \frac{\partial f_1}{\partial g_2} & \dots & \frac{\partial f_1}{\partial g_N} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_6}{\partial g_1} & \frac{\partial f_6}{\partial g_2} & \dots & \frac{\partial f_6}{\partial g_N} \end{bmatrix}$$

2.2 Newton-Raphson Numerical Method

Because of the number of parameters involved as well as the number of error sources involved, the kinematic problem becomes very complicated. No analytical solution can be obtained but numerical solution. The 6-dof precision measurement system, as a general case of parallel robots, its forward kinematic problem is, therefore, very complicated. The Newton-Raphson method as an effective numerical method can be applied to calculate the forward problem, with an accurate Jacobian matrix obtained.

Newton-Raphson method is represented by

$$X_{n+1} = X_n - [F'(X_n)]^{-1} \cdot F(X_n) \quad (7)$$

With the six chain equations obtained before, the following can be obtained

$$[F'(X_n)]^{-1} = INV \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial \beta} & \frac{\partial f_1}{\partial \gamma} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial \alpha} & \frac{\partial f_2}{\partial \beta} & \frac{\partial f_2}{\partial \gamma} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial \alpha} & \frac{\partial f_3}{\partial \beta} & \frac{\partial f_3}{\partial \gamma} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial \alpha} & \frac{\partial f_4}{\partial \beta} & \frac{\partial f_4}{\partial \gamma} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial \alpha} & \frac{\partial f_5}{\partial \beta} & \frac{\partial f_5}{\partial \gamma} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial \alpha} & \frac{\partial f_6}{\partial \beta} & \frac{\partial f_6}{\partial \gamma} \end{bmatrix}$$

This equation is applied to calculate the forward kinematic problem existed in the process of calibrating the system and carrying out error analysis.

The 6-dof measuring system [Zhu and Cui, 2001] consists of the device shown in Figure 1, data acquisition, and application software. Optical linear encoders (from Heidenhain) of submicron resolution are used in measuring the linear displacements. The top platform is connected to any object with its motion to be measured, usually a tool holder or robotic end-effector. The device can be set up quickly and its configuration can be updated in the setup process.

3. Error Budget

When the SVD decomposition is completed and a linearly independent set of error model parameters determined, the Error Budget can be determined. The mathematical description of the error budget is as follows:

$$\begin{aligned} J &= U \cdot S \cdot V^T \\ dX &= J \cdot dg = U \cdot S \cdot V^T \cdot dg \\ U^T \cdot dX &= S \cdot V^T \cdot dg \end{aligned}$$

Assume $U^T \cdot dX = d\bar{X}$ and $V^T \cdot dg = d\bar{g}$. So we have $d\bar{g} = d\bar{X} / S_{ii}$, finally,

$$dg = (V \cdot U^T \cdot dX) / S_{ii}$$

Thus if the dX is given as the accuracy of the system, the error budget dg can be determined. The six chain equations are created for the six link lengths, as follows:

$$F = \begin{bmatrix} f1(link1_points, TCP_points) \\ f2(link2_points, TCP_points) \\ f3(link3_points, TCP_points) \\ f4(link4_points, TCP_points) \\ f5(link5_points, TCP_points) \\ f6(link6_points, TCP_points) \end{bmatrix}$$

Where $TCP_point = f(px, py, pz, \alpha, \beta, \gamma)$
and ϵ is a collection of all the design parameters. Thus,

$$F = \begin{bmatrix} F1(\epsilon, px, py, pz, \alpha, \beta, \gamma) \\ F2(\epsilon, px, py, pz, \alpha, \beta, \gamma) \\ F3(\epsilon, px, py, pz, \alpha, \beta, \gamma) \\ F4(\epsilon, px, py, pz, \alpha, \beta, \gamma) \\ F5(\epsilon, px, py, pz, \alpha, \beta, \gamma) \\ F6(\epsilon, px, py, pz, \alpha, \beta, \gamma) \end{bmatrix}$$

An error model is developed based on the system of equations as described above. All parameters are defined to represent the entire system. All parameters include all the D-H parameters for the links, as well as the coordinates (x, y, z) of the 6 points at both edges of the 6 links, respectively.

4. Conclusion

A 6-dof precision measuring system is introduced. The mathematical models are developed for setting up, calibrating, and error analysis of the system. Newton-Raphson numerical method is used and the Jacobian matrix is also derived. The method can be used in calibrating the device and compensating various types of errors including thermal, dimension and setup errors. The system features low cost, easy and quick setup, and relative large work volume.

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