DESIGN AND ANALYSIS OF HYDORSTATIC GAS BEARINGS FOR VACUUM APPLICATIONS

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INTRODUCTION

Despite the apparent incongruity of operating bearing systems requiring a constant supply of pressurized gas in a vacuum environment, numerous systems of this type have been successfully operated to date [1]. Hydrostatic gas bearings provide the same benefits in vacuum that have made them a mainstay of precision engineering in atmospheric applications. Operation with minimal friction and complete freedom from stick-slip make them ideal for fast high precision servo applications. To some extent, the benefits are greater in vacuum, given the tendency of that environment to exacerbate the stick-slip phenomenon, combined with the limited selection of grease and oil lubricants available for vacuum applications, and their deleterious impact on delicate vacuum components such as cathode materials. While magnetic levitation devices have more recently been developed for similar applications, gas bearings have developed a significant track record in this field, including rotary and linear vacuum feed through bearings as well as large travel X-Y stages for accommodation of 300 millimeter silicon wafers.

A common characteristic of all hydrostatic gas bearings is the employment of two conforming surfaces in close proximity. One moves over the other with contact being prevented by a forced flow of pressurized gas. Typically, the gas exits the bearing at the periphery of the smaller of the two surfaces. In vacuum applications the surface of the smaller element is extended beyond what would normally be the periphery *(Fig. 1)*, and the gas is collected in a groove where that periphery might normally be. The majority of the gas is then vented outside the vacuum chamber. Leakage into the vacuum chamber is limited by the necessity of the gas to traverse the small bearing gap between the groove and the new periphery. It is common practice to employ multiple collection grooves, with those closest to the periphery being vented to progressively higher degrees of vacuum in the pumping system.

A common feature of these systems is that flow conditions under the majority of the bearing are characterized by Poiseuille flow, dominated by the viscous properties of the gas. In some region a transition occurs to a flow regime dominated by free molecular properties. Analysis is complicated by the fact that no suitable closed form theoretical framework exists for the analysis of transitional flow between these two regimes. In typical bearing applications, pressure varies over a wide range about the transition. In such applications it has proved useful to make the simplifying assumption that the transition occurs abruptly. Continuity of flow provides an analytical constraint useful in the subsequent analysis. Energetic considerations provide a second constraint. The two simultaneous constraints are sufficient to define both the location and pressure at which the transition occurs.

ANALYSIS

A typical thrust bearing is shown in *Figure 1*.

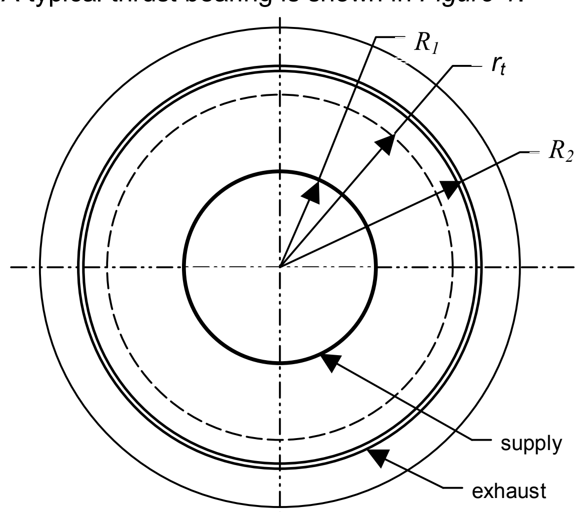

FIGURE 1. Typical circular vacuum thrust bearing configuration with exhaust collection groove and extension beyond.

High pressure gas at pressure P_1 enters the bearing at R_1. Gas flows radially outward between the bearing and its opposed surface producing a gap between the two surfaces of height h. The gas exits into the exhaust collection groove located at R_2, at low pressure P_2. At some radius r_t close to R_2, the nature of the flow transitions from Poiseuille flow to free molecular flow. The change in the character of the flow is due to the fact that at low enough pressure the mean free path of the gas molecules exceeds the bearing gap, and molecules collide with the bearing surfaces before having a chance to collide with each other.

Governing Equations

The viscous flow [2] from R_1 to r_t is governed by the equation:

$$Q = \frac{\pi h^3}{12\,\mu \ln \frac{r_t}{R_1}} \left(P_1^2 - p_t^2 \right) \qquad (1)$$

It is well established that the flow in this type of bearing is isothermal [3]. For isothermal flow conditions, the conserved quantity of flow, Q is defined as the product of pressure and volume at any point in the flow. The parameter μ is the viscosity of the gas.

The free molecular flow [4] from r_t to R_2 is governed by the approximate equation:

$$Q = \frac{\pi R_2 h^2 \overline{v}}{2} \frac{\ln \frac{2\left(R_2 - r_t\right)}{h} - \frac{1}{2}}{R_2 - r_t} \left(P_t - p_2 \right) \qquad (2)$$

where $\overline{v}$ is the average molecular velocity.
From statistical mechanics [5] we have:

$$\overline{v} = \left(\frac{8kT}{\pi m} \right)^{\frac{1}{2}}$$

where k is Boltzmann's constant, T is the absolute temperature and m is the molecular mass.

Equation (2) is based on flow through a rectangular slot of width $2\pi R_2$ and hence is only valid when $\frac{r_t}{R_2} \sim 1$. As will be apparent going forward, in practical bearing designs the transition to free molecular flow tends to occur near the outer edge of the bearing R_2, hence this limitation is generally not a serious one. Equation (2) is subject to the further limiting approximation that $\frac{R_2 - r_t}{h} >> \frac{1}{2\,e^{\frac{1}{2}}}$, that is: $R_2 - r_t >> 0.3\,h$. This also does not represent a serious limitation, since when not satisfied, the transition occurs essentially at the outer edge of the bearing, and the properties of such a bearing are virtually identical to those described by equation (1) alone.

Lower case symbols have been used to represent the transition radius and pressure in recognition of the fact that these are the free variables for which we will be seeking solutions. All other symbols represent parameters. Equating the flow in the two regimes of the bearing, we have:

$$\frac{h}{6\,\mu\,\overline{v}} \; \frac{R_2 - r_t}{R_2 \ln \frac{r_t}{R_1}} \Phi = \frac{p_t - P_2}{P_1^2 - p_t^2} \approx \frac{p_t - P_2}{P_1^2} \qquad (3)$$

where we have made the substitution:

$$\Phi = \frac{1}{\ln \left[\frac{2\left(R_2 - r_t\right)}{h} \right] - \frac{1}{2}}$$

To establish the values of p_t and r_t independent of each other, a second relationship is required. From equations (1) and (2), two separate expression for $\frac{\partial p_t}{\partial dr_t}$ may be determined:

$$\left. \frac{\partial p_t}{\partial dr_t} \right|_{viscous} = -\frac{6\mu}{\pi h^3} \frac{Q}{p_t r_t}$$

and

$$\left. \frac{\partial p_t}{\partial dr_t} \right|_{fm} = \frac{2Q}{\pi R_2 h^2 \overline{v}} \left(\Phi - \tfrac{1}{2}\Phi^2 \right) \approx \frac{2Q}{\pi R_2 h^2 \overline{v}} \Phi$$

Based on the restrictions described above $\Phi << 1$. Therefore the quadratic term may be safely dropped.

The two partial derivatives are of opposite sign since as the hypothetical value of r_t changes, one flow region increases in length while the other decreases. A second important difference is that while $\left.\frac{\partial p_t}{\partial dr_t}\right|_{viscous}$ is inversely proportional to p_t, the other partial, $\left.\frac{\partial p_t}{\partial dr_t}\right|_{fm}$ has no dependence on p_t. In *Figure 2*, the absolute values of the two partial derivatives are represented as a function of the variable p_t.

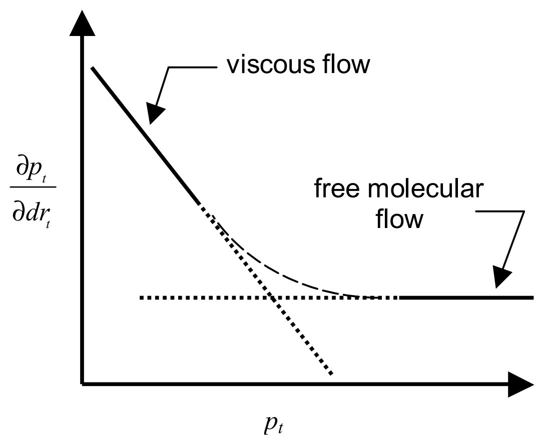

FIGURE 2 Plots of the magnitude of $\partial p_t / \partial dr_t$ for asymptotic viscous flow and free molecular flow . The point of intersection represents an estimate of the theoretical point at which a hypothetical abrupt flow transition occurs.

The solid lines of the plot represent the asymptotic behavior of the two flow regimes. Empirical data on transitional flow, suggests that a smooth transition occurs as pressure varies by a factor of two or three on each side of the asymptotic intersection. This is represented by the curved dashed line. Short of full up simulation involving individual calculations of the trajectories of very large numbers of molecules or ensembles of molecules, no reliable model is available to describe this transitional behavior.

The present analysis treats the flow as if it abruptly transitions at the point of intersection of the asymptotes. Since in realistic bearing designs, the pressure is expected to vary over more than two decades, the resulting error is expected to be small. In the final analysis the validity of this assumption must be tested empirically. It is also noted that a transition point located at the intersection of the asymptotes represents the solution that entails the least pressure drop, hence minimum energy dissipation for any given flow undergoing an abrupt transition.

The second relation between p_t and r_t may then be found from:

$$\left.\frac{\partial p_t}{\partial dr_t}\right|_{fm} - \left.\frac{\partial p_t}{\partial dr_t}\right|_{viscous} = 0$$

leading to:

$$p_t = \frac{3\mu\bar{v}}{h}\frac{R_2}{r_t}\Phi \qquad (4)$$

Substitution of the approximate value of equations [4] into [3] yields the following expression for r_t independent of p_t:

$$\frac{3\mu\bar{v}}{h}\frac{R_2}{r_t} - P_1^2\frac{h}{6\mu\bar{v}}\frac{1-\frac{r_t}{R_2}}{\ln\frac{r_t}{R_1}}\Phi + P_2 = 0 \qquad (5)$$

While this equation does not lend itself to solution for r_t in closed form, the root between R_1 and R_2 is easily found numerically once the other parameters are set. Substitution of the value of r_t thus determined, into equation [4] yields p_t.

<u>Dimensionless Representation</u>

The quantity $\frac{3\mu\bar{v}}{h}$ that appears in equations (4) and (5) has the dimensions of pressure. The viscosity may be represented in terms of statistical mechanical variables as:

$$\mu = \frac{m}{3\sqrt{2}\,\pi d^2}\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}$$

where d is the molecular diameter. Combined with the expression for $\bar{v}$ introduced earlier we have:

$$\frac{3\mu\bar{v}}{h} = \frac{8}{\pi h}\frac{kT}{\sqrt{2}\,\pi d^2}$$

Meanwhile the mean free path traversed by gas molecules at a pressure P, is $\frac{kT}{\sqrt{2}\,\pi d^2 P}$.

It follows that $\frac{3\mu\bar{v}}{h}$ is the pressure associated with a mean free path of $\frac{\pi}{8}h$:

$$P_{\left(\frac{\pi}{8}h\right)} = \frac{3\mu\bar{v}}{h}$$

This pressure may be viewed as a characteristic pressure of any vacuum bearing pad operating at bearing height h. It is useful to normalize the other pressures in the calculation to this characteristic pressure:

$$\mathbf{P}_1 = \frac{P_1}{P_{\left(\frac{\pi}{8}h\right)}} \qquad \mathbf{P}_2 = \frac{P_2}{P_{\left(\frac{\pi}{8}h\right)}} \qquad \mathbf{p}_t = \frac{p_t}{P_{\left(\frac{\pi}{8}h\right)}}$$

where the bold font represents the normalized value. With this normalization scheme, the dimensionless supply pressure will typically be a number much greater than unity, while the dimensionless transition pressure will be of order unity.

It is also useful to normalize the various radiuses in the bearing to the exhaust radius R_2:

$$\mathbf{R}_1 = \frac{R_1}{R_2} \qquad \mathbf{r}_t = \frac{r_t}{R_2}$$

while establishing a scale factor $\mathbf{S}$ for the bearing:

$$\mathbf{S} = \frac{R_2}{h}$$

With these changes in variable, equations (4) and (5) may be represented in dimensionless form:

$$\mathbf{p}_t = \frac{\Phi}{\mathbf{r}_t} \qquad (6)$$

$$\frac{1}{\mathbf{r}_t} - \frac{\mathbf{P}_2^2}{2}\frac{1-\mathbf{r}_t}{\ln\frac{\mathbf{r}_t}{\mathbf{R}_1}}\Phi + \mathbf{P}_2 = 0 \qquad (7)$$

where

$$\Phi = \frac{1}{\ln\left[2e^{-\frac{1}{2}}\mathbf{S}\left(1-\mathbf{r}_t\right)\right]}$$

The value of $\mathbf{p}_t$ is plotted in figure 3. While this plot is accurate at the scale shown, limitations of the model lead to inaccuracy on the scale $\mathbf{r}_t \approx 1 - 1/\mathbf{S}$. Examination of the equations for free molecular flow through thin apertures [4] suggest that in the limit $\mathbf{r}_t \to 1$ we have $\mathbf{p}_t \to 1$ while

$$\Phi \to \frac{\mathbf{P}_1^2}{2\,\mathbf{S}\ln\frac{1}{\mathbf{R}_1}}$$

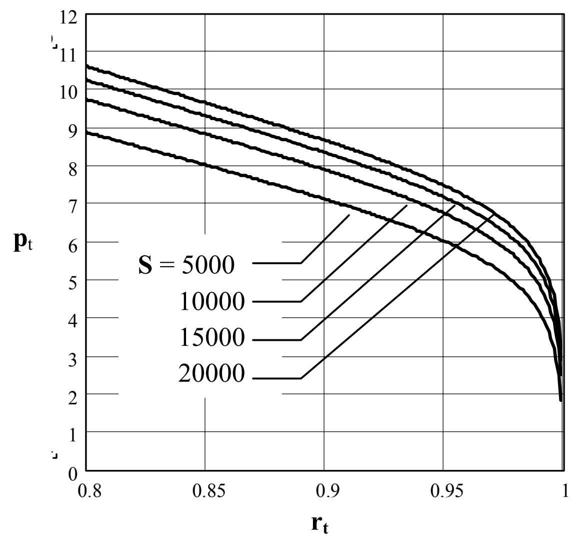

FIGURE 3. Normalized transition pressure $\mathbf{p}_t$ as a function of normalized transition radius $\mathbf{r}_t$ and scale factor $\mathbf{S}$. In the limit $\mathbf{r}_t \to 1$, $\mathbf{p}_t \to 1$.

The root $\mathbf{r}_t$ of equation (7) is plotted in *Figure 4* for the particular value $\mathbf{S}$ = 15000.

<u>Load Capacity</u>

The load capacity of a thrust bearing is simply the area integral of the pressure difference between the top and bottom of the bearing

$$W = 2\pi\int_{R_1}^{R_2} P(r)rdr + \pi R_1^2 P_1 - \pi R_2^2 P_2 \qquad (8)$$

The pressure distribution $P(r)$ under the viscous portion of the flow may be found from equation (1):

$$P_{vis}(r) = P_1\left(1 - \left(\frac{p_t}{P_1}\right)^2 \frac{\ln\left(r/R_1\right)}{\ln R_2/R_1}\right)^{\frac{1}{2}} \qquad (9)$$

The pressure distribution under the free molecular portion of the bearing may be found from equation (2):

$$P_{fm}(r) = p_t\left(1-\frac{P_2}{p_t}\right)\frac{r-R_2}{r_t-R_2} \quad (10)$$

This free molecular flow contribution to load capacity is frequently small enough to be neglected.

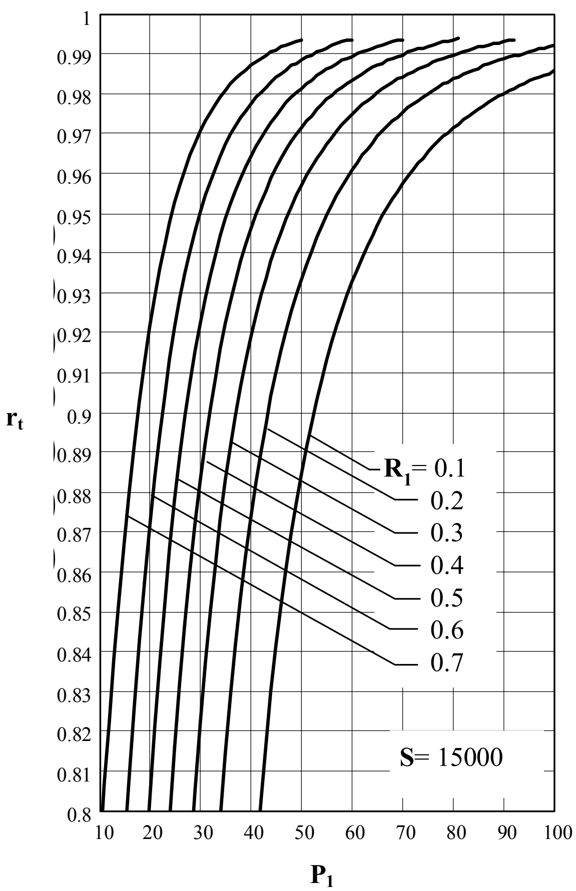

FIGURE 4. Normalized transition radius $\mathbf{r_t}$ *as a function of normalized inlet pressure* $\mathbf{P_1}$*. This sample calculation is done at* $\mathbf{S} = 15000$

Inertial Effects

The Poiseuille flow equation (1) used to describe the flow in the viscous flow portion of the bearing is based on the assumption that kinetic effects in the gas flow are small compared to viscous effects. As a general rule, this requires that the Reynolds number $Re < 1$. In the case of gas bearings, a modified Reynolds number, Re* has historically been identified [3] as determining the ratio of kinetic to viscous forces:

$$\mathrm{Re}^* = \frac{\rho v h^2}{\mu\left(R_2 - R_1\right)}$$

where ρ is the gas density and v is the velocity. Through statistical mechanics [5] it may be shown that $\rho = \frac{3P}{\overline{v}^2}$, where $\overline{v}$ again is the average molecular velocity. The free molecular flow velocity v through a thin aperture may be shown [4] to be is $\frac{\overline{v}}{4}$. This yields the following upper limit for the modified Reynolds number at the entrance to free molecular flow:

$$\mathrm{Re}^* \le \frac{9}{8}\frac{\mathbf{p_t}}{\mathbf{S}\left(1-\mathbf{R_1}\right)} \quad (11)$$

For reasonable bearing designs Re* is reliably several orders of magnitude less than unity, confirming the validity of equation [1] even when exhausted into partial vacuum.

Vacuum Pump Requirements

As a practical matter it is important to evaluate the vacuum pump capacity required to evacuate the flow, at suitably low pressure, from gas bearings as discussed here. Suitable backing pumps are generally of the constant flow variety, characterized by approximately fixed volume flow independent of pressure. An appropriate generic pump pressure to consider is the characteristic pressure of the bearing $\frac{3\mu\overline{v}}{h}$. This represents the lower limit for the transition pressure, and would represent a reasonable choice for the final pressure P_2. The flow at this pressure may then be evaluated from equation [1].

$$\dot{V}_{pump} = \frac{\pi h^2 \overline{v}}{4 \cdot \ln\frac{\mathbf{r_t}}{\mathbf{R_1}}}\left(\frac{\mathbf{P_1}^2 - \mathbf{p_t}^2}{\mathbf{P_2}}\right) \quad (12)$$

In this case we have $\mathbf{P_2} = \mathbf{p_t} = \mathbf{r_t} = 1$. For room temperature air, $\overline{v} \approx 460$ m/sec. In a bearing operating at h = 3 microns the characteristic pressure at the pump is 8.3 x 10^3 Pa. A reasonable example value for P_1 in a useful bearing is 2 x 10^5 Pa. yielding $\mathbf{P_1}$ = 24. With a modest value of $\mathbf{R_1}$ = 0.5 the pump rate is a very modest 0.16 liters per minute. Extraneous considerations may dictate a lower value of $\mathbf{P_2}$. Setting $\mathbf{P_2}$ an order of magnitude lower at 0.1, we may find the value of $\mathbf{r_t}$ from equation (3). Assuming $\mathbf{S}$ = 15000, $\mathbf{r_t}$ may also be found to good approximation from *Figure 4*, yielding the value $\mathbf{r_t} \sim 0.87$.

The value of $\mathbf{p_t}$ may then be found from equation (6) or *Figure 3*, yielding $\mathbf{p_t}$ = 9. The pump rate evaluated from equation (12) is then found to be a still modest 1.7 liters per minute.

DISCUSSION

A substantial number of vacuum air bearings have been built over the past several decades ranging from simple rotary vacuum feedthroughs, to planar feedthroughs as large as a meter in diameter, to gantry type X-Y stages using numerous cylindrical gas bearings entirely inside the vacuum chamber. As a general rule it has been the practice to exhaust the air bearing flow to atmospheric pressure in the first collection groove, while pumping a rough vacuum in the second collection groove. Typically this involves a transition to free molecular flow in the passage between the first and second collection grooves. With such an approach, conventional bearing design equations such as equation (1) may be used to design the bearing proper. The equations developed here may be used to characterize the flow in the passage between the first and second groove.

On the basis of the above analysis, the possibility is raised of venting the main bearing flow directly to vacuum. The results developed here suggest that with bearing clearances in the range of three to five microns, flow rates are small enough that the pumping requirements are within reason.

The benefit of this approach becomes apparent when limited force is available to preload the bearing. In the case where the moving load is massive, gravity may be used to provide a substantial preload. In high speed applications, it is useful to minimize the mass of the moving element. At the same time, dynamical requirements generally require high bearing stiffness. The bearing stiffness k can generally be characterized as $k = \kappa \frac{W}{h}$ where κ is a dimensionless constant characteristic of the bearing design, W is the load and h is the bearing height. With hydrostatic gas bearings operating in atmosphere, κ may be of the order unity. When the mass of the moving element is low, W may be increased using vacuum or magnetic preloading. When the same bearing operates in vacuum, preload options are limited to the magnetic approach. The value of κ is reduced by the approximate multiplicative factor $\left(1 - P_2/P_1\right)$. This is because that fraction of the load is borne by *unopposed* static pressure P_2 when P_2 is much greater than the background vacuum. This part of the supporting force does not change with bearing height, hence does not contribute to stiffness. When the exhaust pressure P_2 is atmospheric pressure this can represent a substantial loss of performance. Reducing the exhaust pressure to rough vacuum levels can, in principle, substantially reduce this deficit.

The necessity of operating at relatively small bearing height introduces substantial challenges to the system designer. The requirements for large servo bandwidth and consequent high resonant frequency, however, tend to drive the system designer further toward the requirement for very stiff bearings, even without the requirements for vacuum operation. Small bearing heights are a natural evolution in this regard.

Experimental Verification

A series of experiments are currently in progress to test the validity of this work. Results will be reported in a follow-on paper.

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