PICOMETER LASER GAUGING TO TEST GENERAL RELATIVITY

Robert D. Reasenberg and James D. Phillips
Smithsonian Astrophysical Observatory
Harvard-Smithsonian Center for Astrophysics
Cambridge, MA, USA

THE EQUIVALENCE PRINCIPLE

The equivalence principle is an essential postulate of general relativity. It has been a subject of interest for 15 centuries, and investigated experimentally for at least four centuries. The universality of free fall (UFF) is an aspect of the equivalence principle. The UFF requires that a body moving solely under the influence of gravity follow a trajectory that is independent of the composition of the body. In an equivalence principle failure, one might have two bodies falling under the same conditions and showing a differential acceleration.

POEM

The SAO principle of equivalence measurement (POEM) is a laboratory test of UFF. It is a "Galilean" test in which a pair of test mass assemblies (TMA), one above the other, are placed in free fall and observed differentially with a laser gauge. The experiment is conducted in a vacuum chamber that is also placed in free fall, maintaining its position with respect to the test masses to better than 0.5 mm. With this approach, there is no need for mechanisms inside the chamber to drive the motion of the TMA and the TMA-observing devices during each toss.

The vacuum chamber is mounted on a commercial slide, a cart that rides along a vertical track, which includes a linear motor and position sensor. In the present (Gen-I) version of the experiment, a single pair of TMA rest on shelves separated by 0.5 m inside the vacuum chamber. (See Fig. 1.) During the upper portion of the chamber motion, the linear motor and its control system serve to enforce a free-fall trajectory, overcoming friction. At the bottom of the free-fall portion of the motion, the chamber encounters a "bouncer" that passively applies an upward force, absorbs the energy of the falling chamber, and returns the chamber to upward motion with a minimum of force required from the motor. This reversal of the moving vacuum chamber takes place in about 0.3 s and the measurement will be repeated every 1.2 s.

In a more advanced (Gen-II) version of POEM, there will be two pairs of TMA with a lateral separation of 7 cm, permitting a double difference observable that cancels many systematic errors. Prominent among these errors are those due to gravity gradient, including the vertical component, $dg/dz = 3\ 10^{-7}$ g / m. Additionally, there are small gravity gradient components that are time dependent, including those due to ground-water variation and parked cars on the nearby street. With a science goal of $\sigma(\Delta g)/g = 5\times10^{-14}$ and a TMA mass that is 30% test substance, we require a measurement accuracy of $\sigma(\Delta g)/g = 1.5\times10^{-14}$ for the TMA. This requirement, when combined with the vertical gravity gradient, implies a requirement for absolute distance measurement with an uncertainty under 0.05 µm. The laser gauge is expected to achieve this.

In the third generation of the experiment (Gen-III), systematic error is mitigated by a series of interchanges: (1) left-right robotic interchange of TMA on a time scale of perhaps 10 minutes; (2) top-bottom interchange of TMA between runs; and (3) manual interchange of test substance half way through the experiment. These interchanges substantially reduce the biasing due to gravity gradient and several small effects.

Coriolis force can produce TMA acceleration far above the intended accuracy level. To address this systematic error, we will use a four-channel capacitance gauge for each TMA. (The capacitance gauge is being developed in a collaboration with Winfield Hill of the Rowland Institute at Harvard.) The POEM error budget requires that the transverse velocity be measured to 33 nm/s in each toss and that the bias in the average of these measurements be under 0.25 nm/s. In order to keep the stability requirement for the capacitance gauge at a reasonable level, we require that the TMA transverse velocity be under 10 µm/s. Since the TMA will pick up the chamber's transverse velocity at the time of launch, when the chamber will be moving upward at nearly 5 m/s, we require that the rail guiding the motion have angular deviations from

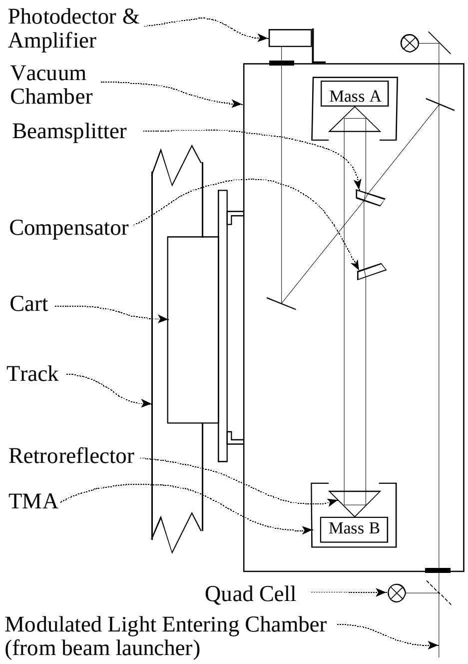

Figure 1. *Optical path in POEM vacuum chamber.*

vertical under 2 10^{-6}. This, in turn, requires both a straight rail and careful leveling, which can be adjusted based on the capacitance gauge measurements.

LASER GAUGES

The tracking frequency laser gauge (TFG) was developed at SAO around 1990 as part of a NASA funded development of a spaceborne optical interferometer, POINTS. The TFG is described in [1]. The requirement then was 2 pm in a minute; now, 1 pm in a second. Before developing the TFG, we considered the standard precision laser gauge, the heterodyne interferometer. Typically, a light source based on stabilized (HeNe) laser provides a beam comprising orthogonally polarized signals that differ in frequency by 10s of kHz to 10s of MHz. In the measurement interferometer, a polarizing beam splitter separates these components so that one travels the path to be measured and the other travels a reference path before they are recombined with a polarizing beam splitter. The recombined beam is detected after passing through a polarizer at 45 deg to the polarization axes of the beam. The phase of the beat signal provides a measure of the change in the difference between the measurement and reference path lengths. A reference phase is obtained by sampling and detecting the dual-component beam before it enters the measurement interferometer.

There are three problems with this laser gauge. First, the heterodyne gauge does not lend itself easily to the measurement of absolute distance, although with enough additional complexity, this can be overcome [2, 3]. Second, the original polarization and the polarizing beam splitters are imperfect, which results in a small portion of each beam traveling the unintended path. This gives rise to a cyclic bias in the laser gauge reading with $\lambda/2$ period and a typical amplitude of one to a few nm. This cyclic bias problem has been partially overcome [4] at the expense of measurement speed and instrument complexity. The bias can also be mitigated by employing spatial separation of the two required beams [5-8], again at the expense of complexity. The third problem with the heterodyne gauge is that it does not operate in a cavity, which can be useful for high-precision applications.

Tracking Frequency Gauge

The TFG is a closed-loop system based on Pound-Drever-Hall locking. In the classical TFG realization (Fig. 2), an optical signal from the Variable Frequency Source (VFS) of an adjustable wavelength λ_{VFS} is phase modulated at a frequency f_m and introduced into the measurement interferometer whose length, L, is to be determined. When λ_{VFS} is away from λ_N, the wavelength at the intensity extremum, the optical signal emerging from the interferometer is amplitude modulated, resulting in an electrical signal at the detector output at f_m with a magnitude and sign that indicate the offset. Synchronous detection at f_m and filtering yield a signal that is used to control λ_{VFS}, driving it back to λ_N. The corresponding optical frequency shift is measured by a frequency counter.

The TFG offers four advantages over the heterodyne gauge. First, it is intrinsically free of the cyclic bias that plagues the heterodyne gauges. It has only one beam and thus it cannot be subject to problems associated with separating beams. Second, it naturally operates in either a cavity or a non-resonant interferometer (Michelson, Mach-Zehnder, etc.) Thus, additional accuracy is accessible when needed. Third, as will be discussed below, the TFG can be used to measure absolute distance with little additional effort. Fourth, the TFG can suppress some additional errors. (a) Many interferometers alter

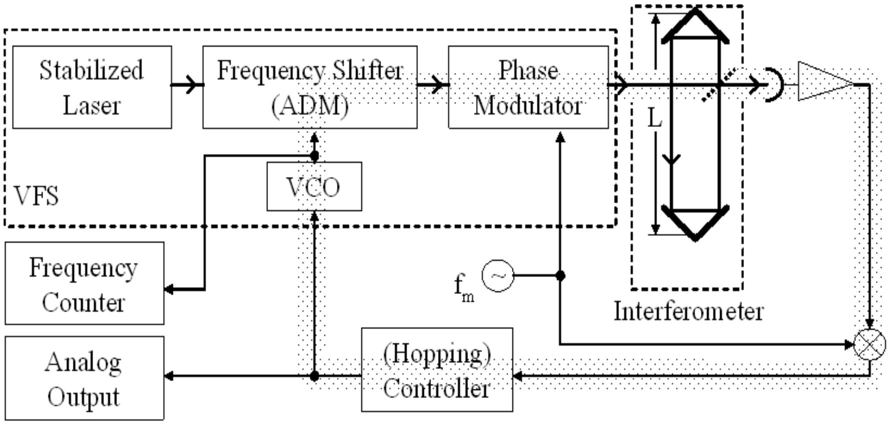

Figure 2. *Classical TFG. The stippled line traces the loop.*

the polarization state of the light because of the S-P phase difference at the mirrors. Since there is no polarizing beam splitter to dictate the polarization state of the incoming beam, that state can be set to yield an extremum of measured distance. (b) Operating in a cavity suppresses distance measurement error due to misalignment. In a cavity, small misalignment will decrease coupling to the principal mode and may permit coupling to other modes. However, if these modes are well separated in frequency from the principal mode, they will not be excited, and they will not corrupt the distance measurement.

The TFG has a disadvantage with respect to the heterodyne gauge: its measurement interferometer must have at least a minimum distance, or distance difference in the case of a non-resonant (e.g., Michelson) interferometer. This minimum is set by the available frequency shift range. For a range of 3 GHz, the minimum path is just over 5 cm, one way.

In considering the precision of the TFG, it is natural to start with the limit set by photon counting statistics. For 1 μW of HeNe (633 nm) power detected from a Michelson interferometer, the limit is 0.06 pm after 1 s. The current TFG is limited by technical noise: σ = 10 pm at either 1 or 10 samples per sec. More recently, we have needed higher performance for POEM: 1 pm in 1s of observing and the ability to sample much faster losing only by $t^{-1/2}$. For this purpose, we are operating in a low finesse cavity.

Since precision is proportional to finesse, it is tempting to seek to gain additional precision by using a cavity of high finesse. However, there are limits to their use. In particular, the storage time is approximately 1/FWHM, and this limits the unit-gain bandwidth of the Pound-Drever-Hall locking (PDHL). This limitation poses a particular problem for the measurement of long distances since it requires the servo bandwidth to be less than about c / (10 F L), where F is the cavity finesse (proportional to the number of round trips). It is necessary to have a servo signal with at least the bandwidth of the laser, in order to stabilize the laser. The distributed feedback (DFB) lasers we are currently using have a linewidth of ~100 kHz. When measuring long distances, the cavity linewidth may be less than this. This problem could be corrected by using one servo to narrow the laser line and a second to implement the PDHL. The measurement laser is beat against a quiet and stable reference laser, and the resulting signal compared to the frequency required by the PDHL using a phase sensitive detector. An error signal from the phase-sensitive detector is fed back to the laser frequency control.

Recent TFG Improvements

As originally implemented, the TFG frequency shift was achieved by passing the light from a stabilized HeNe laser twice through an accusto-optical deflector-modulator (ADM). The second pass doubles the frequency shift and cancels the deflection, yielding a beam that can be introduced into fiber. That ADM operated over a frequency range of 100 MHz, which yielded a distance range of 0.2 μm over a 0.5 m path. More recently, we have introduced an ADM with a frequency range of 125 MHz and passed the light through it four times. This yields a distance range of 0.5 μm over the same path. Since the measurement interferometer's mode spacing is $\lambda/2$ = 0.316 μm, one mode is always accessible with the available frequency shift, and often there are two.

To further increase the measurement range, we have introduced a non-linear aspect to the TFG loop controller. It detects that it is running out of the frequency shifter's range and hops to a mode at the far end of the range, shifting the optical frequency by the free spectral range, $\Phi = c/2L$ = 300 MHz. The hop is fast enough (about 1 μs) to be "unobserved" by the classical portion of the loop controller. We have demonstrated a rate of 5×10^4 hops/s, which corresponds to a linear velocity of 16 mm/s. Hopping allows the TFG to measure over a distance range that is essentially without limit (at least a factor of two in *L*).

The original TFG had a unit-gain bandwidth of 100 Hz. This was adequate for the task of showing that it could meet the POINTS mission requirements. However, POEM requires a more agile laser gauge, so as to follow vibrations when the TMA are not in free fall, to reacquire lock rapidly should it be lost, and to follow accurately the TMA acceleration while in free fall. We have rebuilt the controller with a unit-gain bandwidth of 50 kHz. In this implementation, the bandwidth is now limited by an acoustic delay in the ADM.

The TFG employs a frequency measurement to determine incremental distance. Standard frequency counters generally have a dead time between measurements. For many purposes, one needs to read out the laser-gauge frequency counter at 10^2 to 10^4 times per second, which is not possible with typical counter dead time. The ideal counter for the TFG does a continuous count of the incoming cycles, sampling the count at precisely defined intervals, and sending those samples to a computer for storage or analysis. Such a device was built for us by Jim MacArthur at the Harvard Physics Dept Electronic Instrument Design Lab. This "advanced frequency counter" has synchronized dual channel operation and a network (100 base-T, UDP) interface to a PC. It has a maximum counting rate of 200 MHz and internal dividers (2x, 4x, 8x) that permit counting rates up to 1200 MHz. The counter can be read continuously, using an ordinary PC, at ~10k samples/s. Timing mismatch between the effective gate times for its two channels is well under 1 ns, and variations of this mismatch are far smaller.

The hopping provides an easy means of measuring absolute distance. By measuring the frequency shift before and after a hop, the TFG measures the free spectral range, Φ, of the measurement interferometer corresponding to the current length *L*. The estimate of *L* is then $c / (2\Phi)$. For the case of a dispersive medium in the path, see the Appendix of [1]. High precision in incremental measurement $\sigma_\tau(\delta L)$ yields an option for absolute distance (which depends on group delay) with no ambiguity length. The precision of the absolute distance measurement is

$$\sigma_T(L) = \frac{2}{\eta}\sqrt{\frac{\tau}{T}}\,\sigma_\tau(\delta L).$$

In the above, η is the fractional bandwidth, $\Delta f / f$, τ is the measurement interval for the incremental distance and *T* is the measurement interval for absolute distance. If one assumes that the TFG error is a constant fraction of the FWHM of the measurement interferometer fringe, then both incremental and absolute distance precision are independent of the length measured.

A sufficiently precise absolute distance measurement permits unambiguous count of fringes. Thus, it can be connected to the precision of the incremental distance measurement. However, some care is required if the system is dispersive. Further, to avoid errors due to fluctuating path, one must either use two lasers to read simultaneously or hop fast. Our original TFG used a HeNe laser and an ADM to shift frequency. It has $\eta = 10^{-6}$. More recently, we have been working with a DFB laser at 1560 nm. If we take the bandwidth limitation as being due to our frequency counter (without an external divider), then $\eta = 10^{-5}$. This is several times better than needed to connect the absolute and incremental measurement. However, with a more elaborate scheme, using the present DFB laser, it would be possible to have a bandwidth of 60 GHz, which would yield $\eta = 3\ 10^{-4}$.

A few years ago, when we added hopping to the TFG, we demonstrated the measurement of absolute distance to low accuracy (0.1 mm) in a preliminary test plagued by technical noise. We anticipate demonstrating a substantial improvement in this capability by using the refined TFG now under development.

Current Direction

As previously noted, we have begun to work with DFB (semiconductor) lasers. These lasers operate in the 1550 nm (190 THz) band. They are available at moderate cost from several vendors, create most of their output in a single longitudinal mode, have adequate coherence length (~100 m), can be purchased with fiber-coupled output, and are rapidly tunable via their injection current. They have been used for atomic spectroscopy for more than a decade [9]. They come already coupled to fiber, and the necessary components for building a TFG are also available in fiber-coupled versions. For a TFG, the rapid tunability will facilitate following rapid motion (e.g., vibration and air turbulence) and permit rapid hopping for absolute distance in the presence of rapid motion. The wide tuning range improves absolute distance accuracy.

The FM response of DFB lasers presents a possible complication. At low frequencies (<~1 MHz), the response is thermal, while at high frequencies, it is due to changes in carrier density. The two responses have the opposite sign. We have tested two examples, and found that the phase lag for the laser and its driver together is <45° from 1 kHz to 1 MHz, permitting a servo to have unity loop gain at 1 MHz (cf, 50 kHz for the TFG based on a HeNe laser and acousto-optic frequency shifter).

MOTION SYSTEM

A key feature of POEM is that the experiment is conducted in a vacuum chamber that is placed in free fall with the TMA. In order to have a precision experiment based on a free fall time of ¾ s, it is necessary to have a large number of repetitions, which suggests a need for rapid recycling. To meet this objective, we built a "bouncer" that catches the falling vacuum chamber and returns it to upward motion with little loss of energy or shock to the instrument inside the chamber. Once the chamber is again moving upward at the required speed (nearly 5 m/s), the TMA must be launched. As noted in the introduction, we require that the TMA transverse velocity be under 10 µm/s, which implies that the rails guiding the motion have angular deviations from vertical under $2\ 10^{-6}$.

Torsion Bar Bouncer

It is our intention to run the laser gauge continuously, including during the bounce period where vertical acceleration reaches about 5 g. For this reason, to ease the job of the motor control servo, and to limit vibration so that the TMA are launched with minimal transverse velocity, it is essential that the bouncer produce minimal shock and vibration. This requirement rules out many obvious candidate solutions. Having the chamber contact a cable (or flexible band) under tension results in a force on the chamber that initially grows linearly with cable deflection. We use a ¼ inch steel cable, 62 cm long. It has a 0.1 kg mass but only 0.05 kg "effective mass" as seen by the falling chamber, which now has a mass of about 40 kg. In addition, the cable probably flexes on initial contact, further reducing the shock to the moving system.

The next question is how to store the energy of the falling chamber so that can be returned to upward motion. Coil springs have internal modes that are problematic. Our initial design, which used a ton of lead, a 5:1 lever, and a long cable running over pulleys, had two problems. First, on initial contact with the falling chamber, the cable experienced a longitudinal acceleration that excited an oscillation with the ton of lead. Second, friction internal to the cable as it ran over the pulleys caused the system to fail to meet the efficiency requirement. The new bouncer uses torsion bars connected to the cable through stiff levers. Internal torsional modes of the bars are above 1 kHz, and to the extent excited, cannot contain much energy since the bar's total rotation rate is modest and its moment of inertia is very small compared to $M_{chamber}\ R^2_{lever}$. Tests show that bouncer energy loss is small and masked by uncertainty in the losses in the present slide.

Air Bearing Slide

The present commercial slide uses instrument grade track rollers running on small (¼ inch) but well supported rails. At full speed, we find micron scale vibration, mostly in the 100 to 200 Hz band, yielding transverse velocity of 1 mm/s scale. We are in the process of implementing an air-bearing slide, as described in our original plans for this experiment.

We plan to use porous graphite bearings running on a granite beam. The undulations of the slope of the granite surface will need to under $2\ 10^{-6}$ in the region where the cart is traveling when the TMA are launched. On linear scales shorter than the 2 inch diameter of the bearings, the requirement can be relaxed because of the averaging that will take place. On large scale (say over 1 m), the requirement can again be relaxed. Micron scale transverse displacement of the chamber, either after or well before TMA launch, has negligible effect on the intended results.

Such surface quality requirements are well within the capabilities of the large-optics industry. However, costs there are high, in part because the facilities are intended for making complex (a-spherical) surfaces, not just the required flat. Fortunately, the requirements are just within the capability of the precision granite industry, which operates at significantly lower cost. As of this writing, we are preparing to place an order for a granite beam. Should we eventually wish to further reduce the transverse motion of the chamber due to the shape of the granite beam, we would consider connecting the air bearings to the moving assembly through an actuator (e.g., PZT pusher) and using an inertial sensor to measure the required correction.

CONCLUSION

We have described the tracking frequency laser gauge in the context of a test of general relativity. The TFG is also applicable to manufacturing and to precision astronomical instruments, particularly those deployed in space. It has picometer accuracy, absolute distance as a natural extension, and it employs a simple modulated light source and a single beam in the measurement interferometer. The test of the UFF aspect of the equivalence principle (and thus of general relativity) establishes a transient microgravity environment in which we will be able to measure forces of 2×10^{-14} newton on macroscopic objects.

REFERENCES

1. Phillips, J. and Reasenberg, R., 2005, Tracking Frequency Laser Distance Gauge, *Review of Scientific Instruments*, 76: 064501.
2. Lay, O., Dubovitsky, S., et al., 2003, MSTAR: a submicrometer absolute metrology system, *Optics Letters*, 28: 890-892.
3. Mason, J., 2003, “Absolute metrology for the Kite testbed,” Proc. of SPIE Conf. on Interferometry in Space, 4852: 336-346.
4. Gursel, Y., 1994, Metrology for spatial interferometry, *Proc. SPIE Conference on Amplitude and Intensity Spatial Interferometry II*, 2200: 27-34.
5. Wu, C.-M., J. Lawall, et al., 1999, Heterodyne Interferometer with Subatomic Periodic Nonlinearity. *Applied Optics* 38: 4089-4094.
6. Lawall, J. and E. Kessler, 2000, *Michelson interferometry with 10 pm accuracy, Review of Scientific Instruments* 71: 2669-2676.
7. Halverson, P. and Loya, F., 2004, Signal processing for order 10 pm accuracy displacement metrology in real-world scientific applications, *ESA SP-554: 5th International Conference on Space Optics*.
8. Zhao, F., Diaz, R., et al., 2003, SIM internal metrology beam launcher development, *Proc. of SPIE Conf. on Interferometry in Space* 4852: 370-379.
9. Wieman, C. and Hollberg, L., 1991, "Using diode lasers for atomic physics," *Rev. Sci. Instr.*, 62:1-20 .