Optimization and Analysis of Variability in High Precision Injection Molding

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Abstract

Injection Molding (IM) is considered to be the most important process for mass-producing plastic products. One of the biggest challenges facing injection molders is to determine the best settings for the controllable process variables (CPVs). Selecting the proper settings is crucial because the behavior of the polymeric material during shaping is highly influenced by the process variables. The difficulty of optimizing an IM process is that the performance measures (PMs), such as surface quality or cycle time that characterize the adequacy of the part for its intended purpose usually show conflicting behavior. Furthermore, in actual molding, the CPVs will vary over some range during molding. This inconsistency of the process variables will lead to variability in the PMs. In high precision manufacturing, in particular for micro and nano scale components and devices, this variability needs to be minimized, and if possible eliminated. Thus, the variability in the PMs needs to be included in the optimization problem. The aim of this work is to demonstrate a method based on CAE, statistical testing, artificial neural networks (ANNs), and data envelopment analysis (DEA) to find the optimal compromises between multiple PMs and their variability to prescribe the values for the CPVs in IM. We present an example where the optimization is carried out in two phases. Phase one uses the PMs that are significantly affected by the injection gate location in order to prescribe two possible injection gates. Phase two of the optimization uses Data Envelopment Analysis (DEA) to find a PM-based efficient frontier for each injection gate considering process variability. These two efficient frontiers are then compared to select the best location. Other possible applications are discussed.

Introduction

U.S. industry is under increasing pressure to develop technology to mass-produce high-end goods with tight tolerances. It is critical that the industry meets these goals to maintain an edge over countries that, by keeping a low labor cost scheme, have been the recipients of a large amount of off-shored manufacturing operations. Injection molding is a process that is particularly well suited to the demands of extreme precision at high production rates. The combination of molding, with its ability to produce complex shapes with good surface finish and fine detail, and thermoplastic resins, which provide strength and durability with light weight and low cost, result in a technology that is unique in its capacity to provide manufacturing solutions to a wide range of design problems.

While thermoplastic injection molding is a well-understood technology, the need to mold complex parts to ever–smaller tolerances is driving industry to find better methods for coupling high precision with mass production. The use of injection molding (IM) in high precision manufacturing relies upon the capability of the process to deliver parts consistently conforming to specifications. Characterizing such capability is a matter of understanding the most important sources of variation in IM and finding ways to provide robustness to the process.

In this work, an optimization strategy previously presented in [**1**,**2**] is applied in a novel fashion to consider variability in the controllable process variables (CPVs), resulting in a two-phase procedure. The first phase involved a sequence of modeling, statistical analysis, and multiple criteria optimization tasks that resulted in a series of attractive injection location points (gates) for a particular IM part. The second phase involves the consideration of variability of CPVs to finally arrive at a single solution for the settings of the CPVs and a single injection point.

Optimization Strategy

Proposed by Cabrera-Rios, et al [**2,3**] the general strategy to find the best compromises between several PMs consists of five steps:

Step 1) Define the physical system. Determine the phenomena of interest, the performance measures, the controllable and non-controllable variables, the experimental region, and the responses that will be included in the study.

Step 2) Build physics-based models to represent the phenomena of interest in the system. Define models that relate the controllable variables to the responses of interest. If this is not feasible, skip this step.

Step 3) Run experimental designs. Create data sets by either systematically running the models from the previous step, or by performing an actual experiment in the physical system when a mathematical model is not possible.

Step 4) Fit metamodels to the results of the experiments. Create empirical expressions (metamodels) to mimic the functionality in the data sets.

Step 5) Optimize the physical system. Use the metamodels to obtain predictions of the phenomena of interest, and to find the best compromises among the PMs for the original system. The best compromises are identified here through Data Envelopment Analysis (DEA).

In the strategy outlined here, the metamodels are empirical approximations of the functionality between the controllable (independent) variables, and the responses (dependent variables). These metamodels are used either for convenience or for necessity. Because DEA requires that many response predictions be made, it is more convenient to obtain these predictions from metamodels rather than more complicated physics-based models. In addition, when physics-based models are not available to represent the phenomena of interest, the use of metamodels becomes essential.

Data Envelopment Analysis (DEA)

Cabrera-Rios et al [**1,2**] have demonstrated the use of DEA to solve multiple criteria optimization problems in polymer processing. DEA, a technique created by Charnes, Cooper, and Rhodes [**3**], provides a way to measure the efficiency of a given combination of PMs relative to a finite set of combinations of similar nature. The efficiency of each combination is computed through the use of two linearized versions of the following mathematical programming problem in ratio form:

Find $\boldsymbol{\nu}, \boldsymbol{\mu}, \mu_0$ **to**

$$\textbf{Maximize} \quad \frac{\boldsymbol{\mu}^T \mathbf{Y}_0^{\max} + \mu_0}{\boldsymbol{\nu}^T \mathbf{Y}_0^{\min}} \tag{1}$$

s.t.

$$\frac{\boldsymbol{\mu}^T \mathbf{Y}_j^{\max} + \mu_0}{\boldsymbol{\nu}^T \mathbf{Y}_j^{\min}} \le 1 \quad j = 1,\ldots,n \tag{2}$$

$$\frac{\boldsymbol{\mu}^T}{\boldsymbol{\nu}^T \mathbf{Y}_0^{\min}} \ge \varepsilon \cdot \mathbf{1} \tag{3}$$

$$\frac{\boldsymbol{\nu}^T}{\boldsymbol{\nu}^T \mathbf{Y}_0^{\min}} \ge \varepsilon \cdot \mathbf{1} \tag{4}$$

$$\mu_0 \quad \textit{free} \tag{5}$$

where, $\mathbf{Y}_0^{\max}$ and $\mathbf{Y}_0^{\min}$ are vectors containing the values of those PMs currently under analysis to be maximized and minimized respectively, $\boldsymbol{\mu}$ is a vector of multipliers for the PMs to be maximized, $\boldsymbol{\nu}$ is a vector of multipliers for the PMs to be minimized, μ_0 is a scalar variable, n is the number of total combinations in the set, and ε is a very small constant usually set to a value of $1\text{x}10^{-6}$. The solutions deemed efficient by the two linearized versions of the model shown above represent the best compromises in the (finite) set of combinations of PMs. A complete description of the linearization procedure as well as the application of this model can be found in any of the references **1 through 4**.

Determination of settings of process variables and injection point:

Consider the part shown in Figure 1a. This part represents a case where the location of the weld lines is critical, and part flatness plays a major role. The part is to be injection molded using a Sumitomo IM machine using PET with a fixed flow rate of 9cc/s. Five PMs were included in this study: (1) deflection range in the z-direction, R_Z, (2) time at which the flow front touches hole A, t_A, (3) time at which the flow front touches hole B, t_B, (4) time at which the flow front touches the outer edge of the part, t_{oe}, (5) the vertical distance from edge 1 to the weld line, d_1, and (6) the horizontal distance from edge 2 to the weld line, d_2. Figure 1b shows how the weld line locations were measured. For production purposes it is desirable to minimize R_Z to control the part dimensions in the critical direction. It is desirable to maximize t_A, t_B, t_{oe}, d_1, and d_2. The variables t_A, t_B, and t_{oe} should be maximized in order to provide a balanced flow and a uniform pressure distribution along the edges of the part. Uniform pressure along the edges will minimize the potential for flash. d_1 and d_2 should be maximized to keep the weld lines away from the corners. It was assumed that the top right and bottom left corners of the part would be subjected to the highest stress during the intended application.

Four controllable variables were varied at the levels shown in Table 1 in a full factorial design. These variables include: (a) the melt temperature, T_m, (b) the mold temperature, T_w, (c) the horizontal coordinate of the injection point, x, and (d) the vertical coordinate of the injection point, y. The injection point location is constrained to be in the region shown in Figure 1a, due to limitation of the IM machine. This point will be characterized by the variables x and y in a Cartesian coordinate system with its origin at the lower left corner of the part.

A finite element mesh of the part was created in Moldflow™ in order to obtain estimates for the performance measures. Following with the general optimization strategy, this initial dataset was used to create metamodels to mimic the behavior of each the performance measures. In general, it is favorable to fit a simple model to the data. In this study, second order linear regressions were initially considered as models for the performance measures. When simple models do not suffice, then more complicated models, in this case artificial neural networks (ANNs), become necessary. In order to measure the prediction capability of the metamodels, a validation set was used. The ANNs provided accurate models for each of the PMs, so they were used to generate the larger dataset necessary for the multiple criteria optimization problem. Further details of the performance of the ANNs are available in any of the references **4** thru **7.**

In order to extract useful information about the dependence of the PMs on the controllable variables, an ANOVA (analysis of variance) was carried out. The results of the ANOVA are shown in Table 2. Notice that several PMs (t_A, t_B, t_{oe}, d_1, and d_2) are only dependent on the location of the injection gate (x and y). With this knowledge, we were able to divide the optimization problem into two phases. The first phase incorporates those PMs that are exclusively dependent on x and y in a multiple criteria optimization problem to determine the set of efficient solutions for the location of the injeciton gate. Since these PMs are not influenced by the controllable temperatures, the ANNs were used to create a new dataset by varying x and y at nine levels each within the experimental region (see Table 1) resulting in a second data set with 81 data points. In the second phase, two of these efficient solutions are compared by incorporating the part warpage in the z-direction and considering variability around the temperatures' nominal values.

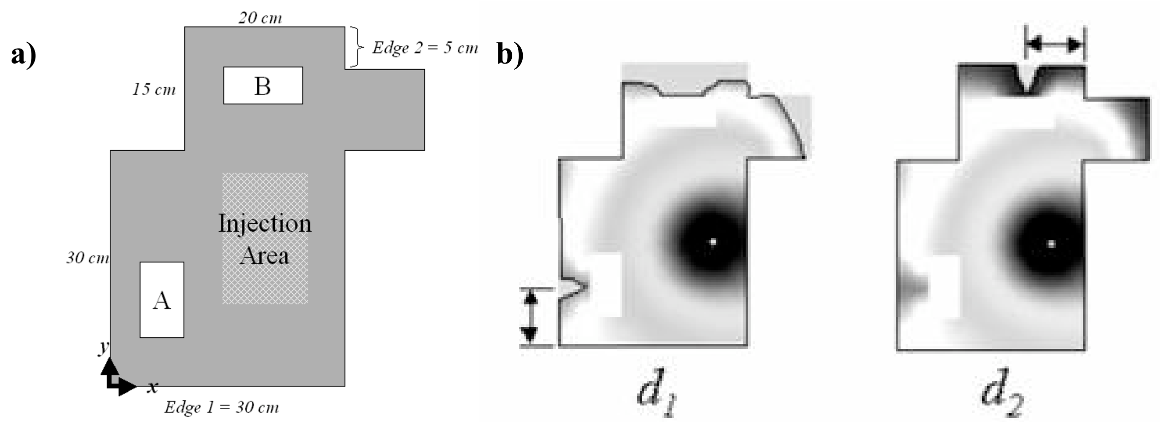

Figure 1: a) Part of constant thickness with cutouts b) Measuring the locations of the weld lines

Table 1: Levels of each of the controllable variables for the initial dataset

	T_m	T_w	x	y
Label	C	C	cm	cm
-1	260	120	15	10
0	275	130	20	17.5
1	290	140	25	25

Table 2: The significant sources of variation (linear, quadratic and second order interaction terms in the linear regression metamodel) to each performance measure

		Performance Measures					
		R_Z	t_A	t_B	t_{oe}	d_1	d_2
Linear Terms	T_w	■					
	T_m	■					
	x		■	■	■	■	■
	y	■	■	■	■	■	■
Quadratic Terms	T_w^2						
	T_m^2						
	x^2		■	■	■		
	y^2		■	■	■	■	
Second Order Interactions	$T_w T_m$						
	$T_w x$						
	$T_w y$						
	$T_m x$						
	$T_m y$	■					
	xy	■	■	■	■	■	■

First Phase of the Optimization Procedure:

DEA was applied to the experimental dataset of 81 points to identify the efficient frontier. The results of the DEA showed that 14 points were efficient. The combinations of PMs that were identified as efficient are shown in Figure 3a. These efficient solutions were then traced back to the corresponding injection locations. The resulting injection locations are shown in Figure 3b. Two of the injection locations on the efficient frontier (1.0,-0.6) and (0.2,-1.0) were chosen as candidates for further analysis in the second phase of the optimization.

Second Phase of the Optimization Procedure:

In the second phase the part deflection in the critical direction (z-direction) was incorporated. In high precision manufacturing applications it is necessary to obtain very tight dimensional tolerances. In injection molding, this may be achieved by decreasing the part deflection. In addition, it is also crucial to achieve a repeatable process that results in a consistent product. With this in mind, it is necessary to incorporate the variability of the PMs in the optimization problem when considering a high precision application.

In a physical IM setup, it is impossible to control certain variables with absolute accuracy. The process variables will vary over some finite range resulting in some variability of the PMs. Traditionally, the variability of a PM is evaluated by running several experimental repeats and then estimating the standard deviation. In our case, this was not feasible since the model of our system is a numerical one. In order to simulate a physical experiment, some perturbations of the process variables were introduced. It was assumed that the mold and melt temperatures could be controlled with accuracies of ±5 and ±10 ^{o}C respectively. For each of the two injection location, an experimental dataset of nine points, referred to as “set points” from this point on, was created by varying the mold temperature and the melt temperature at the three levels shown in Table 1. At each of “set points”, the artificial variability was introduced by varying the mold temperature by ±5 ^{o}C and the melt temperature by ±10 ^{o}C. This was done to simulate the process variables varying over a finite range in a physical system. This resulted in nine combinations of what we will refer to as “experimental points” at each of the “set points.” Table 3 shows the variability introduced for one of the “set points.”

From these perturbations the average, μ_{Rz}, and standard deviation, σ_{Rz}, were estimated for all nine set points for each injection location. This resulting dataset is shown below in Table 4 for one injection location. Similar data was obtained for the other injection location.

From these data second order regression models were used to model the functionality between the new PMs (μ_{Rz} and σ_{Rz}) and the CPVs. Since the second order regression models provided sufficiently accurate predictions in the experimental region, there was no need to to fit ANNs. These second order regression models were then utilized to create a larger dataset to be the subject of the multiple criteria optimization problem by varying the two temperatures in increments of five degrees over the experimental region. This full factorial resulted in 35 data points.

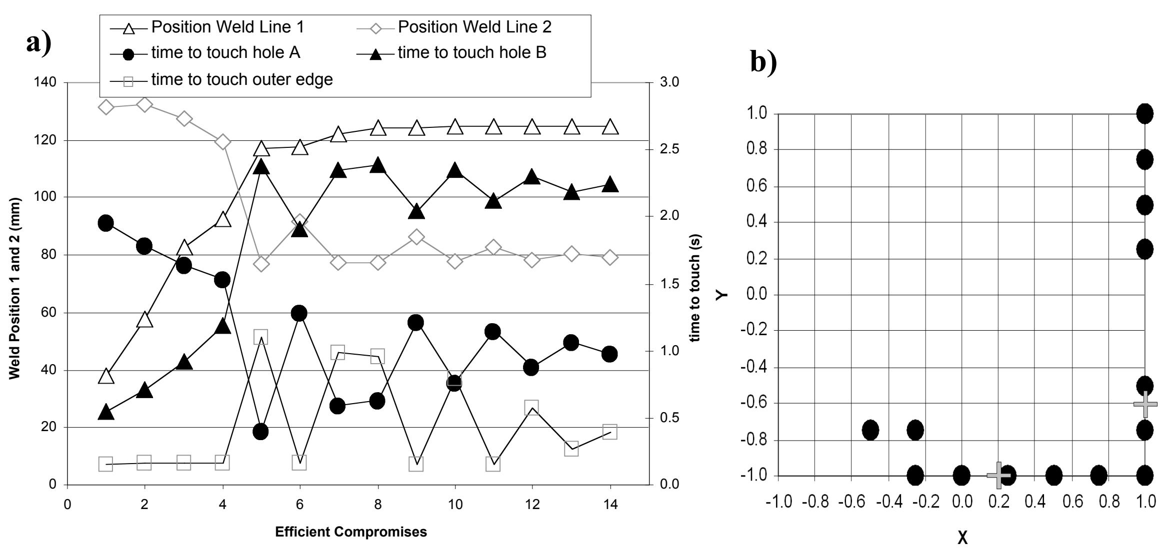

Figure 3: a) Combinations of PMs making up the efficient frontier b) Efficient injection gates shown with respect to the experimental region (cross marks indicate the points that were chosen for the second phase of the optimization)

Table 3: Perturbations introduced to simulate physical system for one "set point"

Set Point				Experimental Points				
x (cm)	y (cm)	T_m (°C)	T_w (°C)	x	y	T_m	T_w	R_Z (mm)
25	16	260	120	25	16	250	115	0.0044
				25	16	250	120	0.0047
				25	16	250	125	0.0050
				25	16	260	115	0.0058
				25	16	260	120	0.0061
				25	16	260	125	0.0065
				25	16	270	115	0.0069
				25	16	270	120	0.0072
				25	16	270	125	0.0075

Table 4: Average and standard deviation of the deflection for each "set point" for one injection gate

X	Y	Tm	Tw	μ_{Rz} (mm)	σ_{Rz} (mm)
1	-0.6	-1	-1	0.0060	0.00075
1	-0.6	-1	0	0.0064	0.00073
1	-0.6	-1	1	0.0070	0.00073
1	-0.6	0	-1	0.0070	0.00049
1	-0.6	0	0	0.0074	0.00048
1	-0.6	0	1	0.0081	0.00064
1	-0.6	1	-1	0.0077	0.00043
1	-0.6	1	0	0.0082	0.00056
1	-0.6	1	1	0.0090	0.00068

In high precision IM applications, it is crucial to minimize both the average deflection and the standard deviation of the deflection for reasons previously discussed. This constitutes a new multiple criteria optimization problem. The two candidate injection locations were analyzed separately. DEA was applied to both in order to identify the efficient frontier of each gate. The efficient frontiers of each candidate injection gate are shown in Figure 5.

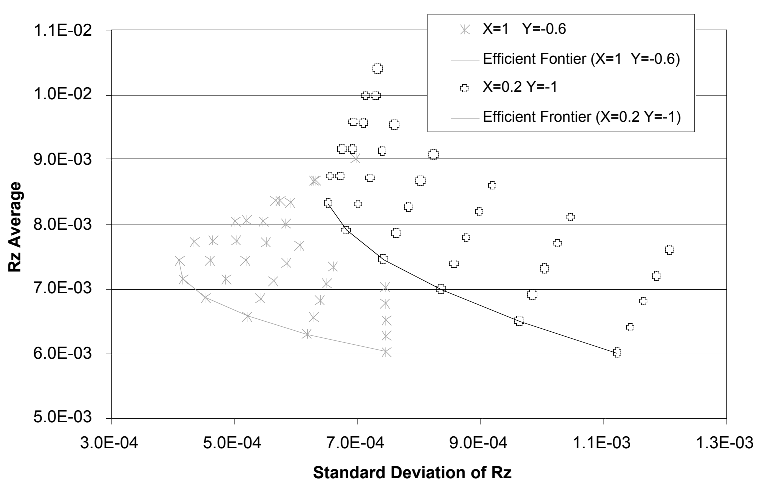

Figure 5: Comparison of the efficient frontiers of the two candidate injection locations

Since it is desirable to minimize both the average and standard deviation of the deflection, it is evident from the graph that the injection location of (1.0, -0.6) results in a more favorable efficient frontier. If a decision was to be made based on the average deflection alone, neither injection gate would hold a distinct advantage over the other, because the minimum deflection of both gates is essentially the same. However, when the variability is introduced in the form of the standard deviation, there is a clear advantage to locating the injection gate at (1.0, -0.6). Choosing this gate would result in a more effective process for high precision IM applications.

Conclusions

High precision injection molding will be possible and marketable only if the process can deliver parts at high production rates with tight tolerances in a consistent manner. This poses a multiple criteria optimization problem in which variability must be considered explicitly as a performance measure. Statistical analysis can help determine the dependency of the performance measures on the controllable process variables. If the analyses can be decoupled, then an optimization in several stages is appropriate as demonstrated in this application. The case where these cannot be decoupled is approached as in the previous publications.

In the case study presented here, injection locations that were otherwise comparable became quite different once variability was considered. This empirical evidence provides further justification to apply the optimization strategy described in cases where high precision is a critical concern.

References

1. Cabrera-Rios, M., Zuyev, K., Chen, X., Castro, J.M., and Straus, E.J. *Polymer Composites*, 23:5 (2002)
2. Cabrera-Rios, M., Mount-Campbell, C.A., and Castro J. M., *Journal of Polymer Engineering*, 22:5 (2002)
3. Charnes, A., W.W., and Rhodes, E. *European Journal of Operational Research,* 2:6 (1978)
4. Castro, C.E., Cabrera-Rios, M., Lilly, B., Castro, J.M., and Mount-Campbell, C.A. *Journal of Integrated Design & Process Science*, 7:1 (2003)
5. Castro, C.E., Bhagavatula, N., Cabrera-Rios, M., Lilly, B., and Castro, J.M., *ANTEC Proceedings* (2003)
6. Castro, C.E., Cabrera-Rios, M., Lilly, B., and Castro, J.M., *ANTEC Proceedings* (2004)
7. Castro, C.E., Cabrera-Rios, M., Lilly, B., and Castro, J. M., *Journal of Polymer Engineering*, accepted for publication (2005)