Design of a Stewart Platform for General Machining Using Magnetic Bearings

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Presented are the details of the design of a Stewart Platform, or hexapod, to be used for generalized machining. Because of the nature of the platform, six degree of freedom machining is possible with a fairly large range of motion, and thus is applicable to a relatively large suite of manufacturing tasks including drilling, grinding, milling, welding and others. Further, the platform is inherently well suited for precision positioning due to its stiffness. This stiffness can be significantly enhanced by the use of magnetic bearings. Together, these features make the design of this platform attractive as a precise positioning tool for machining purposes.

Stewart Platforms

The Stewart Platform is useful to study because it is a widely accepted design for a motion control device, largely due to its wide range of motion and accurate positioning capability. It provides a large amount of stiffness enabling the system to provide a significant source of positional certainty.

Kinematics of the Stewart platforms are well known and provide a framework for this advanced design. The Stewart platform involves six actuators to provide six degrees of freedom motion of an end effector while providing clear workspace to the tool. The Stewart Platform was originally designed as a flight simulator but it is also commonly used for automotive, transportation, and machine tool technology. The Stewart Platform design is also used for positioning of satellite communication dishes and telescopes and in applications such as shipbuilding and bridge construction.

The kinematics and the kinetics of the platform indicate the use of magnetic bearings as an actuation technique for fine and precise positioning while gross platform positing is accomplished via hydraulic systems.

By allowing general machining tasks via the Stewart platform, reduced number of part re-positionings or cell changes need be accomplished. This saves not only manufacturing time but also supervision time for the task. More accurate machining and reduced vibration will greatly reduce the number of tool changes required due to lower levels of tool wear and also tool breakage. Flexibility in the system by allowing more than one machining task enhances the value of the system.

The design of the platform captures these key advantages in an industrially relevant device for general machining in a manufacturing environment. It is estimated that the cost of the platform is comparable to that of current more static manufacturing systems. Presentation of the design is intended as a proof of concept stage for prototype development and eventual machining devices.

Traditionally, a common method for designing the controller for the Stewart Platform required manipulating complicated equations that modeled the physical components used to solve the mechanical equations. Then, the engineer had to solve these equations using complex numerical integration techniques.

Magnetic Bearings

Magnetic bearings as actuators can allow extremely fast and responsive control of vibration and displacement in mechanical systems. Their drawback is their inherent instability and related need for active feedback control. In addition, in order to achieve high performance, a fully multivariable and well-deigned control needs to be in place. Offsetting this, are reduced need for lubrication and significant vibration control. Details of the control design are considered with respect to robustness and nominal performance achievement. Control is considered using various techniques including modern H-infinity methods and using structured singular values to account for unmodeled vibration modes.

Control Systems Design

One way to describe control system performance is in terms of the size of certain signals of interest. For example, the performance of a tracking system could be measured by the size of the error signal. The H_2 and H_∞ performance measures are presented as system norms whose usefulness for evaluating system performance is demonstrated given their signal interpretation. Controllers to achieve optimality with respect to these norms will be developed. The Riccati operator is introduced as algebraic Riccati equations, which are used to compute state space realizations for both H_2 and H_∞ optimal control laws. Optimization is introduced with the general H_2 problem statement and review of optimization theory in optimal control of linear time invariant systems with a quadratic performance criterion. Review of the general H_∞ -optimal synthesis theory is also treated together with the small gain theorem.

Actual engineering control problems invariably include a set of conflicting requirements on time response, frequency response, actuator activity, disturbance rejection, and robustness to changing conditions and plant uncertainty. The key to successful application of optimal control is to construct formal cost functions that tend to cause these genuine system requirements to be satisfied. Although the H_2 -norm may be a meaningful performance measure and although LQ theory can give efficient design compromises under certain disturbance and plant assumptions, the major limitation of the theory is the lack of formal treatment of uncertainty in the plant itself. The result is a controller adept at attenuating impulse disturbances but with potentially weak robustness characteristics. Much of modern control theory relies upon a mathematical model of the plant to be controlled. In the real world, there is always uncertainty in any such model; the actual response of the plant may be quite different from the assumed model. Furthermore the plant may change as it ages or as operating conditions vary. H_∞ optimization focuses on the minimization of system output energy to unknown, but bounded, energy inputs. H_∞ control can be formulated to give robust stability to model uncertainty. The H_∞ -optimal controller results in a highly robust system; however, one which can be notably deficient in providing optimal nominal performance.

The mixed H_2/H_∞ control design methodology reviewed in this paper, allows H_2 and H_∞ performance objectives to be addressed simultaneously. It establishes a link between H_2 and H_∞ -optimizations, allowing a trade-off between their objectives. In this manner it is desired to synthesize an admissible robust state feedback controller that minimizes an H_2 performance objective subject to a constraint on the H_∞ -norm of a different transfer function matrix.

There are many ways in which plant uncertainty descriptions and feedback design problems can be cast as optimization problems. It is very useful therefore to have a general framework into which any particular problem can be manipulated. All systems considered in this paper are modelled as finite dimensional linear time-invariant (FDLTI) operating in continuous time with a state space and transfer function representation in the Laplacian s given by:

$$G(s)=\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad G(s)=C(sI-A)^{-1}B+D$$

where A *in* $\Re^{n\times n}$, B *in* $\Re^{n\times m}$, C *in* $\Re^{l\times n}$ and D *in* $\Re^{l\times m}$, where $\Re^{i\times j}$ represents a real matrix of dimension $i\times j$. The system G is said to be stable if the state space matrix A has no eigenvalues in the closed right half-plane. The general framework to be used here will be referred to as the General Control Configuration (GCC).

The objective is to design a controller K, for the plant G such that the input/output transfer characteristics from the external input vector w to the external output vector z are desirable according to some engineering specifications. The exogenous input w, typically consists of command signals, disturbances and sensor noise; u is the control signal; the output to be controlled z, which customarily consists of the so-called "error" signals which are to be minimized in some sense to meet control objectives; and y the measured output. The system is described by

$$\begin{bmatrix} z \\ y \end{bmatrix}=G(s)\begin{bmatrix} w \\ u \end{bmatrix}=\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix}$$

$$u=K(s)y$$

with a state space realization of the generalized plant G given by

$$G=\begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

where $G_{ij}=C_i(sI-A)^{-1}B_j+D_{ij}$.

Both G and K are real rational and proper. K is constrained to provide internal stability, with state models of G and K defined. Then *internal stability* will mean that the states of G and K go to zero from all initial values when $w = 0$. Since controllers are restricted to being proper and real rational which are also stabilizable and detectable, these properties will be assumed throughout. Thus, the term controller will be taken to mean a controller that satisfies these properties. Controllers that have the additional property of being internally stabilizable will said to be admissible. The realization of the transfer matrix G is taken to be of the form

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix}.$$

Notice the special off-diagonal structure of D: D_{22} is assumed to be zero so that G_{22} is strictly proper; also, D_{11} is assumed to be zero in order to guarantee that the H$_2$ problem is properly posed. The following additional assumptions are made for the output feedback H$_2$ problem:

i) (A, B_1) is stabilizable and (C_1, A) is detectable;
ii) (A, B_2) is stabilizable and (C_2, A) is detectable;
iii) $D^T_{12}\ [\ C_1\ D_{12}\ ] = [\ 0\ I\ ]$.
iv) $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

The first assumption is made for a technical reason: together with ii) it guarantees that the two Hamiltonian matrices associated with the H$_2$ problem belong to *dom*(*Ric*). Assumption ii) is necessary and sufficient for G to be internally stabilizable. Assumption iii) implies that C_1x and $D_{12}u$ are orthogonal so that the penalty on $z = C_1x + D_{12}u$ includes a non-singular, normalized component on the control u. Assumption iv) is dual to iii) and concerns how the exogenous signal w enters G. The H$_2$ problem statement is then summarized as:

H$_2$ Problem: *The H$_2$ control problem is to find a proper, real rational controller K which stabilizes G internally and minimizes the H$_2$ –norm of the transfer matrix* $F_\ell(G,K)$ *in* $z = F_\ell(G,K)w$.

By Lemma 1, the Hamiltonian matrices

$$H_2 := \begin{bmatrix} A & -B_2B_2^T \\ -C_1^TC_1 & -A^T \end{bmatrix}, \quad J_2 := \begin{bmatrix} A^T & -C_2^TC_2 \\ -B_1B_1^T & -A \end{bmatrix}$$

belong to *dom*(*Ric*), and, moreover, $X_2 := Ric(H_2)$ and $Y_2 := Ric(J_2)$ are positive semi-definite. Define

$$F_2 := -B_2^TX_2\ , \quad L_2 := -Y_2C_2^T$$

and

$$A_{F_2} := A + B_2F_2, \qquad C_{1F_2} := C_1 + D_{12}F_2$$

$$A_{L_2} := A + L_2C_2, \qquad B_{1L_2} := B_1 + L_2D_{21}$$

$$\hat{A}_2 := A + B_2F_2 + L_2C_2$$

$$G_c(s) := \begin{bmatrix} A_{F_2} & I \\ C_{1F_2} & 0 \end{bmatrix}, \qquad G_f(s) := \begin{bmatrix} A_{L_2} & B_{1L_2} \\ I & 0 \end{bmatrix}.$$

The unique optimal controller is $K_{opt}(s) := \begin{bmatrix} \hat{A}_2 & -L_2 \\ F_2 & 0 \end{bmatrix}$

In practice, it is usually not necessary to obtain an optimal controller for the H$_\infty$ problem, and it is often computationally (and theoretically) simpler to design a suboptimal one. Let $\gamma_{\min}$ be the minimal value of $\|F_\ell(G,K)\|_\infty$ over all stabilizing controllers *K*. The problem considered in this subsection is the suboptimal H$_\infty$ control problem:

Suboptimal H$_\infty$ Problem: Given a $\gamma > \gamma_{\min}$, find all admissible *K* such that $\|F_\ell(G,K)\|_\infty < \gamma$.

For the GCC with assumptions i-iv, the H$_\infty$ solution involves two Hamiltonian matrices

$$H_\infty := \begin{bmatrix} A & \gamma^{-2} B_1 B_1^T - B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{bmatrix}, \qquad J_\infty := \begin{bmatrix} A^T & \gamma^{-2} C_1^T C_1 - C_2^T C_2 \\ -B_1 B_1^T & -A \end{bmatrix}.$$

There exists a stabilizing controller *K* such that $\|F_\ell(G,K)\|_\infty < \gamma$ if and only if the following three conditions hold:

i) $H_\infty \in dom(Ric)$ and $X_\infty := Ric(H_\infty) \geq 0$.

ii) $J_\infty \in dom(Ric)$ and $Y_\infty := Ric(J_\infty) \geq 0$.

iii) $\rho(X_\infty Y_\infty) < \gamma^2$

where *ρ(M)* is the spectral radius or maximum absolute valued eigenvalue of *M*. Moreover, when these conditions hold, one such controller is

$$K_{sub}(s) := \begin{bmatrix} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix}$$

where

$$\hat{A}_\infty := A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$$

$$F_\infty := -B_2^T X_\infty, \quad L_\infty := -Y_\infty C_2^T, \quad Z_\infty := (I - \gamma^{-2} Y_\infty X_\infty)^{-1}.$$

If full state information is available the design is simplified. Consider an open loop system transfer matrix

$$G_{SF/\infty}(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ I & 0 & 0 \end{bmatrix}$$

with the same assumptions inherited from the output feedback problem. There exists a stabilizing controller *K* such that $\|F_\ell(P,K)\|_\infty < \gamma$ if and only if $H_\infty \in dom(Ric)$ and $X_\infty := Ric(H_\infty) \geq 0$. The resulting suboptimal controller in this case is then

$$K_{sub}(s) := \begin{bmatrix} 0 & 0 \\ F_\infty & 0 \end{bmatrix}.$$

Summary

The problem of designing a general machining system with a fairly large operating space is addressed in a conceptual stage manner in this work.

Shown is the viability of a Stewart platform for this task and the associated issues of using such a device. These issues include kinematics and kinetics of the device and subsequent control. It is noted that the forward kinematics problem is relatively difficult while the inverse kinematics has a unique closed form solution for most operating conditions. This implies that the solution for a control is reasonable. Development of an optimal and robust control is considered in theory by presenting H_2 and H_∞-optimal control theory.

Progress towards manufacture of the device is ongoing.

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