Relay Tuning of Controllers in Machine Axes For High-Speed Tracking

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Abstract

This paper introduces a new PID tuning procedure based on a modified relay tuner and frequency response design techniques. This tuning procedure is intended for use on second order systems common in the motion control industry and emphasizes optimal bandwidth and high low frequency gain. Experimental results from a servo motor application are included.

1. Introduction

The proportional-integral-differential (PID) controller has remained a foundation for control algorithms in the majority of industrial controls and has proven [1,2] to provide excellent control of machines. Frequency response design techniques have been successfully used in high performance controller design for many years, and with focus on some basic concepts can be used in the design of simple but effective high-speed tracking controllers. These industry proven techniques still offer outstanding performance on modern, high performance machines if properly applied. While designing a superior PID controller for a known, well identified plant remains a relatively simple task, tuning a PID controller for an unknown, unidentified plant offers many difficulties. Many tuning methods have been developed to combat this complex task, but these methods tend to oversimplify the PID control and neglect many advantages of the PID controller, such as increased bandwidth potential and high low frequency gain. Many tuning algorithms often result in an inferiorly tuned PID controller.

Relay feedback tuning was introduced in the early 1980's by Aström and Hägglund [3]. Similar to the established Ziegler-Nichols tuning [4,5] relay tuning uses a limit cycle to find the ultimate frequency and corresponding ultimate gain. Within the past 20 years many variations to Aström and Hägglund's original relay tuning have been explored [7-11]. The focus of these efforts has been directed towards improved plant modeling and improved controller tuning. While much of this research has enhanced the performance and capability of relay tuning, not much attention has been paid to using the relay to find higher frequency dynamics beyond the frequency where the plant has a phase lag of 180 degrees. Even the most accurate tuning procedure will have difficulty producing a high performance PID controller for an electromechanical machine axis without obtaining some knowledge of these high frequency dynamics.

This paper introduces a modification to Aström's relay method that improves the tracking of PID controllers. This new tuning procedure utilizes frequency response design techniques and is intended for use on nominally second order systems common in motion control systems. The goal of this new tuning procedure is to produce a well-tuned PID controller with optimized system bandwidth and increased low frequency gain.

2. Control System

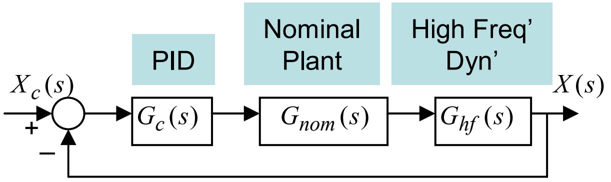

Figure 1: Control System Block Diagram

Figure 1 is a block diagram representation of the classical control loop considered in this work. G_c is a PID controller as given in (1). The nominal plant, G_{nom}, is a type one, second order system as given in (2). The plant will include higher frequency dynamics, G_{hf}, not captured in the nominal plant model. These dynamics are modeled as normalized dynamics with a DC gain of one. They may represent any number of effects including mechanical resonance and/or motor amplifier dynamics. In one the case study at the end of this paper they will represent the closed loop dynamics of the current control loop in the motor amplifier of a motor test stand , given in (3).

$$G_c = \frac{K_d\left(s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}\right)}{s} = \frac{K_d(s+z_1)(s+z_2)}{s} \qquad (1)$$

$$G_{nom} = \frac{K}{Js^2 + bs} \qquad (2)$$

$$T_i = \frac{\omega_n(s+z)}{z(s^2 + 2\zeta\omega_n s + \omega_n^2)} e^{-t_d s} \qquad (3)$$

The second order nominal plant dynamics in (2) are representative of numerous electromechanical machine axes. In combination with the higher frequency dynamics they represent the plant as given in (3a). The higher frequency dynamics represent the, usually unknown, limiting dynamics in a controller design.

$$G_p = G_{nom} G_{hf} \qquad (3a)$$

3. PID Design Philosophy

When designing for tracking performance with frequency response techniques there are two major attributes of the open loop gain to optimize: the 0 dB crossover frequency and low frequency gain. Optimizing both will result in a controller with low tracking errors. By driving these two quantities up the error response is driven down, as depicted in Fig. 2. Note that the error is the approximately the reciprocal of the open loop gain in regions where the gain is much greater than one and that the bandwidth of the closed loop system is approximately the crossover of the open loop gain. The crossover will be limited by the high frequency dynamics. These dynamics are not effectively controlled by PID and usually pose a crossover limit for stability. However, since the overall phase lead of PID is 90 degrees, it is possible to drive the crossover up near these higher frequency dynamics. If the phase lead of PID is not utilized then the bandwidth of the system may be much lower than is potentially possible.

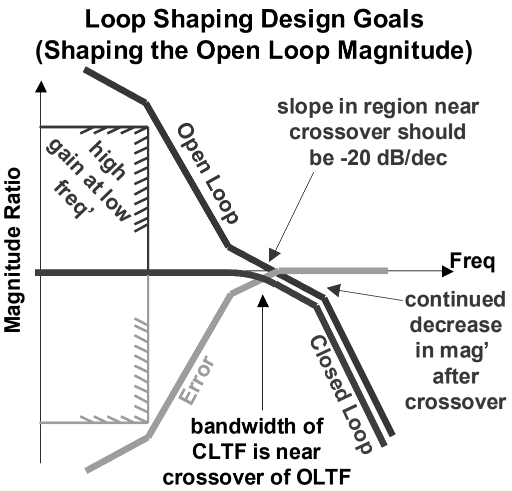

Figure 2: PID Design Goals

The two zeros of the PID controller help to increase this low frequency gain when they are placed at as high a frequency as possible. They should be placed near the crossover frequency, but far enough before crossover to pull the phase up for good phase margin at crossover.

An error bandwidth definition offers a more accurate measure of system performance in tracking than the standard closed loop transfer function (CLTF) bandwidth definition. Therefore throughout the analysis in this paper the -3 dB error bandwidth will be also used to describe controller performance. This is the frequency where the error rises to -3 dB from below.

4. Modified Relay Tuner

Standard relay tuning incorporates an relay in a closed loop with the plant, as shown in Figure 3. The relay will output a constant positive value for a positive error, *e*, and will output a constant negative value for a negative error input. This relay will force the type of plant considered in this work into a stable limit cycle. The plant's sinusoidal output, *X*, will be -180° out of phase with the relay signal, *U,* and will be related to the size of the relay output by the magnitude of the plant at that frequency.

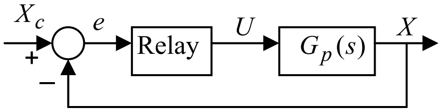

Figure 3: Standard Relay Tuner

This standard relay tuner will find a single frequency datum where the plant phase has dropped to -180°. This frequency is the ultimate frequency, ω_u, and the proportional gain required to achieve this crossover is the ultimate gain, K_u. The two poles of the nominal second order plant will bring the plant phase asymptotic to this -180°. It takes the contribution of higher frequency dynamics to drop the phase below -180°. The overall effect of a PID controller is to increase the open loop phase by +90°. Therefore, with a PID controller, a system can be controlled to a higher frequency than the ultimate frequency found by standard relay tuning. Using a standard relay tuner can severely limit the bandwidth and resulting performance of the system.

This paper describes a modification to the standard relay tuner. Because the limiting dynamics that drop the phase to -270° need to be identified, a derivative has been added after the plant in the relay tests. To help model the plant and refine gain calculations a variable delay has also been included after the relay. This allows multiple frequency response points to be identified. The new arrangement is shown in Figure 4.

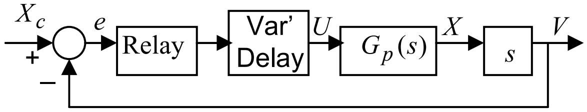

Figure 4: Modified Relay Tuner

With the variable time delay set to zero, the added derivative will increase the open loop phase by +90°, simulating the overall phase contribution of a PID controller. The plant output is now velocity, *V*, and the ultimate gain can be related to the derivative gain of the PID controller, K_d. The velocity output is -180° out of phase with the relay output, but this phase now includes -90° from the higher frequency dynamics. Using the discussed PID design philosophy, a PID controller can be tuned to achieve good tracking capabilities. It is important that the derivative be placed after the plant since, in any practical system, the theoretical impulse resulting from the derivative of the relay output will be limited by saturation. A real system will usually not oscillate if the derivative is placed after the relay. Also, in a practical system it is important to use the derivative approximation or velocity measurement that will be used in the final PID controller. This will account for any effects from this derivative calculation in the final system.

Resonances can pose problems for any relay tuning process. A resonance located near the ultimate frequency will provide inaccurate gain estimations from a single frequency response datum. Using a delay with the relay to obtain multiple frequency points helps in dealing with these problems. This tuning procedure utilizes a simple digital delay with the relay output.

5. Tuning Procedure

In this section the tuning process is presented as a four step procedure.

5.1 Step 1 (Implement the modified relay tuner)

The relay test is run with the derivative and no delay. The oscillation frequency of the plant output velocity, *V*, is the ultimate frequency, ω_u. The gain, K_u, at this ultimate frequency can be calculated from the ratio of the magnitudes of the plant output, *V*, and the relay output, *U*, as shown in (4). Although the relay output is a square wave, using a Fourier approximation, the fundamental component of this square wave is used to approximate the magnitude of the plant at the oscillation frequency.

$$K_u = \frac{4U}{\pi V} \tag{4}$$

5.2 Step 2 (Add delay)

To find plant gains at lower frequencies delays are added. With a time delay the closed loop system will oscillate at a different frequency, ω_j, and give an estimate of the plant magnitude at this frequency using (6).

$$K_j = \frac{4U_j}{\pi V_j} \tag{6}$$

The time delay is increased until a region of the plant magnitude plot is found where the slope is approximately -20 dB/dec. This ensures that the subsequent gain calculations are not affected by a resonant peak near the ultimate frequency.

5.3 Step 3 (Choose the crossover and PID zeros)

The ultimate frequency, ω_u, gives the limitations of the controllable system and where the uncontrollable, higher frequency dynamics begin to become significant. The open loop crossover, ω_c, is selected based on this ultimate frequency. The ratio between the crossover and the ultimate frequency is a measure of the aggressiveness of the controller. Table 1 lists suggested open loop crossovers for a range of designs:

Table 1: Bandwidth Ranges

Desired PID Controller	**Crossover (ω_c)**
Aggressive	$0.65\ \omega_u$
Midline	$0.3\ \omega_u$
Conservative	$0.1\ \omega_u$

In a commercial system implementing this tuner, a slider bar in a graphical user interface might be used at this step of the procedure.

The two zeros of the PID controller should be placed at a frequency, ω_z, relative to the selected open loop crossover frequency. As stated earlier, it is important for a stable, high performance controller that these zeros are placed at a frequency that offers a high gain at low frequencies while maintaining a good phase margin. This can be achieved by placing the zeros at approximately 1/10th of the selected crossover.

5.4 Step 4 (Calculate gains)

The PID gains can be calculated using the last collected delayed relay frequency, ω_j, and the corresponding gain, K_j, from Step 2. Equations (11) through (13) give the required formulae. Equation (11) assumes a -20 dB/dec slope in the region considered in the calculation and (12) and (13) can be found using (1) assuming both zeros are real and placed at ω_z.

$$K_d = \frac{\omega_c}{\omega_j} K_j \tag{11}$$

$$K_p = K_d (2\omega_z) \tag{12}$$

$$K_i = K_d (\omega_z^2) \tag{13}$$

6. Experimental Results

This tuning procedure has been tested on several systems including a motor stand with the nominal dynamics in (2) and higher frequency dynamics given by a motor amplifier modeled with (3). The stand consists of a brushless DC motor powered by a current controlled amplifier. A DSP card interfaces this stand with a PC and provides all of the I/O and control calculations.

In order to test the accuracy of using delay to find various frequency points and model the system, the modified relay tuner was run over a wide range of delays. These results were then compared to the experimentally obtained and theoretical open loop transfer function (OLTF) magnitude plots for the velocity output, V. Figure 5 compares the relay obtained data with these open-loop models.

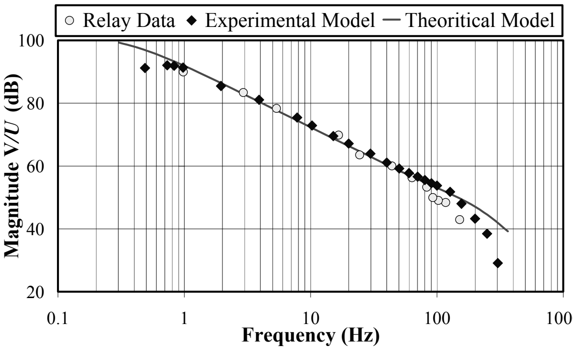

Figure 5: Motor Test Stand OLTF Models

As can be seen in Figure 5, the modified relay accurately models the system. At higher frequencies, poor velocity measurements lead to small inaccuracies in gain estimations with the relay. These can be seen occurring around 100 Hz in Figure 5. The small, fast oscillations in the velocity output of the tuner at these high frequencies can pose resolution problems, and obtaining accurate plant output magnitudes can become difficult. However, implementing the delay and modeling the plant helps to alleviate this problem. The controller gain calculations are based on a constant -20 dB/dec slope, accurately obtained in Figure 5, and are not based on single point relay identification.

A standard standard relay tuned (implementing Ziegler-Nichols rules) PID controller, and a designed PID controller were compared with a PID controller tuned using this modified relay tuning procedure. Figure 6 plots the experimentally obtained CLTF for the tuned controllers and designed controller.

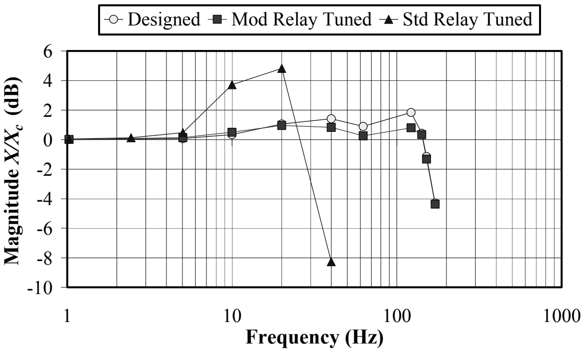

Figure 6: Motor CLTF Models

The modified relay tuned controller outperforms both the standard relay tuned and Ziegler-Nichols tuned controllers, both of which produce controllers with very large overshoot. With a bandwidth of approximately 160 Hz, the modified relay tuner performs very close to the designed controller.

Figure 7 compares the error bode plot for these same controllers. The modified relay tuner maintains very low error, keeping very close to the designed controller with an error bandwidth of 65 Hz. As predicted by the CLTF, the standard relay tuned controller has a significantly lower error bandwidth at about 15 Hz.

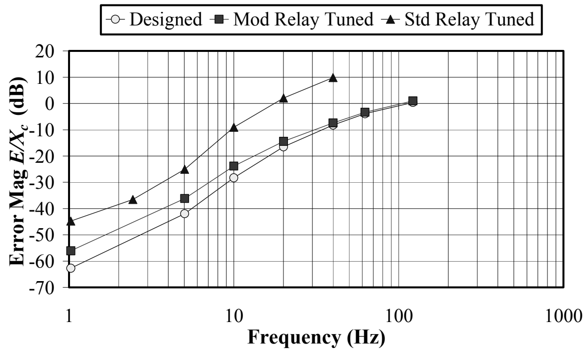

Figure 7: Motor Error Plots

To test the tracking capabilities of this tuned controller, a trapezoidal velocity profile can be used to shape the position command for the controller. This move profile consists of an acceleration portion, constant velocity portion, and deceleration portion as shown in Figure 8. The maximum error is recorded as the tracking error measure. It usually occurs at the point of maximum acceleration. Figure 9 contrasts the tracking error created during this s-curve move for these controllers on the motor test stand.

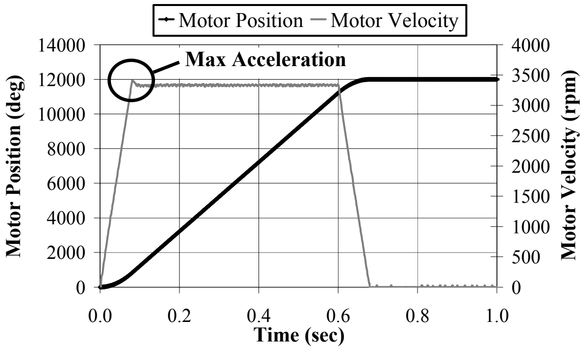

Figure 8: Typical S-Curve Move

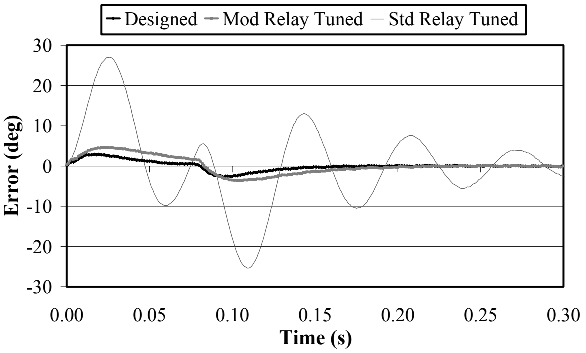

Figure 9: Tracking Error

The modified relay tuned controller performs very close to the designed controller with comparable tracking error. The standard relay tuned controllers provide undesirable levels of tracking error compared to the designed controller for this s-curve profile. Table 2 summarizes the performance of each tuned PID controllers and the designed PID controller.

Table 2: Controller Performances

	Std. Relay Tuned	**Mod. Relay Tuned**	**Designed**
Bandwidth	*33 Hz*	*160 Hz*	*165 Hz*
Error Bandwidth	*15 Hz*	*65 Hz*	*72 Hz*
Tracking Error	*27 deg*	*4.7 deg*	*2.9 deg*

7. Conclusions

Standard relay tuning tends to produce low performance PID controllers for nominally second order systems. The modified relay tuning procedure introduced in this paper provides a solution to this inherent problem with the standard relay. This tuning procedure is intended for use on PID controlled second order systems, such as those commonly encountered in motion controls. It has been shown through experimental results to provide a high performance PID controller comparable to a well designed controller.

8. Acknowledgements

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