

# Exploratory study on MIMO control for motion systems

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**Abstract**— This presentation gives an overview of the results of an exploratory study on multivariable feedback and feedforward control for electromechanical motion systems. Industrial cases are presented where multivariable control may be beneficial. Multivariable control problems due to plant interaction and coupling in external disturbances and performance parameters are studied. Tools are presented to quantify multivariable properties in practical multivariable systems. Also, it is demonstrated how scalar manual loopshaping techniques may be extended to handle multivariable control problems.

## I. INTRODUCTION

Electromechanical multivariable motion systems are common practice in today's industry. Typical applications are XY-tables, vibration isolator tables, robotics and many master-slave positioning devices. Most industrial systems are statically decoupled and controlled with decentralized (a set of independent scalar) controllers. As modern control software offers the possibility to implement multivariable controllers and specifications become tighter, industry has a strong desire to exploit multivariable control design freedom where possible. At the same time, it is desired from an engineering point of view to develop physical insight into properties of industrial multivariable systems. This study gives an overview of motion control applications where multivariable control may be beneficial. Scalar, frequency response based, loopshaping techniques, see e.g. [10], are extended to handle many common multivariable control problems.

In multivariable systems, more design freedom is available than in conventional scalar systems as aside from algebraic and analytical tradeoffs, spatial tradeoffs may be possible, see Figure 1. Also, bandwidth limitations as flexible modes may be attacked using redundancy of actuators and sensor pairs, see [9]. In this work, special attention is paid to specific multivariable properties in a generalized plant control configuration, Figure 2. Herein,  $G$ ,  $K$ ,  $G_d$ ,  $G_z$  denote the plant, controller, disturbance model and performance model respectively.  $r$  is the reference trajectory and  $u$  the control output.  $d_w$  is a white noise disturbance, leading to an output disturbance  $d_o$ .  $z$ ,  $e$ ,  $y$  are the performance variables, servo

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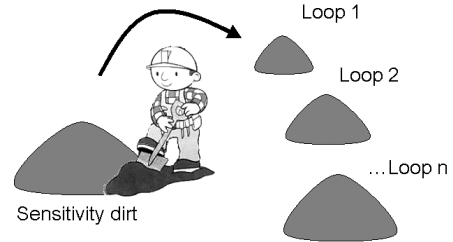


Fig. 1. Artist impression of the MIMO waterbed effect

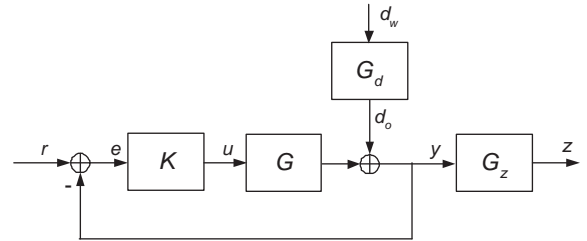


Fig. 2. Multivariable control configuration used in this work. Multivariable properties of  $G$ ,  $G_d$  and  $G_z$  must be considered in the design of  $K$

error and controlled variables respectively.

When interaction is present in the generalized plant, non-diagonal design of the controller  $K$  may be desired. Hence, multivariable properties of the plant, external disturbances and specific performance objectives are discussed.

## II. PLANT INTERACTION

In most multivariable applications, interaction of plant dynamics are the first reason to consider multivariable control. This is because, under decentralized control, closed loop stability may be harmed when the openloop transfer is not diagonal dominant around the bandwidth, [7]. Also, it is not straightforward to derive closed stability properties from frequency response data when the open loop transfer function is not diagonal dominant. In this case, direct extensions of scalar loopshaping techniques are cumbersome and often lack physical insight.

In common industrial motion control problems, kinematic relations are used to decouple the motion system in rigid body modes. Decoupling in this rigid body approach often

works satisfactory as flexible modes are designed as stiff as possible in mechatronic design leading to dominant rigid body behavior in low frequencies. However, flexible modes still limit achievable bandwidth.

With some particular actuator and sensor configurations, a mixed manual-modal decoupling procedure facilitates independent control of flexible modes. As these modes are controlled independently with scalar techniques, bandwidth may be placed above resonance frequencies. Hence, performance may be increased or structural design requirements may be relaxed.

In literature, an extensive number of measures are presented to quantify interaction in plant dynamics, see e.g. [4], [6]. One of these measures is the Relative Gain Array (RGA) which can be evaluated per frequency on frequency response data. The RGA is defined as, see also [6];

$$\Lambda(G) \doteq G * (G^{-1})^T \quad (1)$$

Where  $G$  is a (complex) transfer function matrix evaluated for a particular frequency and ‘\*’ denotes element wise multiplication. A scalar expression of the RGA can be found using;

$$\Gamma_{RGA}(G) = \|\Lambda(G) - I\|_{sum} . \quad (2)$$

This *RGA-number* gives a quick insight in the quality of the decoupling method. When the RGA-number is small, the system is well decoupled. As the RGA-number increases, more interaction is present. For a  $2 \times 2$  mechanical motion system, the RGA number is plotted before and after rigid body decoupling, see Figure 3. This shows that decoupling

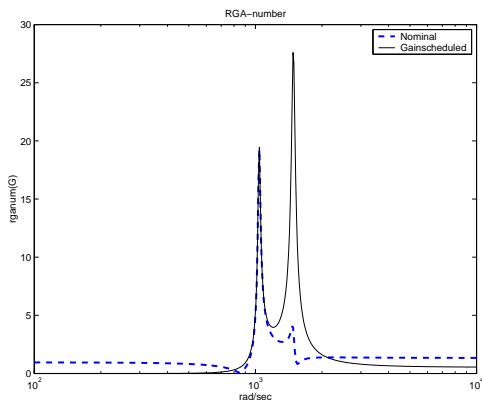


Fig. 3. RGA number  $\Lambda_{RGA}(G)$  plotted for a  $2 \times 2$  mechanical motion system, before and after rigid body decoupling.

the open loop transfer function does not come without costs. When flexible modes are coupled, low frequency modal contributions may lead to tracking problems and cross-talk. A multivariable jerk derivative feedforward controller may then be applied to improve tracking performance, see Figure 9,

Figure 10 and [2]. Also, it is experienced that disturbances may be distributed among various controlled variables. Hence directionality properties of the disturbance are altered. This may lead to degradation of performance.

### III. DISTURBANCE INTERACTION

A new area, quite unexplored, is the use of the multivariable character of disturbances. In many practical applications disturbances are highly coupled as they often relate to the same underlying physical process, e.g. floor vibrations, pumps, reaction forces on metro frames, etc. [1]. A multivariable property of these disturbances is that they often have a fixed direction, the ratio in which they are distributed among controlled variables. A pragmatic method can be used to reconstruct disturbances from servo error measurements during standstill. The directionality of this reconstructed disturbance can then be incorporated in multivariable control design. In this case, interaction in the open loop transfer function is introduced in order to handle interaction in the disturbance model(!).

An industrial example was studied where all (decoupled) controlled axes suffered from the same 24Hz output disturbance. Triangular control was applied using directionality properties of this disturbance. The effect of this disturbances could be reduced with a factor 2, see Figure 4. Note that herein, solely multivariable control design freedom is used. Hence, there are no associated SISO costs using this technique.

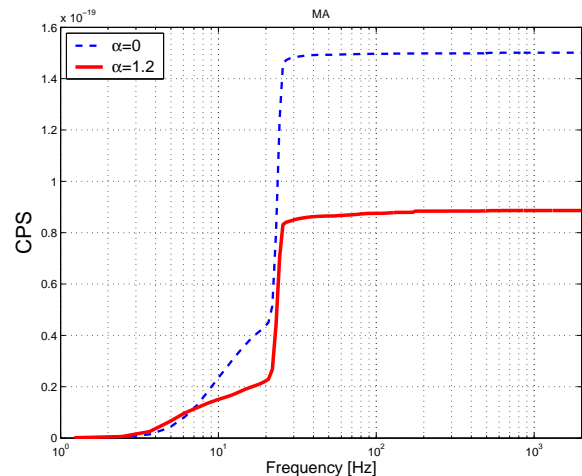


Fig. 4. Cumulative power spectrum (CPS) of the Moving average filtered servo error in  $y$ -axis. The disturbance at 24Hz is effectively compensated using triangular control ( $\alpha = 1.2$ ).

### IV. PERFORMANCE OBJECTIVES

In many industrial applications, performance is not evaluated at the location of the measured variables. When disturbances are not collocated with control inputs and performance

variables are not collocated with measured variables, the sensitivity function,  $S_o = (I + GK)^{-1}$ , may not describe performance/stability tradeoffs, see [3]. This also holds for the scalar system depicted in Figure 5. From engineering

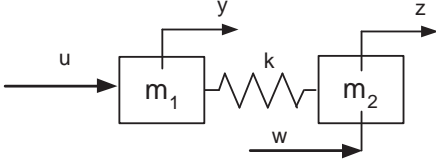


Fig. 5. Scalar plant example where  $\det(\tilde{G}) \neq 0$ ,  $m_1 \neq m_2$

insight, it is known that bandwidth is not necessarily limited by the flexible mode. However, bandwidth will not be placed above the anti-resonance frequency because of performance objectives at the location of  $z$ . This can easily be explained considering the generalized plant of this scalar example, given as;

$$\begin{bmatrix} z \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix}}_{\tilde{G}} \begin{bmatrix} w \\ u \end{bmatrix} \quad (3)$$

Where  $z$ ,  $y$ ,  $w$ ,  $u$  are the performance variables, controlled variables, exogenous disturbances and control outputs respectively. It is assumed that all elements of  $\tilde{G}$  are non-zero. When feedback control is applied, the transfer function from the exogenous disturbances to the performance variables can be calculated as;

$$T_{zw} = G_{zw} + G_{zu}K(I + KG_{yu})^{-1}G_{yw}. \quad (4)$$

Where from now on,  $T_{zw}$  is called the *closed loop disturbance response*. Improvement compared to no feedback is accomplished when  $T_{zw}$  is smaller than the open loop disturbance response  $G_{zw}$ . Note that  $T_{zw} = G_{zw}S$  when  $\det(\tilde{G}) = 0$ . A measure on 'when to use' the sensitivity function is given in [3]. This measure is defined as;

$$\Gamma = \frac{G_{zu}G_{yw}}{G_{zw}G_{yu}} \quad (5)$$

When  $\Gamma$  is close to unity,  $\det(\tilde{G})$  is close to zero, hence  $T_{zw} \approx G_{zw}S$ . In Figure 6,  $\Gamma$  is plotted per frequency for the plant in Figure 5. It is visible that due to rigid body behavior  $T_{zw} \approx G_{zw}S$  in lower frequency ranges. In mid and high frequencies however,  $z$  does not follow  $y$  anymore hence  $\Gamma \neq 1$ . Therefore the sensitivity function  $S$  does not describe the disturbance response at these frequencies. This is supported by the engineering insight not to increase bandwidth above the anti-resonance behavior of the plant.

In a multivariable sense, location of performance may also lead to non-diagonal design of  $K$ . An industrial example is depicted in Figure 7. As in this particular case, the location of  $z$  introduces one-sided coupling, a triangular control design of  $K$  is sufficient.

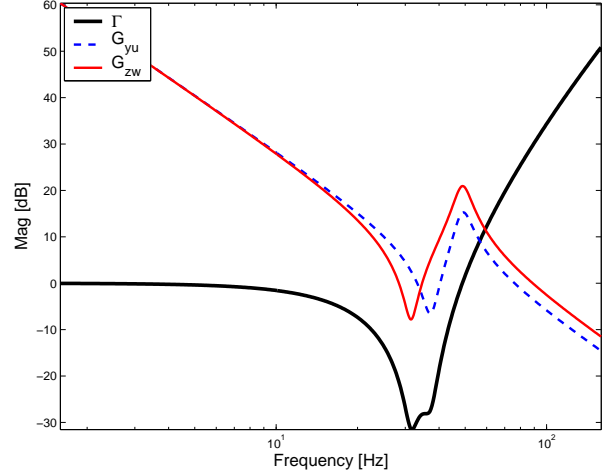


Fig. 6.  $\Gamma$  plotted for the scalar plant example.  $G_{yu}$  denotes the plant,  $G_{zw}$  the open loop disturbance response.

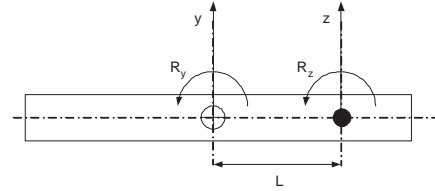


Fig. 7. Rigid body motion system where performance location does not coincide with center of gravity.

## V. MISO/SIMO CONTROL

Systems with only one multivariable input or output are fundamentally more easy to design than square multivariable systems. This is because

$$\det[I + GK] = \det[I + KG] \quad (6)$$

holds. Hence, stability can be evaluated at the loop breaking point where the transfer function is scalar. Therefore, many powerful extensions of scalar loopshaping techniques still provide valuable engineering insight, see e.g. [5],[8]. A useful property of MISO/SIMO systems is that bandwidth limiting effects (e.g. flexible modes) can be made unobservable. Although flexible modes are not attacked in this manner, still the possibility rises to increase bandwidth, see Figure 8. As scalar frequency response loop shaping techniques can be used, MISO/SIMO systems offer a relatively simple way to use the power of multivariable control in a practical environment.

## VI. CONCLUSIONS

It is illustrated that multivariable control design can be beneficial in many commonly encountered industrial motion control applications. Apart from handling bandwidth limitations (e.g. flexible modes), spatial design freedom

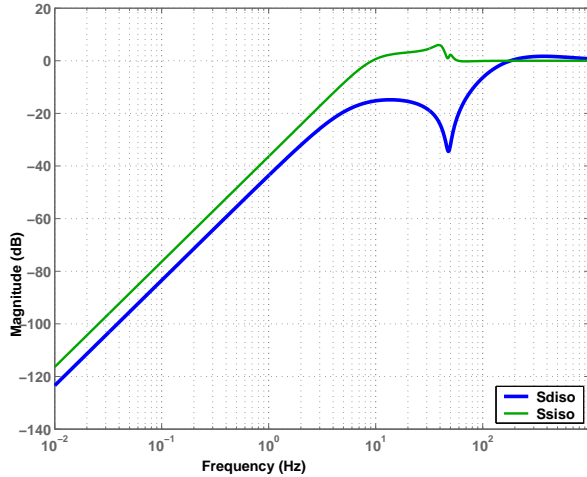


Fig. 8. Sensitivity of SISO design ( $S_{SISO}$ ) and DISO PQ design ( $S_{DISO}$ ).

may be exploited to handle interaction in the generalized plant. Therefore interaction due to disturbances, performance objectives and (the more conventional) plan interaction can be accounted for, often without SISO costs.

Future work will focuss on incorporating results from model based techniques in industrial multivariable motion control. In particular, research will be conducted on methods that exploit multivariable properties of disturbances in multivariable feedback control design.

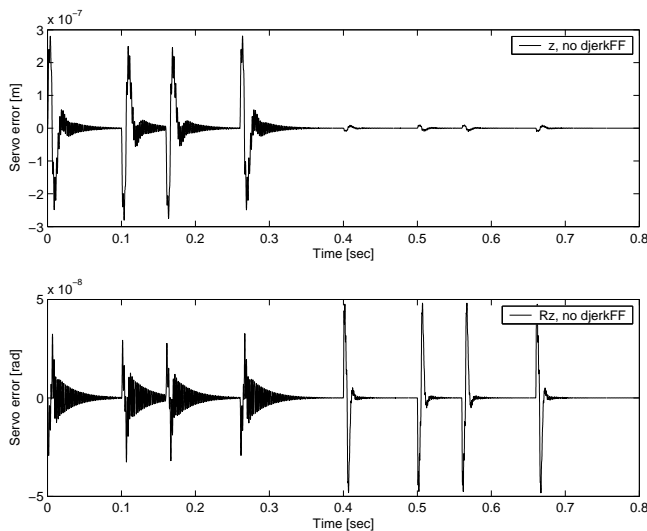


Fig. 9. Servo error in  $z$  axis (top) and  $R_z$  axis (bottom) during motion in  $z$  axis and  $R_z$  axis subsequently. Only acceleration feedforward is used. Due to low frequency modal contributions, tracking errors appear during jerk phases of a motion. As these modal contributions are coupled, cross-talk occurs.

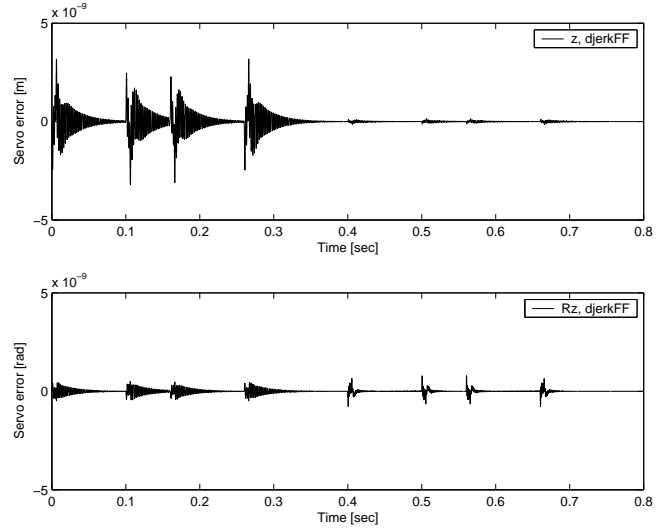


Fig. 10. Servo error during motion in  $z$  axis and  $R_z$  axis subsequently using MIMO jerk derivative feedforward. The coupled low frequency modal contributions are compensated for, resulting in a significant reduction of the tracking error (also less cross-talk). Residual tracking errors are caused by excitation of resonance dynamics (as jerk derivative feedforward does not invert high frequency resonance modes).

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