

# SUBMICROMETER FRICTION COMPENSATION USING VARIABLE-GAIN SLIDING MODE CONTROL

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## ABSTRACT

The paper discusses a sliding mode control suitable for compensation of nonlinear microdynamic friction and parameter changes in a ball-screw driven slide system. The conventional, fixed-gain sliding mode control has a limited range of performance in the presence of varying nonlinear friction in sub-micrometer trajectory tracking. In this work, an algorithm that effectively calculates variable switching gain based on the observation of parameter variation and friction disturbance is proposed. To verify the effectiveness of the proposed algorithm, the comparison with the conventional slide mode control is presented and experimentally verified. It is shown, from the result of this work, that a variable switching gain was critically important in compensating for varying nonlinear friction in the sub-micrometer motion range for ball-screw driven systems.

## 1. INTRODUCTION

Friction in mechanical system is a nonlinear phenomenon that complicates the control design process and reduces the positioning and tracking accuracy in motion control applications. Much work has been done to characterize and compensate for the nonlinear friction behavior (Armstrong-Helouvry et al., 1994).

As the requirement in positioning and tracking accuracy becomes more stringent, a need for a model that accurately predicts the behavior of friction in microdynamic range (usually below 1.0 micrometer for precision ball-screw systems) became apparent. Futami et al. identified the three dynamic regions (microdynamic, transition, and macrodynamic) of friction characteristic, using a single-axis stage mechanism and achieved nanometer positioning by designing a controller around each region (Futami et al., 1990). In Ro and Hubbel's work (1993), a piecewise linear friction model taken from a hysteretic torque-displacement curve was used in devising a model reference adaptive control of a ball-screw driven system both in microdynamic and macrodynamic regions. Nanometer positioning under static loads was also achieved using a parallel mechanism of lead-screw driven table fitted with a secondary voice coil actuator (Awabdy, Shih, and Auslander, 1998). In the same work, the

microdynamic behavior was modeled by a modified Dahl friction relationship and mainly dealt with a secondary voice coil actuator that has good linear input-output characteristic.

Canudas et al. proposed a new nonlinear analytic friction model that captures most of the frictional behaviors that have been observed experimentally (de Wit et al., 1995). The behaviors include Coulomb friction, Stribeck effect, hysteresis, and spring-like characteristic under microdynamic stiction region. Canudas and Lischinsky illustrated the application of adaptive friction compensation on a DC motor servomechanism based on their model (de Wit and Lischinsky, 1997). Submicrometer positioning and tracking for a ball-screw driven slide system was achieved by compensating for the friction using the new friction model (de Wit et al., 1995), with good results (Ro, Shim, and Jeong, in press; Awabdy, Shih, and Auslander, 1998).

For a typical mechanical system, inertial parameters are known only to a certain degree, and the change of actual friction is very difficult to be modeled accurately. In the current research, an algorithm that effectively calculates the switching gain based on observation of parameter variation and external disturbance is proposed. Nonlinear friction is modeled based on the friction model (de Wit et al., 1995), and sliding mode control with an effective algorithm to vary the switching gain is proposed for positioning and tracking for a ball-screw driven slide system. Performances of the proposed algorithm are verified through experiments.

The paper is organized as follows: In section 2, the experimental setup and preliminary tests for identifying the friction behavior of the system are described. The corresponding dynamic model of the system is described based on the friction behavior. Then, Section 3 shows the derivation of the sliding mode control based on the system model and the Canudas' friction model. The experimental results verifying the performance of the proposed control algorithm are shown in Section 4. The conclusions are then followed in Section 5.

## 2. SYSTEM MODEL

A simple conceptual model for the system was developed in Figure 1 that shows the idealized model of the mechanical components of the system. In the current system setup, as seen in Figure 1, the slide position,  $x_2$ , is the only state measured by a laser interferometer. The slide velocity,  $\dot{x}_2$ , is gathered digitally by first order difference of  $x_2$ . The nut position and its velocity,  $x_1$  and  $\dot{x}_1$ , are not measurable. The built-in tachometer can be used for measuring the angular velocity of the ball-screw but the signal output is very noisy. The tachometer signal is usually good for motor speeds orders of magnitude greater than that used in submicrometer motion. The ball-screw rotation and its angular velocity,  $\theta$  and  $\dot{\theta}$ , are thus estimated. The unmeasurable state variables,  $x_1$  and  $\dot{x}_1$ , are estimated by a Kalman filter (Ro, Shim and Jeong, in press).

The significant parameters for the dynamic system are listed in Table 1.

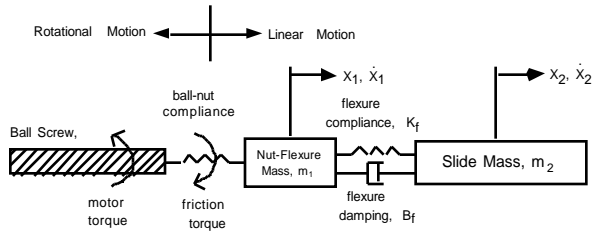


Fig. 1. Conceptual model of mechanical components in the system

The rotational dynamics of ball-screw may be describes as

$$(J_{rot} + (m_1 + m_2)l_p^2) \frac{\ddot{x}_2}{l_p} + F(\cdot) = u(t) \quad (1)$$

where  $J_{rot}$  is the total rotational inertia of ball-screw and motor,  $l_p$  is the lead pitch of the ball-screw,  $F(\cdot)$ , is an equivalent frictional torque, and  $u(t)$  is input torque to the system. Since the frictional torque on the ball-screw dominates the system and the motor torque transmission mechanism has high stiffness, the following reduced-order (nominal) model for the rotational motion is investigated for the controller design:

$$J\ddot{\theta}(t) + F(\cdot) = u(t) \quad (2)$$

where  $J$  is an effective inertia of ball-screw, motor, and equivalent rotational inertia of the linear masses;  $F(\cdot)$  is the equivalent friction torque caused by motor brushes, support bearing, and nut interfaces; and  $u(t)$  is the motor torque generated by a current controlled servo-amplifier driven by voltage command from a DSP.

Table 1. Experimental system parameters

$J_{rot}$	Rotational Inertia	$5.94 \cdot 10^{-5} \text{ Kg} \cdot \text{m}^2$
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$M_1$	Nut-Flexure Mass	0.653 Kg
$M_2$	Slide Mass	11.79 Kg
$l_p$	Lead Pitch	$7.958 \cdot 10^{-4} \text{ m/rad}$
$K_f$	Flexure Stiffness	$7.1 \cdot 10^6 \text{ N/m}$
$B_f$	Flexure Damping	41.59 N/(m/sec)
$K_{Nut}$	Nut Rigidity	$8.828 \cdot 10^7 \text{ N/m}$

For the electromechanical system shown, friction in the ball-screw and nut interfaces is the major nonlinear disturbance that complicates the control design process. The static friction parameters can be estimated from the friction-velocity plot measured during constant velocity motions under a simple PI control. At each discrete point in Figure 2, the corresponding control voltage was measured under control velocity command. By incrementing the velocity command, a well-known static friction profile was obtained as shown in Figure 2. However, as indicated by Figure 2, the control voltage that would initiate the slide displacement has a bias toward the positive-input direction. Figure 3 shows there is a small displacement due to the control voltage of 100 mV, much below the threshold value of 400 mV as shown in Figure 2. Such spring-like behavior appears to result from elastic deformation of the contact patches on the balls (Futami et al., 1990; Ro and Hubbel, 1993). However, as the applied voltage increases, a friction breakaway is observed and the apparent slide displacement begins to occur under open-loop control. The friction behavior of this system is to be modeled as the bristle type friction proposed by (de Wit et al., 1995):

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} \sigma_0 z \quad (3)$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

where  $\sigma_0$  is the stiffness,  $\sigma_1$  is a damping coefficient, and  $\sigma_2$  is a viscous friction coefficient. Friction parameters for controller implementation were estimated based on the experimental result (Ro, Shim and Jeong, in press).

### 3. SLIDING MODE CONTROL WITH VARIABLE SWITCHING GAIN

The inertial parameter of the system is not perfectly known. Thus, the equation of motion is developed based on the nominal values of the system.

$$(\hat{J} + \tilde{J})\ddot{\theta} + F = u \quad (4)$$

where  $\hat{J}$  is the nominal inertia,  $\tilde{J} = J - \hat{J}$  is the estimation error, and  $F$  is the equivalent friction torque on the system.

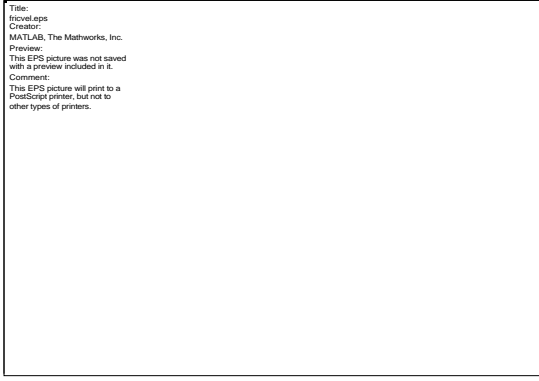


Fig. 2. Friction-Velocity map

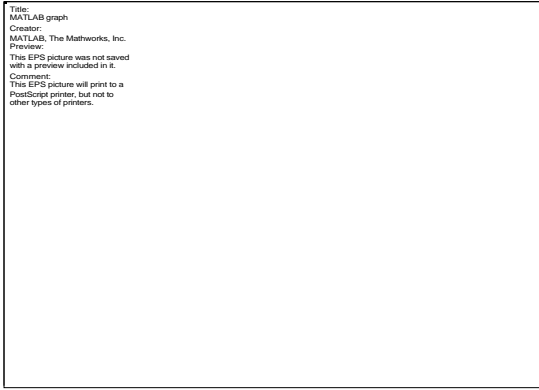


Fig. 3. Open-loop response to 100mV input

In order to have the system track the desired trajectories at the same time, a sliding surface,  $S$ , is defined as the weighted sum of the tracking error,  $e$ , and its derivative:

$$S = \hat{J} \left( \dot{e} + 2\lambda e + \lambda^2 \int e dt \right), \quad (5)$$

where  $e = \theta_d - \theta$ ,  $\dot{e} = \dot{\theta}_d - \dot{\theta}$ , and  $\ddot{\theta}_d$  is the desired reference displacement twice differentiated. While in sliding mode, the dynamics can be written as  $\dot{S} = 0$ . By solving the equation (5) formally for the control input, an expression so-called the equivalent control,  $u_{eq}$ , is obtained from the following procedure.

From equation (5),

$$\begin{aligned} \dot{S} &= \hat{J} \left( \ddot{e} + 2\lambda \dot{e} + \lambda^2 e \right) \\ &= \hat{J} \left( \ddot{\theta}_d - \ddot{\theta} \right) + \hat{J} \left( 2\lambda \dot{e} + \lambda^2 e \right) \\ &= \hat{J} \ddot{\theta}_d - \left( u - F - \tilde{J} \ddot{\theta} \right) + \hat{J} \left( 2\lambda \dot{e} + \lambda^2 e \right) \end{aligned} \quad (6)$$

The equivalent control,  $u_{eq}$ , that would achieve  $\dot{S} = 0$  is thus

$$u_{eq} = \hat{J} \ddot{\theta}_d + \hat{F} + \hat{J} \left( 2\lambda \dot{e} + \lambda^2 e \right) \quad (7)$$

where  $\hat{F}$  is the friction estimate based on the bristle type friction model.

In order to satisfy sliding condition despite the uncertain dynamic disturbance,  $F$ , a term

discontinuous along the surface,  $S = 0$ , is added to  $u_{eq}$ , and overall sliding mode control law is proposed as follows:

$$u = u_{eq} + K \operatorname{sgn}(S) \quad (8)$$

where  $K$  denotes the switching gain, used to guarantee sliding on the surface,  $S(t)$ .

From the sliding condition,

$$\frac{1}{2} \frac{d}{dt} S^2 = S \dot{S} = 0 \quad (9)$$

substituting equations (6), (7), and (8) into (9) gives

$$\begin{aligned} S \dot{S} &= \left[ \left( \tilde{J} \ddot{\theta} + \tilde{F} \right) - K \operatorname{sgn}(S) \right] S \\ &= \left( \tilde{J} \ddot{\theta} + \tilde{F} \right) S - K |S| = 0 \end{aligned} \quad (10)$$

where  $\tilde{F} = F - \hat{F}$  is the friction estimate error.

To maintain stability and guarantee certain performance bounds, the switching control gain in equation (10) must satisfy the following condition:

$$K \left| \tilde{J} \ddot{\theta} + \tilde{F} \right| \quad (11)$$

That is, the switching gain must be larger than the absolute bound related to friction disturbance estimate error and parameter uncertainties. Typically, in the design process, it is critical to choose a suitable set of values for control parameters, such as switching gain,  $K$  and weighting factor,  $\lambda$ , to guarantee good performance. Because of the nonlinear nature of the problem, there is no straight-forward method to determine these values. And trial-and-error is usually the presiding norm (Liu and Handroos, 1999). In this work, an algorithm that effectively calculates switching gain based on the observation of changing inertia and friction estimation error is proposed.

Since a friction model that estimates the friction disturbance is obtained as in (de Wit et al., 1995; Ro, Shim, and Jeong, in press), the control for the nominal system model is calculated using,

$$\hat{J} \ddot{\theta} + \hat{F} = u_m \quad (12)$$

The control,  $u_m$ , for the system model is directly calculated from (12), where  $\ddot{\theta}$  is calculated as  $\ddot{\theta}(k) = \frac{\dot{\theta}(k) - \dot{\theta}(k-1)}{T}$ , and  $T$  is a sampling period.

The bound for switching gain can be obtained by subtracting equation (12) from equation (2), which results in

$$\tilde{J} \ddot{\theta} + \tilde{F} = u - u_m \quad (13)$$

$$K \left| u - u_m \right| \quad (14)$$

Therefore, if the switching gain is selected as the difference between the control input to the system and the calculated control input based on the nominal model (12), the value of switching gain changes according to the parameter variation and friction estimation error. In practical control implementation, there is always a computational delay in a digital control environment. The control signals can be easily stored at each sampling period. So, the real implementation signal of equation (13) is given by

$$u(t) = u(t-T) - u_m(t-T) \quad (15)$$

where  $T$  is the sampling period. A strictly positive constant,  $\eta$ , can be added to the equation (14) to account for the unstructured dynamics to equation (2) and decrease the time constant for reaching the sliding surface. Then, the sliding condition (9) results in (Slotine and Li, 1991),

$$\frac{1}{2} \frac{d}{dt} S^2 = S \dot{S} - \eta |S| \quad (16)$$

The control algorithm is serviced at a fast rate of 5 kHz. Therefore, the controlled system using equation (15) performs very close to the ideal case of using equation (13). The approach suggested here is inspired by (Ishiguro et al, 1992; Hsia and Jung, 1995).

#### 4. EXPERIMENTAL RESULTS

The proposed controller was verified for positioning and tracking motion control for the ball-screw driven slide system. The tracking performance of the proposed control algorithm for 1  $\mu\text{m}$ , 2 Hz sinusoidal tracking command is shown in Figure 4, and its corresponding control voltages during the implementation are shown in Figure 5. The control input does not have high frequency adverse effect of chattering. The variable switching gain of the proposed control algorithm is also gathered during implementation and is plotted in Figure 6. The measured control voltages illustrate that, under the proposed control, the control voltages have the approximate positive bound of 0.4 Volt while the negative bound is -0.3 Volt, which are less than the values shown in static friction-velocity relationship of Figure 2. In the work of (Iwasaki and Matsui, 1996; Iwasaki et al., 1999), it is claimed that the nonlinear friction behaved as the simple Coulomb friction under a dynamic table drive motion. However, the result in Figure 5 proves that the nonlinear friction under high precision motion control behaves as the bristle-type friction. The positioning performance of the proposed controller was tested for 1  $\mu\text{m}$  step command, and its response is plotted in Figure 7. The response shows smooth transient behavior and no steady-state error. The step response for 2.5  $\mu\text{m}$  is plotted in Figure 8. The proposed control algorithm shows consistent performances for tracking and positioning.

When the steady-state error is examined from Equations (4), (7), and (8), it can be rewritten as:

$$\hat{J} (\ddot{e} + 2\lambda\dot{e} + \lambda^2 e) = (\ddot{\theta} + \tilde{F}) - K \text{sgn}(\dot{e}) \quad (17)$$

With the proposed control algorithm, switching gain  $K$  is adjusted to compensate for parameter variation and friction estimation error. For instance, the sign of  $S$  in the tracking control is positive as seen in Figure 10, and adjusted switching gain  $K$  also has positive values seen in Figure 6. With the method in equation (13), the resulting error dynamics (17) can be rewritten as:

$$\ddot{e} + 2\lambda\dot{e} + \lambda^2 e = 0 \quad (18)$$

The resulting steady-state error is expected to have a critically damped behavior. The step command responses show the critically damped behavior as seen in Figures 7 and 8.

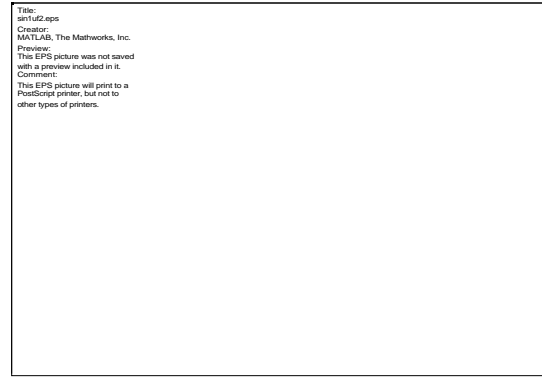


Fig. 4. Tracking performance of the proposed sliding mode control algorithm for 1  $\mu\text{m}$ , 2 Hz command (dashed : reference, solid : experiment)

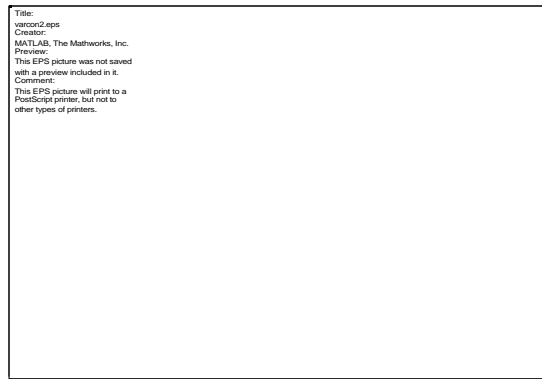


Fig. 5. Control voltages of proposed sliding mode controller with variable switching gain

In comparing with the conventional sliding mode, the appropriate selection of a fixed switching gain was not easy. Therefore, the averaged value of variable switching gains,  $K=0.1$ , was selected from the result of Figure 6. The performance of the conventional sliding mode controller with the selected switching gain value of 0.1 is shown for 1  $\mu\text{m}$ , 2 Hz sinusoidal tracking command in Figure 9. The response shows poor behavior of tracking lag and steady state error. The response also shows poor transient behavior at the start of motion. The corresponding control voltages are shown in Figure 10. The highly biased responses in Figure 9 are attributed to the control peaks in Figure 10 when the control gets close to the friction threshold voltage, seen in Figure 2. The result suggests the separate switching gain for either direction be used for further improvement in conventional sliding mode control for the current experimental system. The response for the step command of 1  $\mu\text{m}$  is shown in Figure 11 with the same switching gain of 0.1. The step response for 2.5  $\mu\text{m}$  command is shown in Figure 12. As in the tracking case, the responses show poor transient behavior and steady state error. For the

increased gain of 0.2, experimental studies did not show any performance improvement.

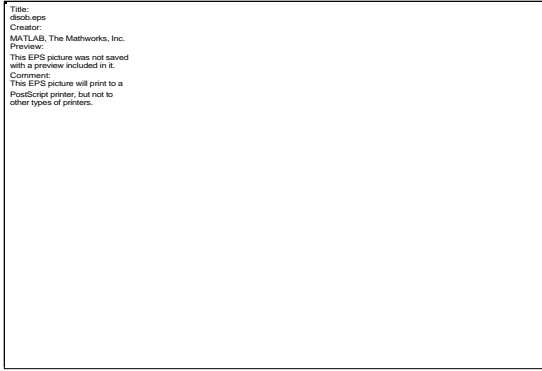


Fig. 6. Variable switching gain during the proposed control implementation

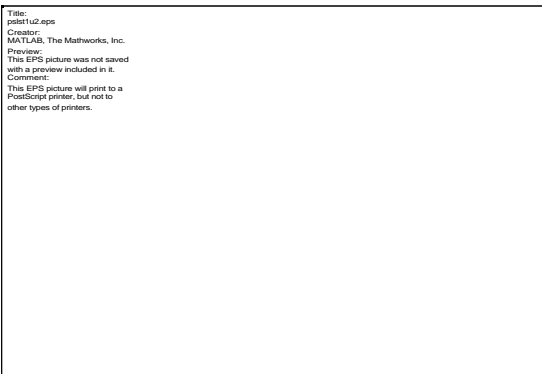


Fig. 7. Positioning response of the proposed controller for 1  $\mu\text{m}$  step command

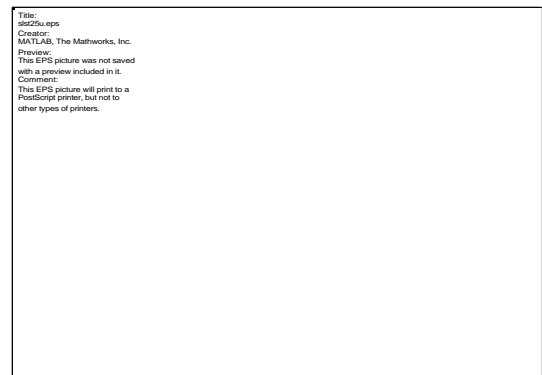


Fig. 8. Positioning response of the proposed controller for 2.5  $\mu\text{m}$  step command

The control voltages of the conventional sliding mode control shown in Figure 10 can be compared with those in Figure 5. The varying switching gain in Figure 6 shows the compensation for the friction estimate error and inertial effect related to uncertain parameters. The positively biased values of the switching gain can also be attributed to the biased behavior of friction relationship of Figure 2.

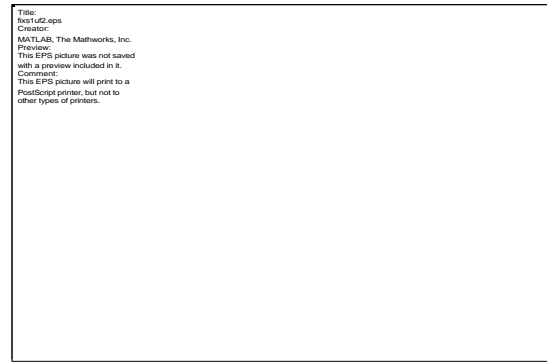


Fig. 9. Tracking performance of conventional sliding mode control for 1  $\mu\text{m}$ , 2 Hz command (dashed : reference, solid : experiment)

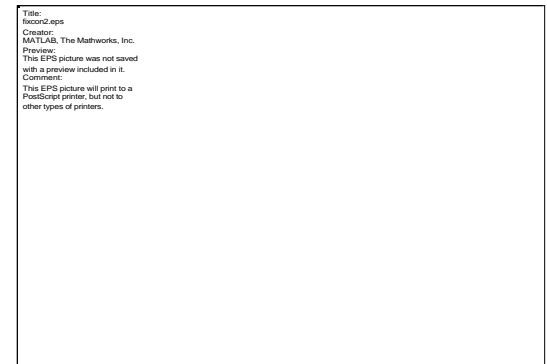


Fig. 10. Control voltages of conventional sliding mode control

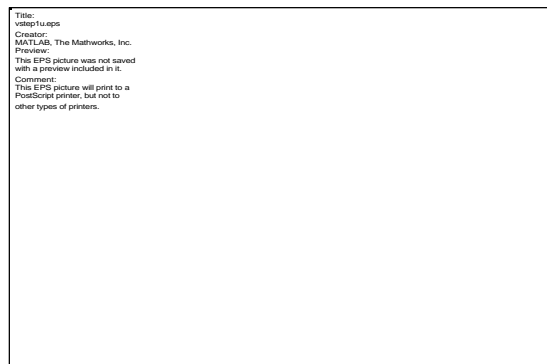


Fig. 11. Positioning response of the conventional sliding controller for 1  $\mu\text{m}$  step command

## 5. CONCLUSION

To enhance the control performance of a ball-screw driven slide system, a sliding mode controller with a variable switching gain based on the observation of parameter change and friction estimation error was proposed. Using the proposed algorithm, control action chatter was eliminated. Experimental results verified the effectiveness of the proposed controller compared with that of a fixed-gain conventional sliding mode control. The proposed controller demonstrates consistent performances with

positioning and tracking motion control of the ball-screw driven slide system. The robustness issue of the proposed controller with large disturbance is under investigation.

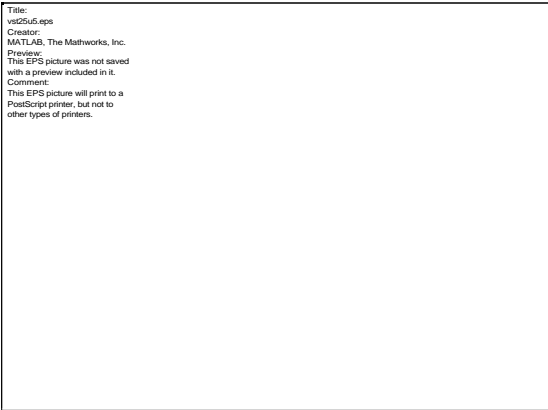


Fig. 12. Positioning response of the conventional sliding controller for 2.5  $\mu\text{m}$  step command

## REFERENCES

- Armstrong-Hélouvy, B., P. Dupont, and C. Canudas de Wit (1994). A survey of models, analysis tools and compensation methods for the control of machines with friction. *Automatica*, vol. 30, no. 7, pp. 1083-1138.
- Awabdy, B.A. W-C. Shih, and D. M. Auslander (1998). Nanometer positioning of a linear motion stage under static loads. *IEEE/ASME Trans. Mechatronics*, vol. 3, no. 2, pp. 113-119.
- de Wit, C.C, et al. (1995). A new model for control of systems with friction. *IEEE Trans. Auto. Contr.*, vol. 40, no. 3, pp. 419-425.
- de Wit, C.C. and P. Lischinsky (1997). Adaptive friction compensation with partially known dynamic friction model. *Int. J. Adaptive Control and Signal Processing*, vol. 11, pp. 65-80.
- Fujimoto, Y. and A. Kawamura (1995). Robust servo system based on two degree-of-freedom control with sliding mode. *IEEE Trans. on Ind. Electr.*, vol. 42, no. 3, pp. 272-280.
- Futami, D., A. Furutani, and S. Yoshida (1990). Nanometer positioning and its micro-dynamics. *Nanotechnology*, vol. 1, pp. 31-37.
- Gutierrez, H.M. and Ro, P.I. (1998). Sliding-mode control of a nonlinear-input system. *IEEE Trans. on Ind. Elec.*, vol. 45, no. 6, pp. 921-927.
- Hsia, T.C. and S. Jung (1995). A simple alternative to neural network control scheme for robot manipulator. *IEEE Trans. on Ind. Electr.*, vol. 42, no. 4, pp. 414-416.
- Ishiguro, A. et al. (1992). A neural network compensator for uncertainties of robotics manipulators. *IEEE Trans. on Ind. Electr.*, vol. 39, no. 6, pp. 565-570.
- Iwasaki, M. and N. Matsui (1996). Observer based nonlinear friction compensation in servo drive system. *Int. Workshop on Advanced Motion Control-AMC*, vol. 1, pp. 344-348.
- Iwasaki, M., T. Shibata, N. Matsui (1999). Disturbance observer-based nonlinear friction compensation in table drive system. *IEEE/ASME Trans. on Mechatronics*, vol. 4, no. 1, pp.3-8
- Kawamura, A., H. Itoh, and K. Sakamoto (1994). Chattering reduction of disturbance observer based sliding mode control. *IEEE Trans. on Ind. Appl.*, vol. 30, no. 2, pp. 456-461.
- Lee, H.S. and M. Tomizuka (1996). Robust motion controller design for high-accuracy positioning systems. *IEEE Trans. Ind. Electr.*, vol. 43, no. 1, pp. 48-55.
- Liu, Y. and Handroos, H. (1999). Sliding mode control for a class of hydraulic position servo. *Mechatronics*, vol. 9, pp.111-123.
- Ohnishi, K. (1987). A new servo method in mechatronics. *Trans. Japanese Society of Electrical Engineers*, vol. 107-D, pp. 83-86.
- Ro, P. I., Shim, W., and Jeong, S. (2000). Robust friction compensation for submicrometer positioning and tracking for a ball screw driven slide system”, **Precision Engineering**, Jour. of the Int’l societies for Precision Engineers and Nanotechnology, Vol. 24, No.2, pp 160-173
- Ro, P.I. and P. I. Hubbel (1993). Model reference adaptive control of dual-mode micro/macro dynamics of ball screws for nanometer motion. *ASME J. Dyn. Sys. Meas. Contr.*, vol. 115, pp. 103-108.
- Slotine, J.-J.E. and W. Li (1991). *Applied Nonlinear Control*, Prentice Hall.