

AC Motor Control in Precision Machines

Michael K. Liebman and David L. Trumper

Massachusetts Institute of Technology
Cambridge, MA 02139

Abstract

The field orientation principle allows us to control AC motors as if they were DC motors. AC motors, such as synchronous and induction motors, have multiple phases and complex transient behavior, which make them difficult to control. DC motors are simple to control: the armature current is directly proportional to the electric torque. The field orientation principle transforms stator currents for an AC motor into field currents, which are analogous to the currents in a DC motor. In this abstract we review the derivation of the field orientation transformations and show how they can be used to control AC motors.

Introduction

The ease with which DC motors can be controlled is due to the fixed arrangement of the main flux and armature flux. The main flux is directed between the two field windings of the DC motor and is also called the direct axis flux. The armature flux is oriented 90 electrical degrees from the main flux and is also therefore called the quadrature-axis flux. The fixed 90° angle between the direct-axis flux and the quadrature-axis flux produces maximum torque. In DC motors the mechanical commutator does the hard work of maintaining these two axes perpendicular to each other as the rotor rotates. The result is an easily controllable motor. In AC motors, we perform the commutation electrically, without the aid of a mechanical commutator. If we can achieve the same effect in AC motors, with the direct and quadrature axes always perpendicular, they will also be easily controllable.

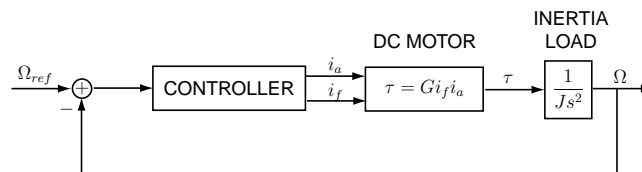


Figure 1: Standard control block diagram showing a DC motor driving a load.

Figure 1 shows a standard control block diagram for controlling the angular position Ω of a load attached to a separately excited DC motor. We assume that high bandwidth current controllers directly set the motor currents. The torque τ produced is proportional to the field current i_f and armature current i_a , as developed in many controls textbooks, $\tau = G i_f i_a$. Thus it seems we can control the motor torque by adjusting either the field current i_f , or armature current i_a , or perhaps both. Typically, a DC motor is operated at maximum field current, and the armature current is adjusted to control torque. A DC motor using permanent magnets instead of field windings to create the main flux can be thought of as a separately excited DC motor with constant field current i_f . Thus, a permanent magnet DC motor is operated only by changing the armature current.

Field Orientation Transformations

In this section we derive the field orientation transformations which are used to simplify AC motor control. The derivation presented here draws upon Leonhard's excellent book, *Control of Electical Drives* [2], and uses similar notation. Another good reference is *Electric Machinery* [1].

We consider a rotary AC motor with a cylindrical cross-section. We will examine the case of a two-pole motor with symmetrical three-phase windings. The three stator currents, $i_{Sa}(t)$, $i_{Sb}(t)$, $i_{Sc}(t)$, may vary in any way provided that they are balanced,

$$i_{Sa}(t) + i_{Sb}(t) + i_{Sc}(t) = 0. \quad (1)$$

Thus, only two of the three currents can be set independently.

We now define $\mathcal{F}(\alpha, t)$ as the number of ampere-turns enclosed by a closed loop magnetic flux path which crosses the air gap radially at an angle α from the a-axis at time t . We will also define an angle γ that represents the offset angle for the other motor phases: for a three-phase motor $\gamma = 120^\circ$. We will model the spatial ampere-turns distribution of each phase as sinusoidal and centered on the phase's magnetic axis. This yields an expression for the ampere-turns of a closed loop flux path crossing the air gap at an angle α ,

$$\mathcal{F}(\alpha, t) = N [i_{Sa}(t) \cos \alpha + i_{Sb}(t) \cos(\alpha - \gamma) + i_{Sc}(t) \cos(\alpha - 2\gamma)]. \quad (2)$$

We can simplify the math by using complex numbers, for example, by substituting

$$\cos \alpha = \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}), \quad (3)$$

and by introducing a time-dependent complex current vector,

$$\vec{i}_S(t) = i_{Sa}(t) + i_{Sb}(t)e^{j\gamma} + i_{Sc}(t)e^{j2\gamma} = i_S(t)e^{j\zeta(t)}, \quad (4)$$

where $i_S(t)$ and $\zeta(t)$ are the time-dependent magnitude and phase of the complex current vector $\vec{i}_S(t)$. Using equations (2), (3) and (4) we can write the ampere-turns wave $\mathcal{F}(\alpha, t)$ as

$$\mathcal{F}(\alpha, t) = \frac{1}{2}N [\vec{i}_S(t)e^{-j\alpha} + \vec{i}_S^*(t)e^{j\alpha}] = Ni_S(t) \cos(\zeta(t) - \alpha). \quad (5)$$

Equation 2 expresses the ampere-turns wave as a superposition of sinusoidal waves centered on the three magnetic axes with magnitudes determined by the three phase currents, i_{Sa} , i_{Sb} , and i_{Sc} . Equation 5 says that the same ampere-turns wave can also be thought of as a single sinusoidal wave with instantaneous magnitude $i_S(t)$ whose peak is instantaneously located at an angle $\zeta(t)$ from the a-axis.

At this point, it should be stressed, that the phase currents can vary in any arbitrary manner provided they are balanced. Although we are using a complex vector $\vec{i}_S(t)$, we have not specialized to steady state operation, and our complex quantities should not be mistaken for complex phasors used to describe steady state oscillation.

As an example of the formalism we have developed, let's consider the special case of steady-state rotation of the stator magnetomotive force at speed ω . If we feed the stator with balanced, three phase currents,

$$i_{Sa}(t) = I \cos(\omega t), \quad (6)$$

$$i_{Sb}(t) = I \cos(\omega t - \gamma), \quad (7)$$

$$i_{Sc}(t) = I \cos(\omega t - 2\gamma), \quad (8)$$

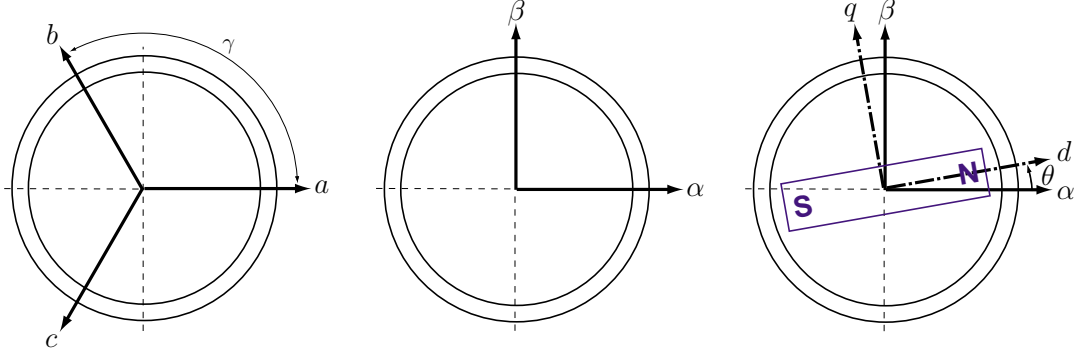


Figure 2: The abc axes form a three-phase AC system fixed in the stator frame. The $\alpha\beta$ axes represent an equivalent two-phase AC system fixed in the stator frame. The dq axes represent an equivalent two-phase DC system that rotates with the rotor flux.

equation (4) tells us that $i_S(t) = \frac{3}{2}I$ and $\zeta(t) = \omega t$. Thus the magnetomotive force wave $\mathcal{F}(\alpha, t)$ has a fixed magnitude, $\frac{3}{2}NI$, and rotates at constant angular velocity ω .

We are now ready to develop the field orientation transformations. The transformations will allow us to express the magnetomotive force and magnetic fields in the motor as two orthogonal components that rotate with the axis of the rotor's flux. We define the direct axis or d-axis to be aligned with the rotor's flux and the quadrature axis or q-axis to lead the direct axis by 90° . The d- and q-axes are shown on the right in Figure 2. For a motor with a permanent magnet rotor, the d-axis is aligned with the permanent magnet and rotates with the rotor.

Figure 2 shows three equivalent frames for describing the stator currents. The field orientation transformations allow us to switch between frames. The $abc \leftrightarrow \alpha\beta$ transformation converts between a three-phase AC system with fixed axes abc and a two-phase AC system with fixed axes $\alpha\beta$. It can be derived by equating the complex current vector $\vec{i}_S(t)$ in the abc frame to the complex current vector in the $\alpha\beta$ frame,

$$i_{S_a} + i_{S_b}e^{j\frac{2\pi}{3}} + i_{S_c}e^{j\frac{4\pi}{3}} = i_{S_\alpha} + ji_{S_\beta}. \quad (9)$$

Equation (9) together with the condition for balanced currents, equation (1), yields the $\alpha\beta \rightarrow abc$ transformation,

$$i_{S_a} = (2/3)i_{S_\alpha} \quad (10)$$

$$i_{S_b} = -(1/3)i_{S_\alpha} + (1/\sqrt{3})i_{S_\beta} \quad (11)$$

$$i_{S_c} = -(1/3)i_{S_\alpha} - (1/\sqrt{3})i_{S_\beta}. \quad (12)$$

The $\alpha\beta \leftrightarrow dq$ transformation converts between a two-phase AC system with fixed axes $\alpha\beta$ and a two-phase DC system with rotating axes dq . The dq system corresponds to a DC motor, and in steady state its quantities are constant. We define θ as the angle of rotation of the dq frame from the $\alpha\beta$ frame as shown in Figure 2. The $\alpha\beta \leftrightarrow dq$ transformation can be derived by equating $\vec{i}_S(t)$ in the fixed $\alpha\beta$ frame and in the rotating dq frame,

$$i_{S_\alpha} + ji_{S_\beta} = (i_{S_d} + ji_{S_q}) e^{j\theta}. \quad (13)$$

The $dq \rightarrow \alpha\beta$ transformation is

$$i_{S\alpha} = i_{Sd} \cos \theta - i_{Sq} \sin \theta \quad (14)$$

$$i_{S\beta} = i_{Sd} \sin \theta + i_{Sq} \cos \theta. \quad (15)$$

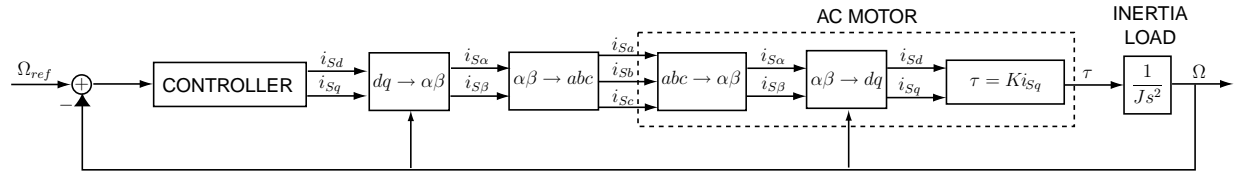


Figure 3: Block diagram of AC motor control in field coordinates. The transformations inside the AC Motor block are part of the motor model.

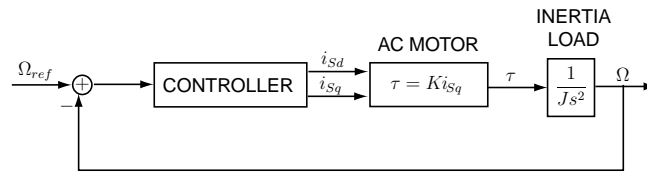


Figure 4: Controller design in field coordinates is analogous to DC motor controller design.

The field orientation transformations reduce the controller design procedure for AC motors to the familiar procedure for DC motors as shown in Figures 3 & 4. Since d-axis flux and q-axis current in an AC motor correspond to the main flux and armature current in a DC motor, the AC motor torque is proportional to i_{Sq} , $\tau = K i_{Sq}$. For an AC motor with a permanent magnet rotor, the rotor flux is fixed and there is no point trying to change it with stator currents, so we set $i_{Sd} = 0$. The controller outputs desired torque which is proportional to q-axis current, i_{Sq} . Then, the $dq \rightarrow \alpha\beta$ and $\alpha\beta \rightarrow abc$ transformations convert i_{Sq} and $i_{Sd} = 0$ into 3-phase currents. The motor model, however, inherently contains the inverse field orientation transformations which turns 3-phase currents into mechanical torque. Thus, for the purposes of controller design, the forward and reverse transformations cancel each other and may be omitted as in Figure 4. The controller for the AC motor can now be designed as if it were controlling a DC motor.

References

- [1] A. E. Fitzgerald, Jr. Charles Kingsley, and Stephen D. Umans. *Electric Machinery*. McGraw-Hill, Inc., New York, 1990.
- [2] Werner Leonhard. *Control of Electrical Drives*. Springer, New York, 1997.