Tachometers, Do We Need Them?

Daniel B. DeBra, AA, Stanford University, CA 94305

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Introduction

To introduce the numbers that are involved consider a carriage of mass m with a direct forcing control and negligible friction. If one nanometer is the desired rms value of the noise nx that would be contributed by a position sensor one can talk about the equivalent velocity noise that could be allowed from a velocity sensor before it became the principal disturbance. Now compare a proportional plus derivative feedback with one that uses a velocity sensor for an inner loop and the same proportional gain for the outer loop. In both cases consider the velocity gain to be large enough that the natural behavior is critically damped and the roots are at Cp/Cd =100 and Cm/m = 1000 rad/sec. The position sensor noise has unity gain to 1000 rad/sec in the first case while at higher frequencies it is attenuated with a minus one slope. By contrast with the first case the position noise starts to be attenuated at 100 rad/sec and decreases with a minus 2 slope above 1000 rad/sec. Unfortunately there is a new noise source from the velocity sensor. Its gain at low frequencies is Cd/Cp. For the velocity noise nv to be small it must be nv<<nx Cp/Cd = 1e-9*1e2=1e-7 m/sec. Or say the velocity noise should be 0.1 of that then for a capstan drive with radius 1 cm we would need a rotational noise less than 1e-6 rad/sec. This is achievable but is not an inexpensive tachometer. It corresponds to about 5 revolutions per year and represents a high quality instrument that may be larger than the motor on which it is mounted. But even in this primitive case it allows one to attenuate the noise from the position sensor above the Cp/Cd frequency. The response of the system to a disturbance on the carriage is the same for both cases. Its gain is 1/Cp and reduces with frequency with a minus one then a minus two slope at Cp/Cd and Cd/m respectively. These slopes in the attenuation may be more or less important for real colored noise.

Life becomes more interesting when the elasticity of the capstan shaft, the contact compliance and the stretch and bending of the draw bar are considered in the capstan driven case. The inertia of the capstan drive and the carriage interact as a fourth order system. The position sensor needs to be on the functional part--the carriage. So the control is not collocated with the sensor leading to some problems in stability if high gain is needed to reduce the effects of disturbances. This is where the introduction of a second sensor on the capstan drive becomes important especially if the disturbance such as friction is associated with the capstan drive e.g. brush friction on a motor. High gains about the capstan drive are now possible with a tachometer both because it is collocated and because it is now a first order system.

Given that velocity information is needed the question is then can one achieve the angular velocity information from an encoder or do we need to depend on the demonstrated precision of the classical tachometer? DTM3 [Bryan 1980], had tachometers on its capstan drives that provided a resolution of 1 rev/5 yrs. With a rapid traverse that was limited by the slew rate of the laser interferometers of the day to a few cm/sec the dynamic range offered by the tachometer was from 3e-8 to 10 rad/sec or greater than 9 orders of magnitude. Furthermore to maintain the resolution of 1e-9 m the orientation equivalent for the 1 cm radius capstan is 1e-7 rad or 60 million bits per revolution. If discrete encoders are used one must do the best job possible to derive the velocity information from this sequence of discrete measurements.

Discrete differentiation

There are a number of ways of approximating the derivative given a sequence of discrete values of a variable. The z-transform can be used to find the coefficients for weighting values of the sequence when the samples are measured at uniform time intervals, T. To develop a control for PD (proportional plus derivative), the Laplace transform, (Cp+Cd*s)*y can be approximated as Cp*y0+Cd*(y0-y1)/T or (Cp+Cd/T)*y0-Cd*y1/T using the definition of a derivative. By zero-pole matching, Cd*(s+Cp/Cd)=>Cz*((z-z1)/(z-z2)) where z2 can be 0 to match the first form or -1 and z1=exp(s1*T) where s1=-Cp/Cd and matching gains at s=0 and z=1, Cz=Cp((1-z2)/(1-z1)). For small T this gives same result as the first method. But neither provides much flexibility in design to minimize the error in approximating the derivative. It is easier to compare methods if the derivative is calculated separately. If the sequence of measured values is fitted with a polynomial

y=a0+a1*t+a2*t^2+....

and t=0 is the current value then the derivative is just the coefficient a1. For t=0, the value measured is y0, for t=-T, y1 et c. for a sampling period T. Given m measurements and a polynomial of degree p-1, for m>=p one can fit the data by least squares. M*a=y , where

M=[1 0 0...; 1 -T -2T...; 1 (-T)^2 (-2T)^2...; et c.]

then for e=M*a-y, minimizing the J=e'*e gives a=(M'*M)\M'*y. But we only need a1 so only the second row of the matrix is of interest. For the unweighted case this second row V contains 1/T. For example for some m and p up to 6 and 4 respectively, V(m,p)=Vmp/T with the Vmp given below (the first few are easily checked by hand)

```
V22 =
  1.0000e+00  -1.0000e+00
V42 =
  3.0000e-01   1.0000e-01  -1.0000e-01  -3.0000e-01
V33 =
  1.5000e+00  -2.0000e+00   5.0000e-01
V43 =
  1.0500e+00  -6.5000e-01  -8.5000e-01   4.5000e-01
V63 =
  5.8929e-01  -3.5714e-03  -3.2857e-01  -3.8571e-01  -1.7500e-01   3.0357e-01
V44 =
  1.8333e+00  -3.0000e+00   1.5000e+00  -3.3333e-01
V55 =
  2.0833e+00  -4.0000e+00   3.0000e+00  -1.3333e+00   2.5000e-01
V65 =
  1.8188e+00  -2.6772e+00   3.5450e-01   1.3122e+00  -1.0728e+00   2.6455e-01
```

One may wish to weight the values closest to time t=0 to preserve the bandwidth and respond more quickly to sudden changes. Then the function to minimize is J=e'W'We which gives the standard result for a=(M'W'WM)\(M'W'W)*y. Again only the second row is of interest and as before, once the sampling time is known the matrix is determined and gives a constant row vector which need only be calculated once at the start.

When the measurements are not weighted, the zeros of the z-transfer function all occur on the unit circle. Increased weighting of current values brings the zeros (except the one at +1) closer to the center of the unit circle.

Figure 1 shows the effect of weighting:

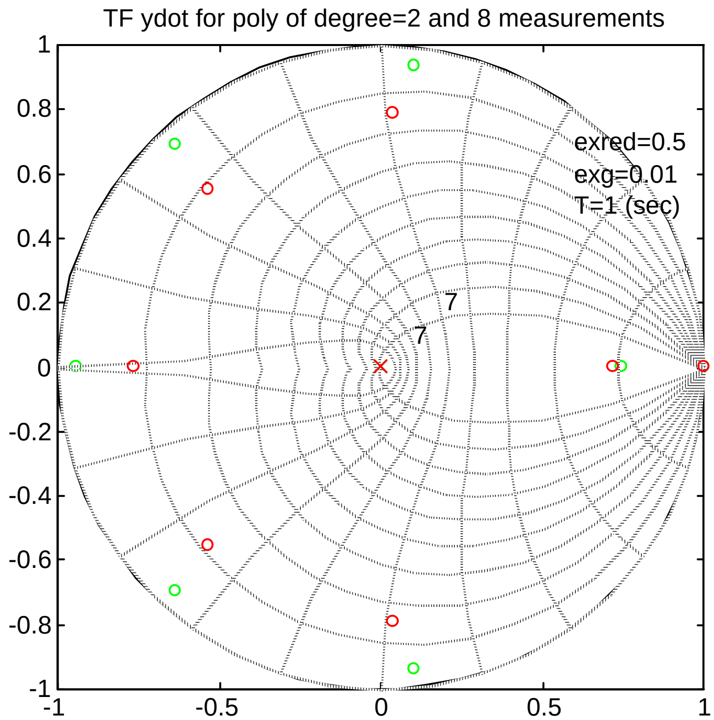

Figure 1, Weighting brings the extra zeros closer to the origin

The polynomial fitted transfer function spreads the zeros around compared to the classical filter thereby preserving the phase, see Fig. 2.

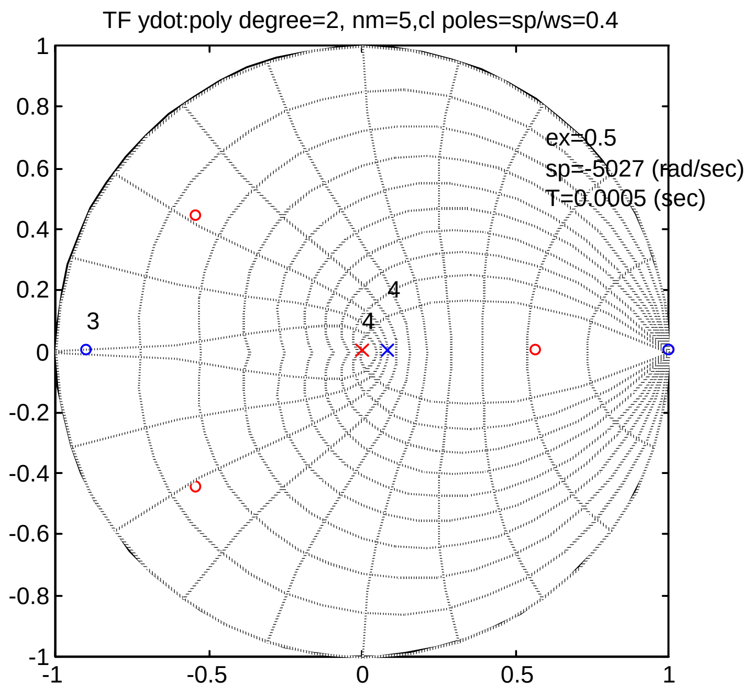

Figure 2. The classical filter has 4 poles placed at 0.4 ws and its zeros at -.9 to avoid the phase singularity as w->ws/2.

The discrete frequency response shows the comparison with the phase preserved better with the polynomial fit than with the classical filter. See Figure 3a and 3b.

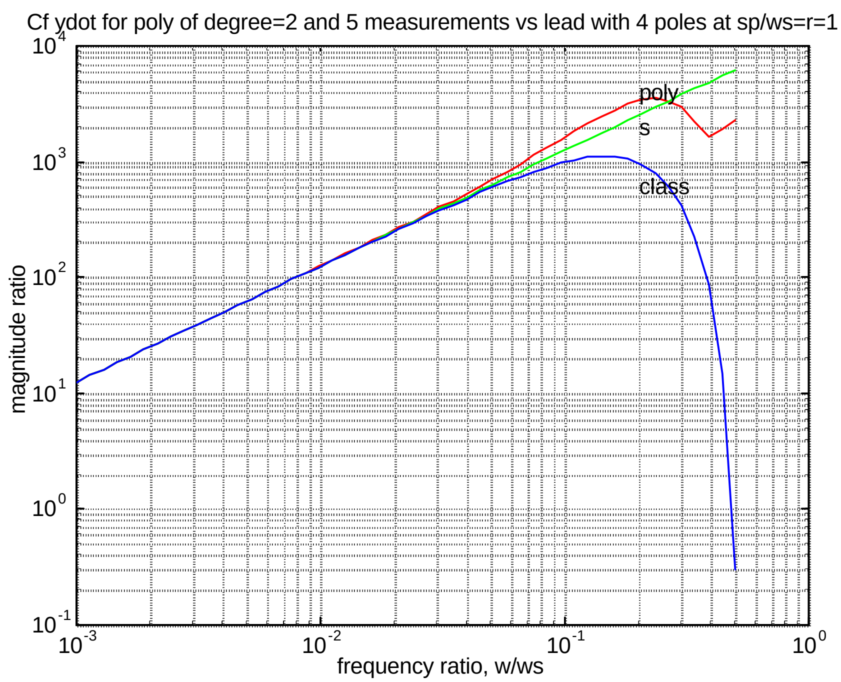

Figure 3a The polynomial filter compared with a perfect 's' and the classical filter.

The slight increase in amplitude slope of the polynomial fit is necessary to preserve the phase as can be seen on Fig 3b. When compared with a classical design with the same number of poles set at a ratio r to the sampling frequency ws, the discrete Bode plot shows the improvement one can obtain with a polynomial fit. Even with r=1, and for p=3 (polynomial of 2nd degree) and m=5 , phase is preserved to within 10 deg up to w/ws=0.07 for the polynomial fit while the classical fit has deviated 10 deg by w/ws=0.01 and falls off sharply reaching a phase error of -65 deg by w/ws=0.07.

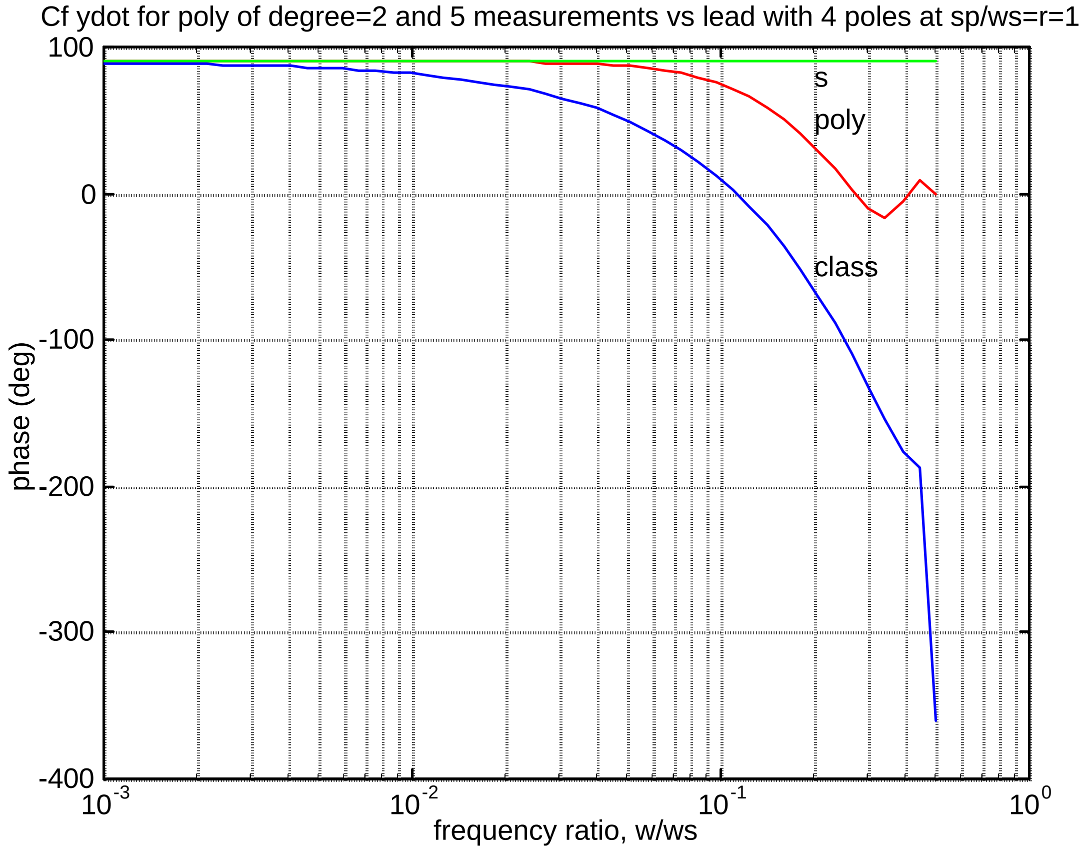

Figure 3b. Phase for the polynomial fit compared with a perfect 's' and a classical filter.

The need for velocity is greatest for an inner loop where the bandwidth is high. It is clear that one needs high sampling rate to preserve the fidelity of the velocity derived discretely. Some improvement can be made by this method of polynomial fitting. But care should be taken not to sample too fast especially if an integral control term is included lest the change each period is smaller than the least significant bit and the integral is lost.

Additional considerations

When the stiction is on the stage as it might be for a carriage in a hard vacuum riding on plane ways there is a good case to make for a direct drive so the position sensor can be collocated. Then the discrete differentiation can improve the bandwidth and is a significant improvement. If the friction is also on a capstan drive, one needs to obtain high bandwidth on the feedback to both inertias: the carriage and the capstan drive. Here one needs to have the carriage position and a good measure of the elastic deformation to use as the ‘control’ on the carriage and use the tachometer on the capstan drive to calculate the control on the inner inertia. The elastic deformation involves the relative displacement so the integral of the tachometer needs to be computed for comparison with the outer stage position. This places additional requirements on the low frequency noise of the tachometer.

Estimation and model based filtering can be very effective as long as the dynamic behavior and models of disturbances are predictable. However when stiction is involved or other disturbances that change suddenly there is no substitute for measurement bandwidth.

Conclusion

When friction or other disturbances require high bandwidth the value of velocity information and collocated sensors on each inertia become important. For precision applications it is still hard to find discrete sensors that can provide the performance of a tachometer. Analog inner loops are still the best choice in many cases and the tachometer should be with us for quite a while for as we push for higher accuracy the very characteristics that makes the tachometer so impressive become more valuable and harder to replace.