# A constant from a mass, a mass from a constant

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## Introduction

The International System of Units (SI) will soon be revised such that all units of measure derive from a set of fixed fundamental constants [1]. Just as the unit of length was once derived from the space between two scratches on a metal bar in Paris, but now can be derived anywhere at any scale using the speed of light, the unit of mass will cease to be derived from a single cylinder of platinum iridium in a vault in Paris, and instead be derived anywhere at any scale using a fixed value of the Planck constant *h*.

The first step, of course, is fixing a value of *h*. This must be done in terms of the existing standard of mass, the International Prototype Kilogram (IPK), to within 5 parts in $10^8$ if the perceived precision of mass is to be preserved after revising the units of measure. Multiple groups have recently crossed this threshold in precision using watt balance [2,3] and x-ray crystal density [4] approaches. At last, the long discussed decision whether to continue deriving *h* from the International Prototype Kilogram, or to instead derive the unit of mass from a fixed value of *h* [5], appears to be drawing to a close. A timeline for redefinition has emerged: a pilot study on the realization of the kilogram from *h* is scheduled to begin in 2015 aiming for revision of the SI in 2018.

NIST recently used a watt balance instrument known as NIST-3 to measure the Planck constant in terms of IPK with a relative uncertainty of approximately 45 parts in $10^9$ [2]. Along the way to this new NIST value of *h*, the instrument was also employed to perform the reciprocal experiment: *h* was “fixed” and the unknown mass of a stainless steel mass standard was “calibrated” with reference only to standards of length, time (frequency), and electrical quantities, all derivable from fixed fundamental constants.

This paper reviews the basic principles of a watt balance experiment and shares the results of a trial dissemination of mass directly from an instrument, rather than from an artifact.

## Watt Balance Principles

As illustrated in Figure 1, a watt balance has many elements in common with an ordinary compensation balance. There is a balance mechanism (a wheel pivoting on a knife edge in this case), a weighing pan suspended from the balance, and a moving coil actuator to apply compensation forces to hold the null position.

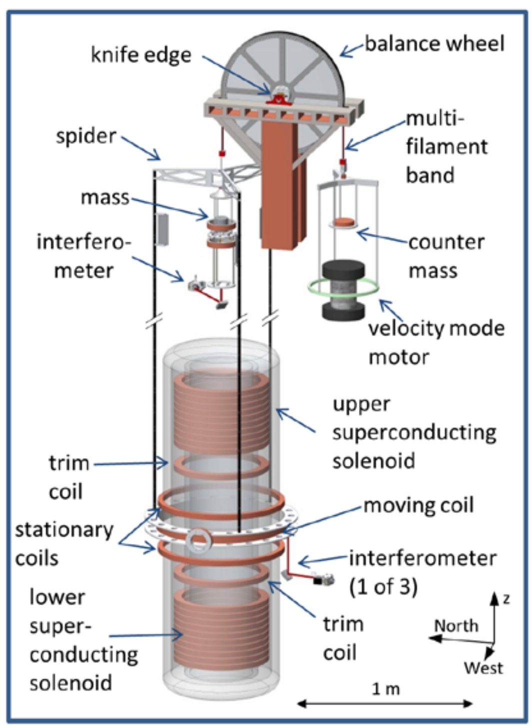


**Figure 1 Schematic drawing of the NIST-3 watt balance.**

In operation, such balances employ the substitution principle. The force on the mass pan is either generated by the gravitational force from the mass set on top of the mass pan or by an electromagnetic force created by the actuator located below the mass pan. However, what sets a watt balance apart from its compensation balance counterpart is its ability to operate alternately in two separated modes.

In the weighing mode, dc current $I$ is fed back through a coil in a magnetic field to produce a force that nulls the position of the balance mechanics and exactly balances the gravitational force acting on the test mass $m$, so that $BLI=mg$, where $B$ is the magnetic flux density, $L$ is the wire length in the coil, and $g$ is the gravitational acceleration.

In the velocity mode, the geometric factor $BL$ is precisely calibrated in terms of length, time, and electrical standards. The calibration is achieved by moving the coil through the field while measuring its velocity $v$ and the voltage $U$ induced across the coil, i.e., $U= BLv$.

Measurements from the two operational modes allow one to compute a virtual comparison of electrical power to mechanical power, $UI=mgv$, both measured in watts hence the name watt balance.

**A constant from a mass**

The connection to the Planck constant comes from two quantum phenomena that were discovered in the 60s and the 80s, respectively. The Josephson effect arises if one irradiates a Josephson junction, i.e., a superconductor non-superconductor, superconductor sandwich with a microwave radiation of frequency $f$. Under certain conditions (bias currents), a voltage of $V=n\ f\ 2e/h$ develops across the device, where $n$ is the number of Josephson junctions on the device. The quantum Hall effect is very special variant of the Hall effect, i.e., a voltage perpendicular to the current and the magnetic field in a semiconductor device. In order for it to be a quantum Hall effect, a two dimensional electron gas and a string magnetic field is necessary. In this case, the energy level of the electrons and hence the hall resistance is quantized at integer steps of $R_K=h/e^2$.

The electrical current is measured by driving it through a resistor, which is measured against a quantum hall resistor, $R=\alpha R_K$. The electrical power can be written as $P=UI=U^2/I$. Since the voltages are measured via the Josephson effect, one obtains for the power

$$P=4\ n^2 f\ h = mgv \text{ or } m = 4n^2\ f\ h/g/v.$$

The local gravitational acceleration $g$ is measured with an absolute gravimeter.

In August 2013, a stainless steel weight with a mass value unknown to the researchers at the watt balance was used on the NIST-3 watt balance. A value of $h$ = 6.626 069 80 x $10^{-34}$ J s that was determined previously using a platinum iridium prototype is used for these experiments. After 25 days of measurement on the watt balance, the mass of the stainless steel weight was determined to be

$$m_{\mathrm{WB}} = 1\ \mathrm{kg} + 342\ \mu\mathrm{g} \pm 70\ \mu\mathrm{g}$$

Before and after the watt balance experiment the NIST mass group measured the mass on their mass comparator and compared its weight to that of the national prototype. The mass group obtained

$$m_{\mathrm{MG}} = 1\ \mathrm{kg} + 324\ \mu\mathrm{g} \pm 14\ \mu\mathrm{g}.$$

The difference of the two values is

$$m_{\mathrm{WB}} - m_{\mathrm{MG}} = 18\ \mu\mathrm{g} \pm 72\ \mu\mathrm{g}.$$

No significant difference between the two ways of measuring mass has been found. This experiment shows that realization of mass at the kilogram level of a watt balance is possible.

## REFERENCES

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