Eliminating Parasitic Error in Dynamically Driven Flexure Systems

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INTRODUCTION
The aim of this paper is to guide designers in determining the optimal number and placement of actuators for driving general multi-axis flexure systems at any desired speed. Although the degrees of freedom (DOFs) of a flexure system are largely determined by the location and orientation of its flexure elements, the system’s stage will tend to displace in unwanted directions (i.e., parasitic errors) while attempting to traverse its intended DOFs if it is not actuated correctly. The problem of correctly placing actuators is difficult because the optimal location changes depending on the speed with which the stage is driven. In this paper we introduce the mathematics necessary to generate geometric shapes, called dynamic actuation spaces, which enable designers to rapidly visualize all the ways actuators could be placed for driving a general flexure system with minimal parasitic error at any speed without having to move the actuators placed. The theory provided here impacts the design of precision motion stages in that it significantly simplifies their control and increases their bandwidth. Example systems that benefit from this research include flexure-based nano-positioners, high-speed assembly stages, and multi-axis micro-mirrors.

CONTRIBUTIONS
In this paper we (i) introduce the concept of dynamic actuation space, (ii) provide the complete library of these spaces, (iii) provide guidelines for placing the correct number of actuators within these spaces, and (iv) introduce the mathematics necessary to generate these spaces as well as calculate the selected actuators’ output force magnitudes for driving a system with a desired DOF at a desired speed.

FUNDAMENTAL PRINCIPLES
Consider the steel flexure system in Fig. 1A ($E=210$ GPa, $G=79$ GPa, $\rho=7700$ kg/m$^3$). Its four wire flexures guide its T-shaped stage to move with three DOFs—two translations and one rotation. These DOFs may be modeled using twist vectors $[1]$, $\mathbf{T}_1$, $\mathbf{T}_2$, and $\mathbf{T}_3$ (Fig. 1B). If we wish to quasi-statically actuate these DOFs, we could do so optimally (i.e., with minimum parasitic error) by pushing on the stage with two linear forces located in the middle of the flexures.
as shown in Fig. 1B and torquing the stage with one pure moment respectively. These actuation actions may be modeled using wrench vectors \([1, W_1, W_2, W_3]\). Their locations correspond with the system’s center of stiffness and are shown a distance, \(D\), from the origin. As long as the system is not driven with any appreciable speed, these actions will minimize the axial loads in the flexures and will thus produce the corresponding DOFs with minimal parasitic error for small displacements.

Although the stage is capable of moving with three DOFs, it may also move with every combination of these DOFs. These motions are visually depicted by the system’s freedom space [2]. Freedom space is a geometric shape that represents all the ways a system may move. The freedom space of this example consists of an infinitely large box of parallel rotation lines (shown red in Fig. 1C) and a disk of translation arrows that are perpendicular to these lines. If \(T_1, T_2,\) and \(T_3\) are linearly combined, this freedom space is generated. For the case of quasi-static actuation, the optimal locations and orientations of the actuators that successfully drive all the motions/twists within this freedom space also lie within a geometric shape called a static actuation space [2]. This space results from the linear combination of \(W_1, W_2,\) and \(W_3\) and consists of a plane of linear forces (shown blue in Fig. 1C) and an orthogonal moment.

The optimal location of this actuation space will remain at \(D=14.5\text{cm}\) only if the DOFs in the freedom space are actuated with quasi-static speeds. If, however, we wish to actuate the motions within the freedom space with an increasing sinusoidal frequency of \(\omega\), the optimal location of the actuation space will displace downwards until it is infinitely far away when \(\omega\) reaches the system’s first natural frequency (Fig. 1D). As \(\omega\) increases to infinity above this natural frequency, the planar actuation space descends from positive infinity until it approaches the stage’s center of mass. The effect of increasing \(\omega\) on the absolute value of the actuation force necessary to displace the stage a sustained-amplitude of 1mm is shown in Fig. 1E.

Given these results, one may conclude that the only way to dynamically actuate a stage with minimal parasitic error is to move the actuators according to the desired actuation speed. This is fortunately not the case. If the wrench vectors within every planar actuation space for all values of \(\omega\) are linearly combined, a new space emerges. This space is called dynamic actuation space. The dynamic actuation space of this example consists of an infinite number of stacked parallel planes that contain linear force wrenches (shown blue in Fig. 1F) and coupled moment/force wrenches as well as a sphere of pure moment wrenches. Unlike the planar static actuation space from Fig. 1C that contains three independent linear forces, the dynamic actuation space of Fig. 1F contains five such forces. Thus, while quasi-static actuation requires only three actuators from the plane of Fig. 1C to drive the system’s three DOFs with minimal parasitic error, dynamic actuation requires at least five actuators from the parallel planes of Fig. 1F. As long as these five actuators’ lines of action are all independent, like \(W_1\) through \(W_5\) shown in Fig. 1F, their output forces may be combined to actuate any motion within the system’s freedom space with minimal parasitic error at any speed without the actuators needing to change locations at any time. A later section provides the mathematics necessary to (i) generate a general flexure system’s dynamic actuation space, (ii) determine the fewest number of actuators that need to be selected from this space, and (iii) calculate the output force magnitudes of the selected actuators for achieving any desired DOF with minimal parasitic error at any speed, \(\omega\).

CONCLUSIONS
This paper introduces the concept of dynamic actuation space as a geometric shape that guides designers in placing static actuators for driving multi-axis flexure systems at various speeds with minimal parasitic error. A comprehensive library of these spaces as well as the mathematics necessary to generate them, have been provided. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344, LLNL-PROC-625332.

REFERENCES