FUNCTIONAL CONSTRAINTS AND THE DESIGN OF A NEW WATT BALANCE

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ABSTRACT
The functional constraints driving the design of the new permanent magnet-driven watt balance (NIST-4) outline the variables and compromises worthy of addressing. Construction according to these design parameters will demonstrate the high precision capabilities of a large, complex system to measure mass with overall uncertainties on the order of 3 parts in $10^8$.

INTRODUCTION
Redefinition of the SI system is a formidable global undertaking. Of the seven SI units, the kilogram unit of mass is the only one still defined via an artifact, a platinum-iridium alloy cylinder casted in 1879 known as the international prototype kilogram (IPK) housed in the BIPM in Paris. The two major shortcomings of any artifact standard are replication and stability.

The watt balance, conceived in 1975 by Bryan Kibble, is a weight measuring instrument designed to virtually compare electrical power to mechanical power. Recent advancements in quantum physics have evolved the watt balance into an instrument capable of realizing the unit of mass.

HOW IT WORKS
The NIST-4 watt balance contains only a few key components. A copper wire (1) coil that is suspended from one side of a diamond turned aluminum (2) wheel balanced on a (3) knife edge along the axis of rotation. This hanging coil is immersed in a 0.55 Tesla (4) magnetic field (some 10,000 times greater than Earth’s magnetic field) generated by a one ton SmCo permanent magnet system.

Theoretically, the instrument operates by linking the unit of mass to Planck’s constant, a fundamental constant of nature as described by quantum physics. Functionally, the instrument achieves this linkage to quantum physics by indirectly comparing electrical power to mechanical power by toggling between two modes during measurement (Velocity Mode and Force Mode).

In Velocity Mode, the coil (wire length $L$) is moved at a vertical speed, $v$, through the magnetic field (flux density $B$) so that a voltage, $U$, is induced. The voltage is then measured precisely in terms of Planck’s constant by comparison to a quantum voltage standard [1].

In Force Mode, the gravitational force of the countermass, $m_sg$, offsets the weight of the coil and main mass, $m_g$. To achieve a balanced state, an upward electromagnetic force, generated by sending an electric current, $I$, through the coil, levitates the main mass stirrup. Measuring the main mass on the same side as the coil ensures equality in the moment generated by a common lever arm, as long as their centers of mass are vertically aligned to gravity. The current, $I$, is measured in terms of Planck’s constant by monitoring the voltage drop across a known resistor, this time making use of both quantum voltage and resistance standards.
Cancelling out the BL factor common to both equations and rearranging the variables, power is revealed and mass is solved for:

\[ BL = \frac{U}{v} = \frac{mg}{l} \]  (scalar version)

(where \( P_{\text{elec}} = UI \) and \( P_{\text{mech}} = mg \cdot v \))

**Combining:** \( UI = mg \cdot v \)

\[ m = UI/g \cdot v \]

This power comparison is the reasoning behind the name of the experiment, since watt is the unit of power. However, it is important to recognize that both types of power are “virtual” because they are not directly measured in either mode. Also worth noting is the presence of \( g \) and \( v \) as vectors, hinting at the significance of physical alignment during measurement.

The advantage of a wheel balance over a traditional beam balance becomes apparent when attempting to design for this “virtual” functionality. A watt balance must independently meet two criteria: (1) balance opposing lever arms about the knife edge and maintain equilibrium in the nominal position, and (2) translate the coil in a purely vertical fashion aligned to gravity. Although both balance designs meet criteria (1), the wheel allows pure vertical motion of the coil as it rotates whereas the beam incorporates a parasitic horizontal motion as it pivots.

**THE MOVING COIL**

The moving coil has six degrees of freedom of motion given by three components \((v_x, v_y, v_z)\) of its velocity \(v\) and three components \((\omega_x, \omega_y, \omega_z)\) of its angular velocity \(\Omega\) about its center of mass [2]. Additionally, the coil can generate a force \(F\) with components \((F_x, F_y, F_z)\) and a torque \(\Gamma\) with components \((\Gamma_x, \Gamma_y, \Gamma_z)\). Of the twelve variables that contribute to the virtual mechanical power, \(F_z\) and \(v_z\) are the only desired components. Note:

\[ UI = F \cdot v + \Gamma \cdot \Omega \]

or

\[ UI = F_x v_x + F_y v_y + F_z v_z + \Gamma_x \omega_x + \Gamma_y \omega_y + \Gamma_z \omega_z \]

To achieve the precision necessary for realization of mass, the ratio of each off-axis term to \(F_z v_z\) must be minimized to 3 ppb. There are essentially three ways to accomplish this: (1) diminish the five off-axis forces, (2) diminish the five off-axis velocities, or (3) reduce both off-axis forces and velocities to diminish the off-axis product \(F v\). For example, if

\[ \frac{F_x}{F_z} = 10^{-4} \quad \text{and} \quad \frac{v_x}{v_z} = 10^{-5} \]

then,

\[ \frac{F_x v_x}{F_z v_z} = 10^{-9}. \]

The off-axis x and y forces and torques are minimized by concentrically aligning the electrical center of the coil to the center of the radial magnetic field. By doing this, the magnetic flux gradients are balanced on opposing sides of the coil. Compliancy in the stirrup system allows for monitoring of the parasitic forces and torques during Force Mode. However, tradeoffs to compliancy are amplified parasitic motions.

**CONCLUSION**

NIST-4 will be used to realize the unit of mass once the redefinition of the SI has occurred with uncertainties of 3 parts in \(10^8\). The full length abstract will include the detailed design and analysis of the NIST-4 wheel, self-centering gimbal, and coil system dynamics.

**REFERENCES**
