Estimation of Volumetric Positioning Errors using Three-face Step-Diagonal Method

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INSTRUCTIONS
To ensure the motion accuracy of entire three-dimensional workspace such as machine tool or coordinate-measuring machine as many as 21 separate setups and measurements are needed for the linear, straightness, angular and perpendicular errors. In recent years, a method namely diagonal measurement following ISO 230-6 has been used for quick check on the volumetric errors of the machine. However, it is impossible to indicate the influence of each error among the linear, straightness and perpendicular errors of each axis from the results of diagonal tests; so, there is not enough information to make error compensation file to compensate volumetric errors.

THREE-FACE STEP-DIAGONAL METHOD
As FIGURE 1 shows, three-face step-diagonal tests in the XY-, YZ-, and XZ-planes and three linear motion errors in the X-, Y-, and Z-directions are used to estimate six straightness motion errors and three perpendicular errors (p indicates that the direction of motion is positive). Hence, 12 errors, including three linear errors, six straightness motion errors, and three perpendicular errors, can be estimated using the six data sets from six measurement paths shown in Fig. 2. In practice, the laser source can be fixed at one corner of the workspace, while the flat mirror is moved to each of the six opposing corners, making the setup and alignment of the measurement process easier and faster. To provide data for the evaluation of the three-face step-diagonal method, experiments were conducted on a three-axis CNC machine with 3D workspace dimensions X:Y:Z = 240×240×60 mm. A direct measurement test for each error will be used to assess the accuracy of the estimates. This machine is based on motion control with a PID filter (PMAC2, Delta Tau) and a linear motor, and hydrostatic bearings are used to reduce the uncertainty of the machine, especially the angular errors. Therefore, it can be assumed

FIGURE 1. The six measurement paths

FIGURE 2. The three face step-diagonal and linear displacements
that the angular errors are negligibly small.

Estimation of perpendicular motion errors

Two-face diagonal measurements based on the ISO 230-6 standard are used to estimate the perpendicular error between two axes. Hwang et al. proved that the perpendicular error between the X- and Y-axes can be estimated not only from the length difference between the two face diagonals $D_{xy}$ and $D_{yp}$ (based on the ISO 230-6 standard), but also from the face diagonal measurement $D_{px}$ and measured positions $P_x$ and $P_y$ on the X- and Y-axes, as shown in Fig. 3a. Where $D_{xy}$, $P_x$, and $P_y$ are measured values including the ideal and error values. By using the processes employed in the XY-plane, the desired angles for the YZ- and XZ-planes can also be calculated from Eq. (1). In this equation, $\gamma_{xy}$, $\gamma_{yz}$, and $\gamma_{xz}$ are the perpendicular errors between X–Y, Y–Z, and X–Z, respectively.

\[
\begin{align*}
\gamma_{xy} &= \frac{D_{xy}^2 - P_x^2 - P_y^2}{2P_xP_y} \\
\gamma_{yz} &= \frac{D_{yz}^2 - P_y^2 - P_z^2}{2P_yP_z} \\
\gamma_{xz} &= \frac{D_{xz}^2 - P_x^2 - P_z^2}{2P_xP_z}
\end{align*}
\]  

(1)

By substituting the face diagonal and linear measured positions at the end points into Eq. (1), the perpendicular errors between X–Y, Y–Z, and X–Z were found to be $-13.3$ $\mu$rad, $-422.8$ $\mu$rad, and $222.9$ $\mu$rad, respectively. On the other hand, using a master square and capacitive sensor, the directly measured perpendicular errors between X–Y, Y–Z, and X–Z were $-14.2$ $\mu$rad, $-420.7$ $\mu$rad, and $220.1$ $\mu$rad, respectively. Therefore, the maximum difference between the calculated and measured errors was $2.8$ $\mu$rad. Compared to direct measurement, the estimation procedure yields less accurate results, but is cheaper and faster in a practical environment.

Estimation of straightness motion errors

For the XY-plane, when the influence of perpendicular error $(2\gamma_{xy}P_xP_y)$ can be eliminated from initial face step-diagonal data by using Eq. (1), the procedure is as follows:

The laser system will provide linear and straightness motion errors for the X-axis when the X-coordinate is moved, and for the Y-axis when the Y-coordinate is moved. The combined motion errors for the X- and Y-axes can be obtained by repeating the measurements along the three-face step-diagonal of the plane. When the linear motion errors for the X- and Y-axes are directly measured, it is possible to estimate the contributions of the straightness motion errors for each axis, as well as the perpendicular errors between the X- and Y-axes. Fig. 3b illustrates the three-face step-diagonal measurement setup for a single step, where $e_{xx}$ and $e_{yy}$, are the linear motion errors and $e_{xy}$ and $e_{yx}$, are the straightness motion errors of the X- and Y-axes, respectively. The first subscript of $eab$ indicates the error direction and the second subscript denotes the axis of error. $\Delta d_{xy}$ and $\Delta d_{yx}$ are the respective diagonal displacement errors when X- and Y-coordinates are moved. $S_x$ and $S_y$ are the respective step lengths for the X- and Y-coordinates. The two angles in the figure can be calculated from Eq. (2).

When the X-coordinate is moved along the step-diagonal, the laser provides the combined data $\Delta d_{xy}$, which includes the linear motion error $e_{xx}$ and the straightness motion error $e_{xy}$. When a mirror alignment error $\delta_{xy}$ exists, the diagonal displacement error is given by Eq. (3).

\[
\begin{align*}
\alpha_y &= \tan^{-1}\frac{S_x}{S_y}, \quad \alpha_y = \tan^{-1}\frac{S_y}{S_x} \\
\Delta d_{xy} &= e_{xy} \sin \alpha_{xy} + e_{xx} \cos \alpha_{xy} + \delta_{xy}
\end{align*}
\]  

(2)

(3)

From Eq. (3), the straightness motion error $e_{xy}$ for the X-axis can be expressed by Eq. (4) for a single measurement step.

\[
\begin{align*}
e_{xy} &= \frac{\Delta d_{xy} - e_{xx} \cos \alpha_{xy} - \delta_{xy}}{\sin \alpha_{xy}}
\end{align*}
\]  

(4)

The straightness motion errors are calculated using Eq. (4) and Eq. (5). All the parameters on the right-hand sides of these equations are already determined, except for the mirror misalignment $\delta_{xy}$. This means that in every measurement step, the estimated straightness motion errors will include the mirror misalignment. Since the measurement step is small, $\delta_{xy}$ will not change greatly during the measurement process.