INTRODUCTION
A method for measuring the moment of inertia of an object using a five-wire torsion pendulum design is described here. Typical moment of inertia measurement devices are capable of 1 part in $10^3$ accuracy and current state of the art techniques have capabilities of about one part in $10^4$ [1]. The five-wire apparatus design, Figure 1, shows the prospect of improving on current state of the art. Current measurements using a laboratory prototype indicate a moment of inertia measurement precision better than a part in $10^4$. In addition, the apparatus is shown to be capable of measuring the mass center offset from the geometric center. Typical mass center measurement devices exhibit a measurement precision of approximately 1 µm. Although the five-wire pendulum was not originally designed for mass center measurements, preliminary results indicate an apparatus with a similar design may have the potential of achieving state of the art precision.

MEASUREMENT APPARATUS
An apparatus designed to generate a rotation about an axis provides the ability to measure an objects mass center and moment of inertia. The measurement apparatus typically attempts to produce a pure rotation about a single degree of freedom. When designing an apparatus to measure the moment of inertia to a high precision, care must be taken to minimize the extra degrees of freedom in the system. The measurements of rotation will have uncertainties when there are significant other degrees of freedom. Bifilar and trifilar pendulums do not constrain the swinging or lateral translation modes. To improve the accuracy of a standard trifilar pendulum, the lateral pendulum modes need to be constrained. Five support wires are sufficient to constrain all but one degree of freedom. In a five-wire pendulum, two additional wires are arranged as shown in Figure 1 to minimize rotations about the other two rotational axes. The design reduces errors due to tilt and horizontal translational degrees of freedom. The three attach points on the platform supporting the inertia to be measured are positioned equidistant from the center of rotation. At one attach point, a single vertical wire is used and the other two attach points consist of two wires. The wire geometry is symmetrical about a plane formed with the vertical wire and a line emanating from the rotation center to the vertical wire attach point. The horizontal components of the wires which are splayed out from a single attach point provide horizontal stiffness to prevent pendulum platform swinging motion.

Moment of Inertia Measurement
In order to measure the moment of inertia, $I$, the object is rotated about an axis and the pendulum natural frequency, $\omega$, is used to determine the radius of gyration, $R_g$, about that rotational axis. The relationship between the torsion pendulum rotational frequency and the radius of gyration is given by:

$$R_g^2 = \frac{I}{m} = \frac{I_o + m_o r^2}{m} = \frac{k}{\omega^2}$$

where, $k$ is the torsion coefficient or stiffness constant of the pendulum and $m$ is the total mass. The full moment of inertia tensor contains six independent terms. As such, a moment of inertia measurement device must be capable of determining the radius of gyration about at least six
different axes of rotation. Since a torsion pendulum generates a rotation about a single axis, the measurement apparatus must be capable of reorienting the measurement object to at least six independent directions.

**Mass Center Measurement**

By measuring the pendulum natural frequency of rotation, the mass center offset from the geometric center of an object is also obtained. The instantaneous moment of inertia, $I$, about the rotation axis consists of the moment of inertia about the objects mass center, $I_0$, plus the parallel axis theorem components of the objects mass, $m_o$, times the square of the distance from the rotation center to the mass center, $r^2$. Placing an object on the pendulum far from the rotation axis amplifies the contribution due to the mass center offset from the geometric center, due to the quadratic dependence on $r$. By changing the orientation of the object with a fixed geometric location relative to the pendulum rotation center, the mass center offset is determined by measuring the change in the natural frequency, $\omega$.

Consider for example an object with a mass center offset from the geometric center of magnitude $\delta$. The object's geometric center is placed on the pendulum at a location of $R$ from the pendulum rotation axis, $\hat{k}$. Refer to Figure 2 for a graphical depiction. The mass center offset within a plane can be determined by rotating the object about an axis parallel to the pendulum rotation axis. The offset is determined by measuring the pendulum oscillation frequency for different rotation angles of $\phi$. For each rotation angle $\phi$, a corresponding change in the pendulum oscillation frequency will be observed.

**REFERENCES**


