DEVELOPING A PREDICTIVE MODEL FOR MILLING STABILITY

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INTRODUCTION

Both physically and economically motivated uncertainties are often present in manufacturing decision situations. To deal with these uncertainties, predictive models can be used. Most methods for predicting functions are statistically based, including regressions and curve fits. However, these methods may be inadequate for the prediction of complex functions, since there is no rigorous method of aggregating information from multiple sources (experimental data, simulation data, and theoretical predictions) and they do not quantify the value of information gathering activities.

Although Bayesian methods for predicting functions can be mathematically complex, they have two major advantages over other approaches. First, any information learned through experiment about the function can be incorporated. This makes these methods more versatile than statistical methods, which are most compatible with simple sets of observables. Second, Bayesian methods enable information to be valued based on its influence on profit, which makes it possible to systematically choose an information gathering scheme that maximizes profit.

Since Bayesian methods have not been widely applied in manufacturing, the focus of this work is to explore their use in this field. To aid in the explanation, stability limit prediction is examined.

BAYESIAN PREDICTION OF FUNCTIONS

These methods are fundamentally different from curve-fitting methods in that they treat a set of predictions rather than a single prediction. Methods which use a single curve as a prediction only answer a single question about an unknown function $g(x)$:

Given an arbitrary knowledge state, what is the most likely $g(x)$?

In contrast, Bayesian methods assign a probability to every function. In other words, for any $g(x)$ the following question can be answered:

Given an arbitrary knowledge state, how likely is it that an arbitrary function $= g(x)$?

The advantage of this approach is that arbitrary information regarding the function of interest can be incorporated into the predictive model. For example, abstract functional assumptions that can't be handled by statistical methods, such as monotonicity, differentiability class, or periodicity, can be incorporated. In addition, multiple informative statements can be aggregated in a mathematically rigorous manner.

Mathematically, probability assignment over a functional space is equivalent to assigning a multivariate distribution over infinite dimensions. This assignment can be simplified by assigning a distribution only over the set of all measurable quantities. The most general set of measurable quantities is the set of all functionals, i.e., any operator which takes a function as an input and produces a scalar, such as a definite integral.

BROWNIAN DISTRIBUTIONS

Suppose that the value of $g(x)$ for an arbitrary $x$ depends only on the value of $g$ in a small neighborhood of $x$. In terms of the conditional probabilities, this is given by:

for any $x_1 < x_2 < x_3 < x_4 < x_5$

$$f_{g(x_3)|g(x_2),g(x_4)} = f_{g(x_3)|g(x_1),g(x_2),g(x_4),g(x_5)}$$

(1)

With this assumption, the probability density functional takes the form:
\[ f_{\mu(x)}(g(x)) = \exp\left(-\int_{-\infty}^{g} k(x, g)(g(x, g) - \mu(x, g))^2 + c(x, g) \, dx\right) \]

where \( k(x, g) \) is the diffusion field, \( \mu(x, g) \) is the drift field, and \( c(x, g) \) is the creation and killing field.

Distributions which take this form are referred to as Brownian distributions. Brownian distributions are ideal for predicting and updating when little is known about the function in question. Provided that any information gained through experiment can be expressed in terms of \( g \) or its slope at each point, any distribution updated from a Brownian distribution will also be a Brownian distribution.

**STABILITY LIMIT PREDICTION USING BROWNIAN DISTRIBUTIONS**

To illustrate how Bayesian methods can be used in a manufacturing application, consider the prediction of the speed-dependent stability limit in milling. The stability limit has a particularly complex structure, since it generally has multiple non-differentiable points (Fig. 1). For the numerical example provided here, the true stability limit is generated using the algorithm described in [2]. The input parameters used for the algorithm are summarized in Table 1.

![Example stability lobe diagram](image)

**FIGURE 1. Example stability lobe diagram.**

**TABLE 1. Parameters used to generate the reference stability limit.**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangential cutting coefficient</td>
<td>2500</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Radial cutting coefficient</td>
<td>750</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Tool stiffness</td>
<td>5x10⁶</td>
<td>N/m</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>Tool natural frequency</td>
<td>2.4</td>
<td>kHz</td>
</tr>
</tbody>
</table>

For this study, a Brownian distribution is used to describe the initial knowledge state (Table 2). In addition, it is assumed that the stability limit is known to occur below an axial depth \( g_{max} \) known \textit{a priori}. While this may be an unrealistic assumption in practice, the bound provides an intuitive link between the imposed constraint and \( c(x, g) \).

**TABLE 2. Parameters defining the Brownian distribution prior to experimentation.**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper bound</td>
<td>( g_{max} )</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>Diffusion field</td>
<td>( k(x, g) )</td>
<td>0.00005</td>
</tr>
<tr>
<td>Drift field</td>
<td>( \mu(x, g) )</td>
<td>0</td>
</tr>
<tr>
<td>Creation and killing field</td>
<td>( c(x, g) )</td>
<td>( \infty ) for ( g &lt; 0 ) \text{ or } \infty ) for ( g &gt; g_{max} ) \text{ otherwise}</td>
</tr>
</tbody>
</table>

Here, new knowledge is gathered using only simple stability testing, where a machinist selects an axial depth-spindle speed combination, performs a test cut, and decides whether the cut was stable or unstable. For this numerical evaluation, the testing is simulated by comparing the selected test parameters \( \{x_{i}^*, g_{i}^*\} \) to the predefined/reference stability limit.

Since simple stability testing updates knowledge of \( g(x) \) directly, the probability density functional for any updated knowledge state is again a Brownian distribution, and can be expressed by “killing” all \( g(x) \) which are not consistent with the experimental data. For an arbitrary number of simple stability tests, the updated creation and killing field is given by Eq. (3).

\[
c_{up}(x, g) = \begin{cases} 
\infty & \text{for } g < 0 \\
\infty & \text{for } g > g_{max} \\
\infty \text{ for } g(x_{st}) < g_{i} & \text{if test } i \text{ stable} \\
\infty \text{ for } g(x_{st}) > g_{i} & \text{if test } i \text{ unstable} \\
0 & \text{otherwise}
\end{cases}
\]

In order to gauge how performing an experiment will affect profit, the decisions that are affected by knowledge of the stability limit should be considered. In this study, the decision situation consists of selecting an axial depth-spindle speed combination \( \{x_{opt}, g_{opt}\} \) at which to mill away a cube of material. Profit for stable operation, \( P(x_{opt}, g_{opt}) \), is calculated using the approach and parameters described in [3]. Profit for unstable operating parameters is assumed to be zero.
For an arbitrary test history, defined in Eq. (4) where \( R_i \) denotes the result of stability test \( i \), the expected profit of a production run, \( P_{\text{exp}}(x_{\text{op}}, g_{\text{op}}) \), can be calculated using the probability tree shown in Fig. 2.

\[
T_i := \{x_i, g_i, R_i\} \quad 1 < i < \# \text{ of tests}
\]

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability of outcome</th>
<th>Value of outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>stable</td>
<td>( p_{\text{st}}(x_{\text{op}}, g_{\text{op}}; T) ) ( P(x_{\text{op}}, g_{\text{op}}) )</td>
<td>( )</td>
</tr>
<tr>
<td>( {x_{\text{op}}, g_{\text{op}}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unstable</td>
<td>( 1 - p_{\text{st}}(x_{\text{op}}, g_{\text{op}}; T) ) ( 0 )</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 2.** Probability tree for calculating the expected profit of a production run \( P_{\text{exp}}(x_{\text{op}}, g_{\text{op}}; T) \).

For a stability test at an arbitrary \( \{x, g\} \), the probability of each outcome, as well as the expected value of each outcome (stable or unstable) can be calculated before actually knowing the outcome of the test. Using the probability tree shown in Fig. 3, the expected value of the production run after the experiment’s result may be calculated. Using larger probability trees, this calculation can be extended to an arbitrary number of tests, which can be optimized to determine an optimal testing policy.

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</tr>
</thead>
<tbody>
<tr>
<td>stable</td>
<td>( p_{\text{st}}(x, g; T_{\text{initial}}) ) ( ) ( T_{\text{st}}(x, g; T_{\text{stable}}) )</td>
<td>( )</td>
</tr>
<tr>
<td>( {x, g} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unstable</td>
<td>( 1 - p_{\text{st}}(x, g; T_{\text{initial}}) ) ( p_{\text{st}}(x, g; T_{\text{unstable}}) )</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 3.** Probability tree for calculating the value of performing an experiment, \( V_{\text{E}}(x, g) \). \( T_{\text{initial}} \) is the current knowledge state, \( T_{\text{stable}} \) denotes \( T_{\text{initial}} \) augmented with an additional row vector \( \{x, g, \text{stable}\} \), and \( T_{\text{unstable}} \) denotes \( T_{\text{initial}} \) augmented with an additional row vector \( \{x, g, \text{unstable}\} \).

Since stability tests can be chosen sequentially, the number of test parameters grows exponentially with the number of tests being considered. To reduce the computational complexity of this optimization, each test was optimized individually, instead of optimizing the full sequence of tests. Using this heuristic, a sequence of twelve experiments were chosen and tested against the reference stability lobe diagram. Figure 4 shows the progression of the predicted stability limit as the results of a sequence of experiments are incorporated into the prediction.

**FIGURE 4.** Progression of \( p_{\text{st}}(x, g; T) \) as experimental data is gathered. The top panel shows the prediction prior to any testing. From top to bottom, the remaining panels show the updated predictions after 4, 8, and 12 tests, respectively.
BUILDING GENERAL DISTRIBUTIONS BY TRANSFORMING A SET OF OBSERVABLES

While Brownian distributions offer the advantage of simplicity, in some cases it is advantageous to describe the function using an underlying model. For example, the stability limit can be inferred from knowledge of the frequency response function (FRF) and force model coefficients [4], which can be used as the set of measurables, instead of assigning a distribution over the stability limit directly. The advantage of this treatment is two-fold. First, a complex correlative structure is implied directly from the underlying dynamical model. Second, experiments which measure these parameters (such as force measurement or impact testing) can be incorporated into optimal experimental design. This provides an important alternative to simple stability testing, since a single FRF measurement enables much more accurate predictions to be made (see Fig. 5).

FIGURE 5. Contour plot of the probability of stability calculated assuming uncertain force model coefficients [3].

While the Brownian correlative structure works best when little is known about the function of interest, a correlative structure built from an underlying model enables predictions to be made when relationships governing the underlying dynamics are known.

CONCLUSION

As efficiency and productivity demands on the manufacturing industry increase, the need for a systematic approach to information gathering is growing. Due to the inherent complexity involved in many manufacturing decision problems, a general treatment of optimized experimentation for an arbitrary pay-off and knowledge state is beyond the scope of traditional statistical methods. However, Bayesian methods are a natural candidate for this task. Not only can they treat arbitrarily complex mathematical objects, but they also associate a value with each knowledge state. As a result, information gathering can be optimized rigorously.

Since Bayesian methods have not previously been widely applied in manufacturing decision making, the primary motivation for this work was to explore possibility of using Bayesian methods to predict functions arising from the complex dynamics involved in milling. First, Brownian distributions were used to explore how Bayesian methods could be used in cases where little is known about the function of interest. Second, the process of incorporating underlying dynamical models into Bayesian prediction was explored. While these methods are still in their infancy, they offer sufficient analysis capability for a systematic treatment of the prediction of complex milling dynamics.

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REFERENCES