INTRODUCTION
The main advantages of hexapod systems are user-friendly compact shipment design and high stiffness. An improvement in the accuracy of hexapod systems can be achieved via better actuator/sensor, cable outlet and joint design. For most applications the use of external 6 DOF sensor systems at the platform is not suitable due to the sensor performance, the size or the price of such sensor systems. Therefore, sensor systems inside the struts have to be used as a reference for each single axis of the hexapod. The platform position will be determined by the strut-sensor systems, the joints – and the mechanical tolerances (which can be calibrated). This paper describes different joints used in nanometer repeatable, parallel-kinematic systems. Such joint designs require large computing power of the controller.

HEXAPOD STRUCTURES

The multi-axis application defines the structure of the hexapod parallel kinematics. Mostly the customer requirements determine size, height and shaping of the systems. Also environmental conditions or special scan routines demand different structure design. [1] Two different parallel-kinematic designs are realized at PI:
- Systems with constant length of the struts:
  - vertical motion of the lower joints
  - horizontal motion of the lower joints
- Systems with changeable length of the struts
  - Stewart Gough platforms

The advantage of systems with changeable strut lengths is that no additional linear guiding parts influence the positioning performance. Such systems should be used preferably for high-accuracy applications. However, systems with movable joints could have better dynamic properties because the drives are better decoupled from the platform. All parasitic forces from the drives can be decoupled with the linear guiding.
DIFFERENT JOINT DESIGNS

The cardan joint with axis offset features a compact joint part between the two links. The two axes can be used as inseparable cylinder parts. In case of high accuracy applications, both bearings inside the joint part (stone) can be realized with long needles. Systems with cardan joints with axis offset provide twice as much stiffness as systems using cardan joint design with crossed axes.

During assembly the cardan joint bearings have to be preloaded or fine adjustment modules should be used for preload. For small angular ranges below 10 degree in each joint, the out-of-plane motion and the roundness of high-precision bearings can reach values better than 50 nm. Hysteresis effects are of high importance. They are caused by the friction and different rolling lines for forward and backward motion. These hysteresis effects can be minimized by medium preload and specially designed spindle bearings.

Sphere joints facilitate the calculation of the inverse kinematics. Our sphere joints consist of ceramics for fully nonmagnetic hexapod systems with piezo actuators. The drawback of such joints is the hysteresis effect due to friction. A very thin lubricant layer is strongly recommended achieving high-precision joints. There are also sphere joints with balls between the joint sockets, but their disadvantage is that boring forces occur during rotation.

Flexure design for joints has the advantage of very small hysteresis effects. It can be used for small angular motion of the joints up to 10 degrees. For bigger rotation angles special super-elastic materials are appropriate. The flexure design can also be used for medium load conditions at the joints. The wire flexure is a simple joint which can take in bending as well as torque forces. The drawback of such structures is the unstable pivot point.

Inverse Kinematics of Hexapods with Cardan Joints

The hexapod with cardan joints with axis offset can be described as a linkage of six 6R serial linkages systems placed between the base plate and the moving platform. Many different solutions were developed in the 80s and 90s of the last century for the solution of inverse kinematics of a general 6R linkage system.


\[ D = T_c \cdot G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6 \]  

(1)

Where the transformation matrixes and geometric properties are:

\[ T_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(u_i) & -\sin(u_i) & 0 \\ 0 & \sin(u_i) & \cos(u_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(2)
A very interesting paper was presented by M. Husty [4] in which he showed that the general problem could be solved by numerical calculation without iteration. The general 6R kinematics of each strut-platform module was cut into two 3R serial chains. The rotation angles $u_i$ of the revolute joints have to be computed. For the upper part of such 3R-chain systems the direct kinematics can be described as

$$G_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_i/2 & 1 & 0 & 0 \\ 0 & 0 & \cos(\alpha_i/2) & -\sin(\alpha_i/2) \\ d_i & 0 & \sin(\alpha_i/2) & \cos(\alpha_i/2) \end{bmatrix}$$

(3)

The lower part of the separated 3R-chain can be set to a just equivalent term. A numeric example in the paper shows that the equation can be solved via a simplified chain structure. This method produces sixteen “solutions” for the inverse kinematics.

For the upper part of such 3R-chain systems the direct kinematics can be described as

$$U_1 = T_1 \cdot G_1 \cdot T_2 \cdot G_2 \cdot T_3 \cdot G_3$$

(4)

$$G_{3i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_i/2 & 1 & 0 & 0 \\ 0 & 0 & \cos(\alpha_i/2) & -\sin(\alpha_i/2) \\ d_i & 0 & \sin(\alpha_i/2) & \cos(\alpha_i/2) \end{bmatrix}$$

(5)

The lower part of the separated 3R-chain can be set to a just equivalent term. A numeric example in the paper shows that the equation can be solved via a simplified chain structure. This method produces sixteen “solutions” for the inverse kinematics.

In our first application in 1993 we used a very similar philosophy but we established an iteration algorithm to solve the two equation parts for the upper and lower linkage system of each strut. Our method provides exactly one solution for each pose. In the past the first algorithm did not rely on the Jacobian matrix for acceleration of the iteration. In the iteration we found a very linear dependence. Also for non pre-settings not more than 3 iteration steps are necessary (typically two steps). We need less than 60 $\mu$s for the direct transformation of all six struts and about 4 ms for the inverse transformation.

It is necessary to indicate that in most of our hexapod systems we use the rotation between spindle and nut for the $5^{th}$ DOF in each strut. Otherwise an additional precise bearing should be inserted between both cardan joints at platform and base-plate.

**Input:** The lower plane $e$ collects the spindle axes, the direction vector of joint $\vec{u}$ and the center point of joint $P'$. In an equal manner plane $d$ collects direction vector $\vec{r}$, point $P$ and the center point of joint $Q'$.

**Output:** All joint angles, the distance $P' - Q'$ and the angle between both planes (spindle/nut).

FIGURE 5. Description of variables at general axes configuration

Two equations describe the angular relations of the upper und lower part of the 3R system.

$$\vec{q} + b(\vec{v} \cos \beta + \vec{w} \sin \beta) = \vec{p} + f \vec{b} + g(\vec{s} \cos \alpha + \vec{t} \sin \alpha)$$

$$\vec{q} + d\vec{u} + c(\vec{v} \cos \beta + \vec{w} \sin \beta) = \vec{p} + a(\vec{s} \cos \alpha + \vec{t} \sin \alpha)$$

The values $a$ and $b$ represent the axis offset. It is assumed that the linkage part axes are rectangular.

With two additional vectors and the scalar product of e. g. $p*w$ six equations are derived. Each iteration step calculates the distance between e. g. point $P'$ and plane $e$. The distance has to be minimized.

$$F_1(\alpha, \beta) = (q_u + sv \cos \alpha + tw \sin \alpha) \sin \alpha - (q_v + tw \cos \alpha + tw \sin \alpha) \cos \alpha = 0$$

$$F_2(\alpha, \beta) = (p_v + sv \cos \alpha + tw \sin \alpha) \sin \beta - (p_w + tw \cos \alpha + tw \sin \alpha) \cos \beta = 0$$

Equations (8) and (9) lead to two independent constraints for the angles alpha and beta.

$$J(\alpha, \beta) = \begin{bmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial \beta} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial \beta} \end{bmatrix}$$

(10)
The sensor consists of the optical head and a small industrial computer with three A/D channels. All three deviations from the initial position are reported with command structure. This sensor can also be used for calibration tasks.

The static and dynamic performance of hexapod systems is measured with Zygo laser interferometers in single axes at the platform.

**SOFTWARE SIMULATION TOOL FOR HEXAPOD SYNTHESIS**

The simulation program is based on identical routines which also used in our controller. It provides hexapod synthesis, working space analysis, load conditions and joint angle calculation and some scanning features. All types of PI’s hexapods with different joint structures are automatically recognized.

**MEASUREMENT RESULTS**

An optical sphere sensor was developed for the measurement of pivot point stability. A small precise ball is placed at the desired pivot point and fixed at the platform. The 3D sensor housing is placed around this position.

**REFERENCES**

[1] Physik Instrumente (PI) GmbH & Co.KG Katalog, Piezo Nano Positionierung, 2009, Pages:4.3 -4.18

