EVALUATION OF HARMONIC DISTORTIONS IN PRECISION INERTIAL SENSORS

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INTRODUCTION
Inertial sensors do not require physical reference bases and relevant fixtures. Their unique property is the ability to measure 6-degrees-of-freedom (6-DOF) displacements relative to the inertial coordinates of the Earth, an ideal reference frame. Other sensors for measuring linear or angular displacement, including laser and lidar trackers, need reference bases such as tripods and measure displacements with reference to these bases [1]. Since reliable reference frames and low-invasiveness of fixtures are critical in precision engineering, inertial sensors have the potential to enable qualitative changes in this field. However, to be a viable element of precision engineering solutions, the accuracy of inertial sensors has to be in the range of at least “parts-per-million” (ppm).

The introduction of Global Positioning System (GPS) greatly diminished the importance of high performance accelerometers, indispensable in Inertial Navigation Systems (INS). This has shifted the focus of research towards small, inexpensive and reliable Microsystems Technology (MST) based designs [1]. A rapid progress in miniaturization and cost reduction that followed has not been matched by an increased accuracy and resolution of the new generation of sensors.

INVESTIGATED SENSORS
Commercially available accelerometers fall short of the requirements of precision engineering. In particular their distortions are high, typically above 0.1%, i.e. 1,000 ppm. However, this level is a result of technological imperfections in fabrication. Simulations of the best accelerometers available today suggest nonlinear distortions well below 1 ppm [2]. There is also an experimental evidence of distortions below 50 ppm. It is therefore justified to investigate, detect, model and attenuate an impact of phenomena that degrade the achievable accuracy of sensors below the level predicted by the relevant physical laws.

A challenging consequence of size scaling in MST based devices is, from the viewpoint of precision engineering, the need for extremely high resolution of motion measurement. Movements of components in MST based inertial sensors are typically in the sub-micron range. Consequently, investigating phenomena occurring in these sensors with a “part-per-million” precision implies femto-meter (10^{-15} m) resolution. Such resolution is feasible in some currently available accelerometers employing capacitive transducers. For example, the digital SiFlex accelerometer [2] features 24 bit “word length”. This is equivalent to the resolution of 0.05 ppm. Its noise floor, although relatively low, limits at present the resolution to 10 pm/√Hz at 25 Hz and 700 fm/√Hz at 100 Hz. However, with narrow band digital filters reaching the needed femto-meter resolution is possible.

In this research we consider a representative pendulous servo accelerometer. Its mechanical design, shown in Figure 1, limits the total displacements of the seismic mass to about ±1 µm. Furthermore, the servo system keeps this mass at a nearly constant distance from the enclosing plates. This is necessary to minimize nonlinear distortions of the capacitive readout system as discussed below.

The seismic mass suspended on flexures exhibits complex spatial dynamics which has to be accounted for in the design and tuning of the servo. Since imperfections of fabrication affect this dynamics, there is a need to identify it for the purpose of monitoring and correcting the fabrication process. Furthermore, if an accurate model of the sensor is identified, such a model can be used for the cancellation of distortions occurring in this sensor.
We further observe that if \( x(t) << x_0(t) \), which is common in servo accelerometers, the output signal becomes a nearly linear function of the mass displacement

\[
V_{xL}[\beta, x_0, x(t)] = \frac{4 \, V_d \, x(t)}{x_0 \beta + 2} \quad (2)
\]

The gap variation \( x(t) \) is a function of the acceleration \( a(t) \) acting on the sensor. In general this relationship is dynamic and nonlinear (e.g., asymmetric gain for the positive/negative accelerations, insensitivity at small signals, hysteresis, etc.). In this abbreviated analysis we neglect the dynamics and use an asymmetric gain to express the acceleration-gap relationship as follows

\[
x(t) = \Psi[a(t)] \, K_a \, a(t); \quad \Psi[\theta] = \begin{cases} 1 & \text{for } \theta \geq 0 \\ \gamma & \text{for } \theta < 0 \end{cases} \quad (3)
\]

This leads to the acceleration-voltage formula

\[
V_a[\beta, \gamma, x_0, a(t)] = \frac{4 \, V_d \, x_0 \, \Psi[a(t)] \, K_a \, a(t)}{(\beta + 2) \, x_0^2 - \beta \, \Psi[a(t)] \, K_a \, a(t)^2} \quad (4)
\]

To facilitate a systematic characterization of the investigated nonlinearity we employ the Taylor series expansion. Assuming a small ratio \( \Psi[a(t)] \, K_a \, a(t)/x_0 \) (i.e., small variations \( x(t) \) of the gap) we expand the above expression around 0 and keep the first 5 terms

\[
V_{aL}[\beta, x_0, a(t)] = \alpha_1 \, \frac{a(t)}{x_0} + \alpha_3 \, \frac{a(t)^3}{x_0^3} + \alpha_5 \, \frac{a(t)^5}{x_0^5}
\]

where

\[
\begin{align*}
\alpha_1 &= \frac{2 \, V_m \, K_g \, [1 + \gamma + (1 - \gamma) \, \text{sgn}(x)]}{(\beta + 2)} \\
\alpha_3 &= \frac{V_m \, K_g^3 \, \beta \, [1 + \gamma^3 + (1 - \gamma^3) \, \text{sgn}(x)]}{(\beta + 2)^2} \\
\alpha_5 &= \frac{V_m \, K_g^5 \, \beta^2 \, [1 + \gamma^5 + (1 - \gamma^5) \, \text{sgn}(x)]}{(\beta + 2)^3}
\end{align*}
\]

An absence of the even terms in the expansion is a consequence of the simplifications made above. In general, all terms are present and their analytical form can be derived using, for example, software capable of symbolic computations.
Equations (4) and (5) show that the level of nonlinear distortions in the considered capacitive transducer depends upon:

1. A relative input strength coefficient, \( \eta \), defined as \( \eta = K_a \ a(t) / x_0 \),
2. Coefficient \( \gamma \), i.e., a nonlinearity of the acceleration-gap relationship,
3. Coefficient \( \beta \), i.e., an impact of the parasitic capacitance \( C_p \).

A knowledge of these quantities facilitates correction of the inherent nonlinearities of the considered transducer. The following section illustrates the estimation of \( x_0 \) and \( \gamma \).

PARAMETER ESTIMATION

To assure the needed accuracy, the above coefficients should be estimated in the conditions closely resembling the actual operation of the sensors. In particular, the tested sensors should be subjected to dynamic excitations. In this research we use precisely guided, servo controlled translational air bearing stages ABL1500 and ABL2000 with brushless DC motors (Aerotech, Inc.). One of the tests setups is shown in Figure 3.

![Tested sensor and Laser](image)

**FIGURE 3. Experimental setup.**

To maximize the accuracy, identification is conducted in a very narrow frequency bandwidth and repeated over the entire frequency range of interest. The stages are driven such that the tested sensor is subjected to sinusoidal acceleration \( a(t) = A_0 \ sin(\omega t) \). Furthermore, rather than using the time domain relationships, Eqs. (4, 5), we employ the trigonometric Fourier series and formulate expressions involving the sought coefficients in the frequency domain as

\[
a_0(\beta, \gamma, \eta) = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} V_a[\beta, \gamma, x_0, a(t)] \ dt
\]

\[
a_\eta(\beta, \gamma, \eta) = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} V_a[\beta, \gamma, x_0, a(t)] \ cos(n \omega t) \ dt
\]

\[
b_\eta(\beta, \gamma, \eta) = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} V_a[\beta, \gamma, x_0, a(t)] \ sin(n \omega t) \ dt
\]

\[
n = 1, 2, 3, ...
\]

(6a-c)

Performing the above integrations yields relationships needed for the parameter estimation. In particular

\[
a_1() = b_2() = a_3() = 0, \ etc.
\]

\[
b_1(\lambda) = \lambda \left( \frac{2 + \beta}{\sqrt{2 + \beta - \beta \eta^2}} + \frac{2 + \beta}{\sqrt{2 + \beta - \beta \gamma^2 \eta^2}} - \gamma - \eta \right)
\]

where

\[
\lambda = \frac{4 \ V_d}{\beta \gamma \eta}
\]

(7)

Expressions for other coefficients are too long to be shown here. Nonetheless it becomes clear that, if we experimentally estimate the coefficients on the left hand side of the obtained analytical expressions, we can then compute the sought parameters \( \gamma \) and \( x_0 \) (provided that \( K_a \) is known). To illustrate this we plot in Figure 4 a relative deviation of the coefficient \( b_1(\lambda) \) from unity, which is its ideal value. The deviation is in terms of \( \gamma \) with \( \eta \) as a parameter. The value of \( \beta \) is set to 2, which is reasonable for the accelerometers under consideration. If we choose, for example, an estimate \( b_1() = 0.04\% \) and \( \eta = 0.4 \), we find out \( \gamma \) as -0.04 % (i.e., \( \gamma = 0.96 \)). In the actual assessment we solve a relevant set of nonlinear equations to simultaneously find all the needed quantities.

EXPERIMENTAL RESULTS

Successful application of the proposed approach to improving precision of inertial sensors hinges upon:

1. The fidelity of employed models of investigated nonlinearities, and
2. The accuracy of experimentally estimated values of Fourier coefficients.

The latter item is of utmost importance, since reliable identification of model coefficients aids a refinement of these models.
Accurate assessment of the Fourier coefficients depends upon (1) the ability of generating precise sinusoidal translational motion, and (2) suppression the impact of unavoidable, residual imperfections of motion on the recorded sensor signals [3]. These imperfections interact with distortions caused by the nonlinearity of investigated sensors and can greatly deteriorate the accuracy of results.

The quality of sinusoidal excitations generated in this research is characterized in Figure 5 by the magnitude spectrum of displacement obtained for 12 Hz oscillations with the amplitude of 120 µm. Of particular interest in an absence of the 2nd, 3rd or 4th harmonics. Figure 6 shows a representative plot versus time of the second harmonic of the excitation which remains in the generated signal when the motion controller can not provide a desirable quality of excitation, e.g. for oscillations with amplitudes below 10 nm. The same figure shows a signal from the output of the tested accelerometer. This latter signal has nearly the same amplitude but a significant phase shift, which indicates a strong interference between distortions introduced by the tested sensor and the impurity of the generated motion. A “true” distorted signal, computed according to the method proposed in [3] is also plotted.

CONCLUSIONS
Model based suppression of distortions is a convenient way for improving precision of MST based inertial sensors. Needed models can be developed by analyzing responses of these sensors to precisely controlled excitations. Robust mechanical integration of guidance, actuation and measurement functions in the air bearing stages used in this research facilitates achieving the quality of motion that is necessary to develop models for the best currently available accelerometers.

REFERENCES