INTRODUCTION
Well known limitations to milling productivity include tool wear, quasi-static and dynamic positioning errors of the tool relative to the workpiece, spindle error motions, fixturing concerns, programming challenges, and the process dynamics. Many research studies have been completed to address these issues. In this work, we are concerned with the limits imposed by the process dynamics. For a particular machine-spindle-holder-tool-workpiece-fixture system, the assembly frequency response function and force model establish the variation in the stability limit and location of the machined surface (due to forced vibrations) as a function of spindle speed. Frequency domain solutions for the stability limit have been available for many years [e.g., 1, 2]. The data is presented graphically in the form of stability lobe diagrams, which separate stable spindle speed-axial depth combinations from unstable pairs. Recently, a frequency domain solution to surface location error, SLE, was also demonstrated in the literature [3]. In this paper, we augment the stability limit with the user selected maximum allowable SLE to produce a new “super diagram” which identifies stable zones that also meet accuracy requirements.

NUMERICAL DEMONSTRATION
The “super diagram” is a combination of the information in a stability lobe diagram and the information obtained from surface location error (SLE) predictions. The predicted stability limit is at a fixed location given the dynamics of the system and the cutting conditions. However, the threshold for the acceptable levels of surface location error is user defined; therefore, user dependent accuracy requirements are incorporated with stability limitations. The parameters for the system selected for a numerical demonstration are shown in Table 1. These include the tool geometry, the system dynamics, and the cutting parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
<td>5x10^6</td>
<td>N/m</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Natural frequency</td>
<td>2400</td>
<td>Hz</td>
</tr>
<tr>
<td>Tool Diameter</td>
<td>6.35</td>
<td>mm</td>
</tr>
<tr>
<td>Helix angle</td>
<td>45</td>
<td>degrees</td>
</tr>
<tr>
<td>Number of teeth</td>
<td>4</td>
<td>teeth</td>
</tr>
<tr>
<td>Tangential cutting coefficient</td>
<td>700</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Normal cutting coefficient</td>
<td>210</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Feed per tooth</td>
<td>0.075</td>
<td>mm/tooth</td>
</tr>
<tr>
<td>Radial depth of cut</td>
<td>3.175</td>
<td>mm</td>
</tr>
</tbody>
</table>

The limits of surface location error used in this example are based on the absolute value of the calculated positive or negative SLE. However, this is not a generic requirement. For example, the user may only care if a part is overcut (too much material removed) because the part is then too small and cannot be corrected by a “finishing pass”. However, if the part is undercut then finishing operations can be performed to obtain accurate dimensions.

In order to complete the combined stability and SLE diagram, we first calculate the stability boundary using the method in [2]. We then compare a grid of test points (within the preselected domain) to the stability boundary to determine their feasibility. Next, we calculate SLE using the frequency domain method defined in [3] for each of the grid points. Penalties are finally applied to
the points as necessary. For example, an unstable point receives a penalty of two, whereas a point that is stable but outside the SLE limit gets a penalty of 1, while points that are stable and within the user’s SLE preference receive no penalty. Example diagrams are shown in Fig. 1 with varying SLE limits, but the same dynamic system and cutting conditions (see Table 1). This helps illustrate the effects of user preferences or requirements on the usable machining parameter space. In this figure, the black regions are unstable, the grey regions violate the SLE limit, and the white areas are the preferred zones for operation. Note that the grey areas are smaller in Fig. 1B due to the relaxed SLE limit.

FIGURE 1. Effects of the user’s SLE limits on the feasible machining domain for errors A) greater than 30 µm and B) greater than 70 µm at a 0.15 mm/tooth feed rate.

The feed rate during the machining process also affects the diagram. Although feed rate has only a second order affect on stability, it is an important parameter in the SLE calculations. For a linear force model, the force amplitude scales proportionally with feed rate. In other words, doubling the feed rate doubles the force amplitude. This, in turn, doubles the forced vibration amplitude, which doubles the surface location error as well. Figure 2 displays the same SLE limits as Fig. 1, but with half the feed rate. Note that Figure 2B shows no SLE in excess of the user’s limit of 70 µm.

FIGURE 2. Effects of the user’s SLE limits on the feasible machining domain for errors A) greater than 30 µm and B) greater than 70 µm at a 0.075 mm/tooth feed rate.

We note that, in general, the SLE varies with spindle speed for a particular axial depth of cut. The source of this SLE behavior is the system frequency response function magnitude and phase with the forcing frequency. Due to the change in phase with tooth passing frequency (spindle speed), the time lag between the force and vibration...
varies. Therefore, the location of the cutter in its vibration cycle when leaving the surface depends on the selected spindle speed. The dependence of SLE on the phase lag between the forcing function and displacement causes significant variation near the natural frequency (considering a single degree of freedom system for simplicity) because, for the lowly damped tool point frequency responses typically observed in practice, the phase changes rapidly in this frequency range.

The variation in SLE with spindle speed for an axial depth of 4 mm is demonstrated in Fig. 3. Here we see significant variation in SLE for spindle speeds near 36000 rpm. This corresponds to a tooth passing frequency of \( \frac{36000 \times 4}{60} = 2400 \) Hz, which is equal to the selected natural frequency for the system dynamics (Table 1). This spindle speed is the “best speed” normally selected for high-speed milling applications due to the increased stability limit. However, there is clearly a tradeoff between accuracy and the increased material removal rate. We also see local SLE variation at integer fractions of this best speed; specifically, 18000 rpm (1200 Hz tooth passing frequency).

Another condition we consider here is when the combination of system parameters and helical endmill geometry leads to a constant (or nearly constant) cutting force condition. Depending on the user’s SLE limits, this may expose a feasible zone that would not be present on a straight flute cutter under the same conditions. The location and viability of this parameter space depends on the helix angle and number of teeth on the cutter (the tool geometry determines the axial depth at which the phenomenon occurs), as well as the system dynamics (these determine if the axial depth is feasible or not at any given spindle speed). This phenomenon is demonstrated in Fig. 4 using the parameters described in Table 1. The user’s SLE limit is 42 \( \mu m \).

The axial depth, \( b \), at which constant force occurs is determined using Eq. 1, where \( d \) is the tool diameter, \( \phi_p \) is the angular tooth spacing, and \( \gamma \) is the helix angle of the cutter. The resulting constant force depth occurs at approximately 4.98 mm, and its multiples, using the selected cutter geometry.

\[
b = \frac{d \cdot \phi_p}{2 \tan(\gamma)}
\]

This results in a horizontal “step” in the SLE behavior at an axial depth of 4.98 mm. For constant force, the deflection calculation reduces to the force divided by the static stiffness and the SLE reduces substantially (Fig. 4).

![FIGURE 3. SLE variation with respect to spindle speed at an axial depth of 4 mm and a feed per tooth of 0.15 mm/tooth. Note the correspondence between sharp changes in SLE with the location of the stable zone peaks in Fig. 1.](image)

![FIGURE 4. Demonstration of the constant force phenomenon. The “step” behavior is observed at an axial depth of 4.98 mm with a 42 \( \mu m \) SLE limit (0.15 mm feed/tooth).](image)
Because the force behavior depends on the selected axial depth for helical endmills, SLE also varies strongly with axial depth (rather than increasing monotonically as we might expect). This is shown in Fig. 5 for a fixed spindle speed of 36000 rpm. Here we see that the SLE increases with axial depth up to about 1.25 mm, where it begins to decrease as we experience the combined effect of a phase change and the approach towards the 4.98 mm constant force depth. The “slope” of sinusoidal variation is caused by the linearly scaling of force (and deflection) with axial depth of cut.

![Figure 5](image1.png)

**FIGURE 5.** SLE variation with respect to axial depth at 36000 rpm and a feed per tooth of 0.15 mm/tooth.

Figure 6 shows the effect of the interaction between these two phenomenon at the first constant force axial depth. Note that despite the fact the SLE still varies with spindle speed the variation is much less than in Fig. 3. These small variations may be a result of the discretization of the tool and parameter space in the SLE algorithm as well as truncation errors of the axial depth at which these were calculated.

![Figure 6](image2.png)

**FIGURE 6.** Variation of SLE with respect to spindle speed at an axial depth of 4.98 micrometers and a feed per tooth of 0.15 mm/tooth.

**CONCLUSION**

This paper presented a method of combining surface location error and stability information in a new “super diagram”. When applied at the process planning stage, this diagram will enable users to make machining parameter decisions that respect the system dynamics.

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**REFERENCES**

