INTRODUCTION
The accuracy of thread components depends on the kinematic errors of lathes and thread grinding machines. Castro and Burdekin [1] have developed a method for kinematic accuracy assessment of lathes and thread grinding machines in screw-cutting mode and free of load. Figure 1 shows a schematic layout of the measuring system to assess the kinematic error on a CNC lathe based on this method. A Hewlett Packard (nowadays, "Agilent Technologies") 5529A laser interferometer system measures the longitudinal displacement of the saddle with respect to a cube-corner affixed on the chuck of the spindle. A laser diode system (hereafter LDS), in conjunction with a precision polygon mounted on the spindle assesses the angular position of the spindle. As the spindle rotates, a string of pulses is generated by the angular trigger device (LDS and a pulse generator). These pulses are used for triggering the laser interferometer. A software package has been elaborated for on-the-fly data acquisition and data analysis. The components of the kinematic error considered are progressive pitch error, the cyclic error and spindle-free float. This paper intends to study the individual measurement uncertainties associated with this laser calibration system. As a result of this analysis, the uncertainty budget and the measurement uncertainty of this calibrator are determined.

UNCERTAINTY COMPONENTS
The uncertainty components of this calibrator can be classified into four categories as follows: 1) uncertainties due to trigger device repeatability; 2) uncertainties inherent to the laser system; 3) uncertainties caused by environmental effects; 4) uncertainties caused by the installation. These uncertainty components can be divided into proportional and fixed terms. In this analysis, the calculation of the standard uncertainties associated with this calibrator is based on Type A and Type B evaluations in accordance with GUM [2].

Trigger Device Repeatability
The repeatability of the angular trigger device is ±0.2 arc sec with a level of confidence of 95% [1]. The standard uncertainty associated with this repeatability is computed by

\[ u_D (\text{arc sec}) = \frac{Re}{t_{s_5}(m)\sqrt{m}} \]  

(1)

where, \( Re \) is the repeatability of the trigger device; \( m \) is the number of measurements; \( t_{s_5}(m) \) is the Student factor for a given value of \( m \) with a level of confidence of 95%. In this case, \( Re = 0.2 \text{ arc sec}, \ m = 5, \ t_{s_5}(5) = 2.776 \). Substituting these values in Eq. (1), it results \( u_D = 0.0322 \text{ arc sec} \). Therefore, the standard
uncertainty in the position of the saddle due to
the repeatability of the trigger device is given by

\[
u_1(\mu m) = \frac{(0.0322s)\text{pitch}(mm)10^3}{(360)(3600s)}
\]  (2)

where, pitch(mm) is the nominal pitch (in millimetre) that is set on the machine for the
screw-cutting test.

**Uncertainties Inherent to the Laser System**

Some sources of uncertainty of the laser
interferometer system are intrinsic to this
equipment and affect the measurement
accuracy. They are considered in the following
sections.

**Laser Wavelength Uncertainty**

The principle of measuring of the laser
interferometer system is based on the value of
the wavelength of the laser light. If this value
changes for any reason, a measurement error
occurs. Lifetime wavelength uncertainty for the
laser heads is ±0.02 ppm (parts-per-million) with
optional calibration to MIL-STD 45662 [3]. In this
analysis, the measured distance \(L\) (hereafter) is
given in metres. Assuming a rectangular
probability distribution for the uncertainty in the
laser wavelength, its standard uncertainty is
calculated by

\[
u_2(\mu m) = \frac{L(m)(\pm 0.02 \times 10^{-6})}{\sqrt{3}} = \frac{0.02L}{\sqrt{3}}
\]  (3)

**Electronic Error**

In the Hewlett Packard (HP) system, the
electronics error is equal to the uncertainty of
the least resolution count. For this set-up, the
measurement resolution is 1 nm. A rectangular
distribution for the resolution of the laser
interferometer system’s measurement display is
assumed. This gives a standard uncertainty of

\[
u_3(\mu m) = \frac{0.001/2}{\sqrt{3}}
\]  (4)

**Optics Non-linearity**

The interferometer can contribute to
measurement uncertainty because of its inability
to separate perfectly the two laser beam
components (vertical and horizontal
polarisation). This error is referred to as optics
non-linearity. For a linear interferometer, the
peak-to-peak phase error is 5.4°, corresponding
to a distance of ±4.8 nm. Using a statistical
model, this value is ±4.2 nm. As this error is
periodic (sinusoidal), the "U"-shaped probability
distribution is assumed [4]. The standard
uncertainty of the optics non-linearity is
computed by

\[
u_4(\mu m) = \frac{0.0042}{\sqrt{2}}
\]  (5)

**Uncertainties caused by Environmental Effects**

These uncertainties are related to the influence
of the atmospheric conditions, thermal
properties of the machine under test and
temperature change of the optics on the
measurement uncertainty of this calibrator.

**Wavelength Compensation**

Since most laser interferometer systems operate
in air, it is necessary to correct for the difference
between vacuum wavelength and the
wavelength in air. This correction is referred to as
atmospheric or wavelength compensation.
Without this compensation, degradation in
system accuracy and repeatability would occur.
Assuming a standard and homogeneous air
composition, a 1 ppm error results from any one
of the following conditions [3]: a) a 1°C change
in air temperature; b) a 2.5 mm of mercury
change in air pressure; c) an 80% change in
relative humidity.

The thermometer and barometer utilized in the
experiments have an accuracy of ±0.2°C and ±2
mm Hg, respectively. The relative humidity was
estimated at an accuracy of ±20%. Assuming a
rectangular distribution, the standard
uncertainties in the assessment of the air
temperature, air pressure and relative humidity
are, respectively, 0.2 ppm/\sqrt{3} , 0.8 ppm/\sqrt{3} and
0.25 ppm/\sqrt{3}. The standard uncertainty of the
wavelength compensation is given by

\[
u_w (\text{ppm}) = \sqrt{(0.2/\sqrt{3})^2 + (0.8/\sqrt{3})^2 + (0.25/\sqrt{3})^2} = \frac{0.862}{\sqrt{3}}
\]  (6a)

\[
u_6(\mu m) = u_wL = \frac{0.862L}{\sqrt{3}}
\]  (6b)
Material Thermal Expansion
Correction for expansion (or contraction) may be necessary due to the variation of dimensions of the machine caused by temperature changes. This correction relates the distance measurement back to a standard temperature of 20°C. The method of correction is to change the laser wavelength compensation by an amount sufficient to correct for thermal expansion. This correction is known as material thermal compensation (MTC) and is defined as

$$MTC = \alpha(T_M - 20)$$

(7)

where, \(\alpha\) is the coefficient of material expansion (ppm/°C) and \(T_M\) is the temperature of the machine in Celsius (°C). MTC is given in ppm. For steel \(\alpha = 11.7\) ppm/°C.

The standard uncertainty of the MTC \(u_s\) is computed in accordance with GUM [2], considering that the input quantities \(\alpha\) and \(T_M\) are independent or uncorrelated. In this computation, the partial derivatives \(\partial \alpha \partial T = \alpha\) are employed. Besides, \(u_r = 0.2/\sqrt{3}\) is the standard uncertainty of the thermometer in °C (Celsius) and \(u_a = 0.1\alpha/\sqrt{3}\) is the standard uncertainty of the coefficient of material expansion \(\alpha\) in ppm/°C. Hence,

$$u_s(\mu m) = \sqrt{\left(\frac{2.34L}{\sqrt{3}}\right)^2 + \left[\frac{T_M - 20 | (1.17L)}{\sqrt{3}}\right]^2}$$

(8)

Optics Thermal Drift
This occurs in the linear interferometer in the form of a change in optical path length with changes in temperature. A typical value for optics thermal drift is 0.5 µm/°C [3]. This error only occurs when the temperature varies during the measurement time. As the tests were of short duration, the temperature did not change. Therefore, the correction for optics thermal drift is null. Assuming a rectangular distribution for the thermometer accuracy (±0.2°C), the standard uncertainty of the optics thermal drift is calculated as follows:

$$u_r(\mu m) = \frac{0.2}{\sqrt{3}} (0.5 \text{ }\mu m/°C) = \frac{0.1}{\sqrt{3}}$$

(9)

Uncertainties caused by the Installation
These sources of uncertainty are related to the installation of the laser interferometer system.

Deadpath Error
Deadpath error is caused by an uncompensated length of the laser beam between the interferometer and retroreflector, with the machine stage at zero position. Deadpath error can be calculated as follows [3]:

$$De = (D)(\Delta WCN)$$

(10)

where, \(De\) is the deadpath error, \(D\) is the deadpath distance and \(\Delta WCN\) is the change in wavelength compensation number during the measurement time. As the tests were of short duration, the deadpath error is negligible due to the fact that there were no significant changes in the atmospheric conditions, i.e. \(\Delta WCN = 0\).

The standard uncertainty of the deadpath correction is calculated by

$$u_s(\mu m) = Du_s = (0.21)(\frac{0.862}{\sqrt{3}}) = \frac{0.1810}{\sqrt{3}}$$

(11)

Cosine Error
The cosine error in ppm, when using the cube-corner reflectors, is approximately equal to

$$S^2/8L^2,$$

where \(L\) is the measured distance in millimetre and \(S\) is the lateral offset of the returning beam in micrometer [3]. In this application, \(S = 300 \mu m\) for a measured distance of 1 m. Hence, for \(L = 1000\) mm, the cosine error is given by

$$Ce(\mu m) = \frac{S^2}{8L^2}(1 m) = \frac{(300)^2}{8(1000)^2}(1 m) = 0.01125$$

(12)

Assuming a rectangular distribution, the standard uncertainty of the cosine error for \(L = 1 m\) is given by

$$u_s(\mu m) = \frac{0.01125}{\sqrt{3}}$$

(13)

Combined and Expanded Uncertainty
The standard uncertainties \(u_s\) and \(u_8\) are a function of \(u_w\). This means that these input
uncertainties are correlated. Assuming all of the other standard uncertainties are independent, the combined standard uncertainty $u_C$ is calculated by [2]

$$u_C = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2 + u_6^2 + u_7^2 + (u_5 + u_6)^2}$$  

(14a)

For $L = 1\, \text{m}$, $T_M = 20^\circ\text{C}$ and $\text{pitch} = 10\, \text{mm}$, the value of $u_C$ is

$$u_C (\mu\text{m}) = 1.48$$  

(14b)

The expanded uncertainty $U_p = k_p u_C = t_p (v_{eff}) u_C \cdot t_p$ is the Student factor for a given value of $v_{eff}$ with a level of confidence $p$ [2]. $v_{eff}$ is the effective degrees of freedom which is calculated by the Welch-Satterthwaite formula [2]. For this application, $v_{eff} = 5.03 \times 10^{15}$. As $v_{eff}$ has a huge value, the t-distribution (Student's distribution) approaches the normal probability distribution [2]. Thus, for a level of confidence of approximately 95%, the coverage factor $k_{95} = 2$, considering the normal distribution. Hence,

$$U_{95} (\mu\text{m}) = 2u_C = 2.96$$  

(15)

**TABLE 1.** The uncertainty budget of the kinematic error calibrator for $L = 1\, \text{m}$, $\text{pitch} = 10\, \text{mm}$ and $T_M = 20^\circ\text{C}$.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Probability distribution</th>
<th>Standard uncertainty ($\mu\text{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger device repeatability</td>
<td>normal</td>
<td>$u_1 = 0.0002485$</td>
</tr>
<tr>
<td>Laser wavelength uncertainty</td>
<td>rectangular</td>
<td>$u_2 = 0.012$</td>
</tr>
<tr>
<td>Electronics error</td>
<td>rectangular</td>
<td>$u_3 = 0.00029$</td>
</tr>
<tr>
<td>Optics non-linearity &quot;U&quot; shaped</td>
<td>rectangular</td>
<td>$u_4 = 0.0030$</td>
</tr>
<tr>
<td>Wavelength compensation</td>
<td>rectangular</td>
<td>$u_5 = 0.498$</td>
</tr>
<tr>
<td>Material thermal compensation</td>
<td>rectangular</td>
<td>$u_6 = 1.351$</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

1) A study of the uncertainty components that make up the uncertainty budget of the kinematic error calibrator has been carried out. The sources of uncertainty that have been considered are the following: trigger device repeatability, laser wavelength uncertainty, electronics error, optics non-linearity, wavelength compensation, material thermal compensation, optics thermal drift, deadpath correction, cosine error;

2) The measurement uncertainty of this calibrator was computed according to GUM [2]. This uncertainty analysis was calculated when this calibrator was applied on a CNC lathe. The expanded uncertainty of this calibrator was $\pm 2.96\, \mu\text{m}$ for a measured distance of 1 m and nominal pitch of $10\, \text{mm}$ with the machine at temperature of $20^\circ\text{C}$. This uncertainty is based on a combined standard uncertainty multiplied by a coverage factor $k_{95} = 2$, providing a level of confidence of approximately 95%.

**REFERENCES**


