MODELING AND SELF-CALIBRATION OF LASER TRACKER SYSTEM USING PLANAR CONSTRAINTS

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ABSTRACT

A kinematics model that describes not only the motion but also the geometric variations of LTS is developed. Through error analysis of the proposed model, it is claimed that gimbals axis misalignments and tracking mirror center offset are the key contributor to measuring errors of LTS. A self-calibration method is presented to calibrate LTS with planar constraints. Various calibration strategies utilizing plane constraints are proposed for different situations. For each calibration strategy, issues about the error parameter estimation of LTS are exploded to find out in which conditions these parameters can be uniquely estimated. These conditions reveal the applicability of the planar constraints to LTS self-calibration. Intensive experimental studies are conducted to check the validity of the theoretical results. The results show that the measuring accuracy of LTS has been increased by 5 times after using the proposed technique for calibration.

KEYWORDS: Laser tracking, Modeling, Self-Calibration, Planar constraints

INSTRUCTIONS

Establishing dimensions and relative locations of object surfaces is one of the basic tasks of geometric metrology. Coordinate measuring machines (CMM) are the most commonly used device for the dimension inspection, but they are severely restricted when measuring very large objects in workshop. Recently, laser tracking measuring technology has been demonstrated commercial success at overcoming this difficulty. Laser tracking system (LTS) is the latest portable 3D large size coordinate measuring system, which can measure three dimensional coordinates with laser beam by tracking a spherically mounted retroreflector (SMR). The LTS can track a SMR automatically in range of 35m. It can scan 1000 points per second with the accuracy of 1×10⁻⁶m. Among the large-scale measuring technologies, such as measuring arms, theodolites, and total station systems, LTS possesses many advantages, such as broad range, high speed, high accuracy and real time. It has been widely used in manufacturing measurement for dimension inspection, robot calibration, machine alignment, surface contour mapping, and reverse engineering, etc[1-2].

Although LTS is very accurate instrument, the measurement accuracy is highly dominated by the geometric errors in the tracking mirror mechanism. The proper calibration of LTS is essential for a good measurement. There have not been any standard device and method with broad range for the calibration of LTS. The purpose of this paper is to develop a kinematics model that describes not only the motion but also the geometric variations of LTS, and to propose a self-calibration method to calibrate LTS with planar constraints.

WORKING PRINCIPLE

LTS is composed of laser interferometer, beam splitter, tracking mirror mechanism, retroreflector, position sensitive detectors (PSD) and control unit as shown in Figure 1.

Figure 1 shows the working principle of LTS. A laser beam emitted from laser interferometer is divided into two orthogonally polarized beams, referred to as the measuring beam having a frequency of f₁ and the reference beam having a frequency of f₂. The reference beam is diverted to the receiver by polarizing beam splitter. The measuring beam proceeds through beam splitter and is directed by the tracking mirror to a retroreflector sited on the measured objects and reflected back by the retroreflector.
The returning beam then goes back to receiver through the tracking mirror and polarizing beam splitter. If the retroreflector changes its position, a frequency shift of \( \Delta f \) occurs and the relative displacement can be measured. Part of the returning beam is directed to the PSD to measure possible offset of the returning beam. The beam offset got by PSD is used to control the angle of the tracking mirror to make sure that the measuring beam is always pointed at the optical center of retroreflector. The change of horizontal and vertical angles of the measuring beam is measured by two angle encoders respectively. At the same time, the moving distance of the retroreflector is measured by a laser interferometer. Thus two angular values and one linear value are good enough to do 3D coordinate measurement of the retroreflector as well as the measured objects accurately [3].

**FIGURE 1. Working principle of LTS**

**MODELING**

Modeling is a process of finding a function to best describe the real system. As shown in Figure 2, three coordinate systems are established to model a LTS. They are the base coordinate system of the gimbal \( \{X_b,Y_b,Z_b\} \), the first link frame \( \{X_1,Y_1,Z_1\} \), and the mirror frame \( \{X_m,Y_m,Z_m\} \). \( O_b \), \( O_1 \), and \( O_m \) denote the origins of the three coordinate systems \( \{X_b,Y_b,Z_b\} \), \( \{X_1,Y_1,Z_1\} \), and \( \{X_m,Y_m,Z_m\} \) respectively. When the LTS is at its home position, \( X_b \) and \( X_1 \) are along the common normal directions of \( Z_b \) and \( Z_1 \). If \( Z_b \) and \( Z_1 \) intersect, \( X_b \) and \( X_1 \) are assigned perpendicular to both \( Z_b \) and \( Z_1 \). \( O_b \) and \( O_1 \) are the intersection points of the common normal of axes \( Z_b \) and \( Z_1 \) with \( Z_b \) and \( Z_1 \) respectively. Angular parameter \( \alpha_i \) and \( \theta_i \) are used to model the misalignment of the gimbal axes. Thus, \( \text{Rot}(x,\alpha_i) \) will align \( \{X_b,Y_b,Z_b\} \) with \( \{X_1,Y_1,Z_1\} \), and \( \text{Tran}(a_i,0,0) \) will bring the resultant frame to be coincident with \( \{X_1,Y_1,Z_1\} \). \( Z_m \) is parallel to the direction of the mirror surface normal and passes through \( O_m \). \( O_m \) is the intersection point of \( Z_m \) with the mirror surface. \( X_m \) may lie arbitrarily on the mirror surface. Two angular parameters \( \theta_1 \) and \( \alpha_2 \) as well as one translation parameter \( e_z \) are thus sufficient to model the case, in which the second rotation axis does not lie on the mirror surface. In other words, \( \text{Rot}(z,\theta_1 + \Delta \theta_2)\text{Rot}(x,\alpha_2) \) will align \( \{X_1,Y_1,Z_1\} \) with \( \{X_m,Y_m,Z_m\} \), and \( \text{Tran}(0,0,e_z) \) will bring the result frame to be coincident with \( \{X_m,Y_m,Z_m\} \).

In summary, the transformation matrix \( ^bT_m \) relating the coordinate system \( \{X_m,Y_m,Z_m\} \) to \( \{X_b,Y_b,Z_b\} \) is:

\[
^bT_m = \text{Rot}(z,\theta_1)\text{Rot}(x,\alpha_2)\text{Tran}(a_1,0,0) \\
\quad \text{Rot}(z,\theta_2 + \Delta \theta_2)\text{Rot}(x,\alpha_2)\text{Tran}(0,0,e_z)
\]

(1)

Nominally, \( \alpha_i = 90^\circ, \Delta \theta_i = 90^\circ, \alpha_i = 90^\circ \) and \( e_z = 0 \). The target coordinates can be computed through the following formula:

\[
r_p = R_r m c_z + t_r - (l_m + l_r - l_i)B(b_c)\vec{b}_r + l_j\vec{b}_i
\]

(2)

Where, \( l_s = -\vec{b}_c \cdot (R_r m \vec{c}_r) / \vec{b}_c \cdot \vec{b}_i \).
\[ B(h_i) = \begin{bmatrix} 2b_{cx}^2 - 1 & 2b_{cy}^2 & 2b_{cz}^2 \\ 2b_{cx}b_{cy} & 2b_{cx}^2 - 1 & 2b_{cz}b_{cy} \\ 2b_{cx}b_{cz} & 2b_{cy}b_{cz} & 2b_{cx}^2 - 1 \end{bmatrix} \]

is the familiar of the mirror image reflection matrix [4].

The beam incident on the mirror represented in the base coordinate system \{b\} is:

\[ c = R_y + m c_r + t_r + l_s b_i \]  \hspace{1cm} (3)

SELF-CALIBRATION

LTS calibration is a process of accuracy enhancement. It encompassed compensation for absolute accuracy deficiencies using compensation software. In this paper, we develop a self-calibration method with planar constraints to calibrate LTS. Planar constraints are chosen because they are the simplest types of surfaces and still provide sufficient constraints for LTS calibration. By restricting retroreflector motion to an arbitrary surface of constraints plane, the calibration problem is reduced to an optimization problem of minimizing a cost function \( C \) of the type [5]:

\[ C = \sum_{i=1}^{m} (n \times b \cdot r_i(x, \theta) - d)^2 \]  \hspace{1cm} (4)

Where, \( r_i(x, \theta) \) is the coordinates of retroreflector at \( i \)th position, which can be calculated from the forward kinematics model of LTS, \( n \) is the plane normal, \( b \) is the laser beam vector, \( d \) is the distance from the origin to the plane, \( x \) is a parameter vector to be calibrated, and \( \theta \) is the angular measurement of LTS.

The schematic to calibrate LTS using a plane constraint is shown in Figure 3. In the LTS, a retroreflector is moved along planar constrains, the gimbal rotation angles and the interferometer relative distance reading between the LTS and retroreflector are both recorded. The constraint plane is defined as a coordinate frame that is different from the base coordinate system. We develop a self-calibration algorithm as following. These steps are iteratively solved until the difference of two consecutive solutions is within a preset limit.

**Step 1.** The forward kinematics model of the LTS is used to compute \( r_i^0(x^0, \theta), i = 1, 2, \ldots, m \), for a given initial values \( x^0 \) of the nominal parameter vector of the LTS and the measuring data \( \theta, d \).

**Step 2.** The result of \( r_i^0(x^0, \theta) \) is then used to update the geometric parameter vectors consisting of \( x^0, b^0, d^0 \), where \( i \) indicates the iteration index. The equation (4) can be computed as:

\[ (n \times b)^T \cdot r_i^0(x^0, \theta) - d_i^0 + [(n \times b)^T]_T \nabla r_i^0(x^0, \theta) dx_i + r_i^0(x^0, \theta) \cdot n \times db - d(d) = 0 \]  \hspace{1cm} (5)

Where,

\[ \begin{bmatrix} \cos \theta_1^0 & \cos \theta_2^0 & -\sin \theta_1^0 & \sin \theta_2^0 \\ -\sin \theta_1^0 & \cos \theta_2^0 & -\cos \theta_1^0 & \sin \theta_2^0 \\ 0 & \cos \theta_2^0 & \cos \theta_1^0 & \sin \theta_2^0 \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \end{bmatrix} = C \begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix} \]  \hspace{1cm} (6)

Then,

\[ J^T \cdot d\rho = \text{Re}s^T \]  \hspace{1cm} (7)

Where, \( \rho = [x^0, \theta, d] \), \( \text{Re}s^T = \begin{bmatrix} (n \times b)^T \cdot r_i^0(x^0, \theta) - d_i^0 \\ \vdots \\ (n \times b)^T \cdot r_m^0(x^0, \theta) - d_m^0 \end{bmatrix} \)

The Jacobean matrix of \( J^T \) can be written as:

\[ J^T = \begin{bmatrix} (n \times b)^T \nabla r_1^0 & (r_1^0)^T C^0 & -1 \\ (n \times b)^T \nabla r_2^0 & (r_2^0)^T C^0 & -1 \\ \vdots & \vdots & \vdots \\ (n \times b)^T \nabla r_m^0 & (r_m^0)^T C^0 & -1 \end{bmatrix} \]  \hspace{1cm} (8)

The error parameters vector \( d\rho \) can be solved from equation (7). If \( |d\rho| < \varepsilon \) (a preset small number), the current \( \rho^i \) is the values of the parameters. If not, the parameter vector can be updated in the following formula:
\[ \rho^{t+1} = \rho^t + d\rho(t = t+1) \]  

(9)

**Step3.** The forward kinematics model of the LTS is used to compute \( r'(x', \theta)(i = 1, 2 \cdots m) \) for a given initial values \( x' \) of the calibration parameter vectors and the measuring data \( \theta, d \). Go back to step2.

Two kinds of self-calibration methods are available, which uses single-plane constraint and multi-plane constraints respectively.

**EXPERIMENTS**

A lot of experiments are done to check validity of the theoretical results. The experiment layouts are shown in Figure 4. The retroreflector is mounted on the Z-axis of a CMM. The CMM that rides the retroreflector moves in space to simulate the constraint planes. The LTS then tracks the retroreflector and measures the coordinates of the retroreflector in real-time.

**FIGURE 4. Experiment layouts**

In the experiments, 100 points are measured on each plane. Figure 5 shows the measuring results. The horizontal axis represents the number of measurement and the vertical axis denotes the measuring accuracy. The red line is the results, in which LTS has not been calibrated by the proposed technique. At the same time, the blue line denotes the results, in which LTS has been calibrated using the proposed technique. As shown in Figure 5, after calibrated, the mean measuring error of the LTS has been decreased from 99.67μm to 21.05μm, which means that the measuring accuracy has been increased by near 5 times.

**CONCLUSIONS**

(1) We develop a kinematics model, which describes not only the motion but also the geometric variations of LTS.

(2) A self-calibration method to calibrate LTS with planar constraints is proposed. Various calibration strategies using plane constraints are proposed for different situations.

(3) The experimental results have proved that using the proposed technique to calibrate a LTS, the measuring accuracy of LTS has been increased by 5 times.

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**REFERENCES**


