ADAPTIVE REPETITIVE CONTROL OF PIEZOELECTRIC ACTUATORS FOR NANO-PRECISION MOTION CONTROL

Chi-Ying Lin and Tsu-Chin Tsao
Mechanical and Aerospace Engineering
University of California, Los Angeles
Los Angeles, California, U.S.A.

INTRODUCTION
Piezoelectric actuators are used in a wide range of precision engineering applications. Nanometer level precision in static positioning has been achieved by using conventional Proportional-Integral-Differential (PID) feedback control with superior position sensors to compensate for hysteresis and creep existed in piezoelectric materials. Achieving nanometer level precision for dynamic motion is beyond the PID control capability. Earlier effort has contended that hysteresis was the main source of error and developed nonlinear hysteresis model and methods to compensate for this effect at low frequency, typically around 1 ∼ 10 Hz range [1, 2, 3, 4, 5]. This paper introduces linear model based high performance digital controllers and implementation results for dynamic positioning for higher frequency signals that contain both deterministic and stochastic components.

EXPERIMENTAL SYSTEM DESCRIPTION
The experimental system consists of a piezoelectric transducer (PZT) driven fast tool servo and a Pentium based PC that host analog interface to an amplifier and sensor signal conditioning electronics. Figure 1 shows the cross section of the fast tool servo. The fast tool servo includes a 20 µm stroke ceramic hollow PZT actuator and a 25 µm range capacitance sensor probe. The sensor signal conditioning electronics generate output voltage with 5 kHz signal bandwidth, 2.4 nm r.m.s. and 25 nm peak-to-peak noise level. The PZT is driven by a voltage amplifier. The digital control schemes were implemented by xPC target (©MathWorks Simulink) computer, which host a data acquisition board with 16-bit resolution in both analog-to-digital and digital-to-analog conversions (NI 6052E). The digital control sampling frequency is 18 kHz. The sampled data of the sensor voltage output have a noise level of 3.9 nm (r.m.s. value) resulting from a combination of sensor electronics and analog-to-digital conversion noise and resolution.

FIGURE 1. Cross section of a fast tool servo actuator system.

DIGITAL MOTION CONTROLS
To perform model based control design, system identification was performed on the system. A seventh order linear time invariant model was obtained by time-domain subspace system identification method. The frequency response of the model is shown in Figure 2, which shows a resonance frequency at 5.8 kHz. The aforementioned 18 kHz. sampling frequency was selected based on the system bandwidth shown here.

FIGURE 2. Bode plot of piezoelectric actuator model. Solid line: Magnitude; Dash line: Phase.
Figure 3 shows the overall control system block diagram, which includes robust feedback control, feedforward control, repetitive control, and disturbance cancelation compensator. The main feature of this control system structure is that the four different control blocks perform synergistically to produce superior dynamic performance while the individual controller design are generally decoupled from one another. In the block diagram shown, $G$ is the plant model, $r$ is the reference command input signal and $w$ is disturbance signal appearing at the plant output. Disturbance injected at the plant input or other location can be represented as equivalent output disturbance. $y$ is the plant output signal and $e$ is the tracking error, the magnitude of which represents the control system performance in its capability to follow the command input $r$ subject to disturbance input $w$. The control performance is thus characterized by the sensitivity functions $e/r$ for tracking and $e/w$ for disturbance rejection. When $G = G$, the following can be obtained from the block diagram:

$$E(z) = S_{fb}S_{rep}S_{ff}R(z) + S_{fb}S_{rep}S_{w}W(z)$$ (1)

As can be seen, the total error rejection transfer function is the product of the sensitivity functions of each control actions, thus the claimed synergism. This control structure has the ‘add-on’ or ‘plug-in’ feature in that each controller may be added on to existing ones and effect the total performance in a multiplicative manner. The individual error rejection (sensitivity function is as follows:

$$S_{fb} = \frac{1}{1 + GC_{fb}}$$
$$S_{ff} = z^{-L} - GC_{ff}$$
$$S_{rep} = \frac{1}{1 + T_{fb}C_{rep}}, \quad T_{fb} = 1 - S_{fb}$$
$$S_{w} = 1 - \hat{G}C_{w}$$

(2)

The absence of a particular control action simply makes the corresponding sensitivity function equal to unity.

**Robust Feedback Control**

A robust feedback controller [6] is designed for the plant model based on discrete-time $\mu$-synthesis to ensure a robust performance under plant uncertainties. The feedback controller typically provides a low bandwidth performance aimed for stabilizing and linearizing the plant dynamics.

**Feedforward Tracking Control**

To enhance the tracking performance, the feedforward compensator $C_{ff}$ can be added onto the feedback controller. The delay block $z^{-L}$ in Figure 3 is introduced to provide look-ahead or preview of the reference command signal. $L$ represents the number of samples in previewing the reference signal. $C_{ff}$ is a stable inversion of the plant dynamics, which can be designed in particular by zero-phase-error-tracking-control (ZPETC) method [7] or in general by optimal solution in [8], where optimal feedforward controller $C_{ff}$ minimizes the following equation with appropriate transfer function norms:

$$\min_{C_{ff} \in RL_{\infty}} \|z^{-L} - \hat{G}C_{ff}\|_{\infty}$$ (3)

**Repetitive Control**

The plug-in prototype repetitive controller $C_{rep}$, can be designed based on the the closed loop plant model $T_{fb}$

$$T_{fb} = \frac{\hat{G}C_{fb}}{1 + \hat{G}C_{fb}}$$ (4)

The repetitive controller includes an internal model and is in the following filter from [9]:

$$C_{REP} = \frac{q(z,z^{-1})z^{-N+1}}{1 - q(z,z^{-1})z^{-N}}K_{rep}(z)$$ (5)

$N$ stands for the period of the reference signal, $l$ is the sum of the plant delay and the controller delay which comes from the inversion of the unstable zero part in the closed loop plant $T_{fb}$. $q(z,z^{-1})$ is a zero phase low pass filter whose form is
where a & b satisfies \( a + 2b = 1 \) for unity d.c. gain and \( n \) is a positive integer. \( K_{\text{rep}}(z) \) can be obtained from the aforementioned ZPETC as an approximate stable inversion of \( T_{fb} \), or from robust control approaches by formulating and solving a \( \mu \)-synthesis problem \([10]\). Although \( q(z, z^{-1}) \) is a non-causal filter, the controller’s causality is still assured because of the cascaded long delay terms \( z^{-N} \) and \( z^{-N+1} \).

Adaptive Disturbance Rejection

The disturbance rejection performance by the robust feedback and repetitive control is effective for low frequency and periodic disturbance respectively. A general class of disturbance can be modeled as colored noise, which is the output of sending a white noise into a filter \( W \). The adaptive disturbance canceller \( C_w \) can be added to compensate for such type of disturbance and minimize an output measure. The minimization problem could be represented as

\[
\min_{C_w \in RH_{\infty}} ||W(1 - C_w \hat{G})||_2
\]  

(7)

In general the disturbance model \( W \) is unknown or varying. In such a case adaptive scheme such as Least Mean Squares (LMS) \([11]\) \( C_w \) may be employed to minimize the mean square value of the output \( y \) (or equivalently \( e \) with the absence of \( r \)). In the following experiments an non-adaptive \( C_w \) \([8]\) calculated for \( W = 1 \) will be included to compare the performance with the adaptive LMS scheme \([11]\).

**EXPERIMENTAL RESULTS**

Two sets of experiments were performed to demonstrate the effectiveness of the digital control schemes. The first one compares the tracking performance of the control action without the presence of disturbance \( w \). The reference signal tracked is a typical step-and-move type scan trajectory traversing at 30 Hz and 300 Hz respectively. Figure 4 shows the time domain experimental data with the reference profile for the case of 30 Hz trajectory. Table 1 shows the steady state tracking error r.m.s. values for both cases. For both trajectories adding feedforward controller to the existing feedback controller reduces at least 50% tracking error. Adding repetitive control substantially improves the tracking performance since at the multiples of fundamental frequency it provides the deep notches in the sensitivity function. For tracking 30 Hz profile case, repetitive control essentially reduces the tracking error to close to the sensor noise level.

In the second set of experiment disturbance \( w \) is introduced and then the disturbance canceller is added on. A sequence of color noises was generated by passing a white noise sequence through a low-pass filter of 50 Hz cut-off frequency. Injecting the disturbance to the control system significantly deteriorates the tracking performance, which is shown in Table 2. Adding non-adaptive disturbance canceller designed for \( W = 1 \) almost did nothing since it does not capture the disturbance dynamics. Adaptive disturbance canceller rejects disturbance inputs and shows superior performance. Figure 5 shows the magnitude of the disturbance input and the effects of adding adaptive disturbance cancellers for tracking the
30 Hz reference trajectory. Again, the adaptive control reduces the tracking error to close to the sensor noise level.

**TABLE 2. Tracking error r.m.s.: effects of adding disturbance canceller**

<table>
<thead>
<tr>
<th>Controls</th>
<th>30Hz profile r.m.s. (µm)</th>
<th>300Hz profile r.m.s. (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback + Feedforward + Repetitive</td>
<td>0.02488</td>
<td>0.02235</td>
</tr>
<tr>
<td>Adding non-adaptive Disturbance Canceller</td>
<td>0.02427</td>
<td>0.02221</td>
</tr>
<tr>
<td>Adding adaptive Disturbance Canceller</td>
<td>0.00579</td>
<td>0.01805</td>
</tr>
</tbody>
</table>

**FIGURE 5.** Experimental results for tracking 5 µm, 30 Hz scan profile subject to stochastic disturbance input. Top: Injected disturbance; Middle: Feedback + Feedforward + Repetitive Controls; Bottom: Feedback + Feedforward + Repetitive Controls + Adaptive Disturbance Canceller.

**CONCLUSION**

A digital control system that includes feedback, previewed feedforward, repetitive, and adaptive control for achieving superior performance in tracking and disturbance rejection in precision motion control is presented. Experimental results on a piezoelectric actuator driven fast tool servo demonstrate the synergistic effects of combining these linear control actions for achieving nanometer level dynamic precision.

**ACKNOWLEDGMENTS**

This work was supported in part by the National Science Foundation under grant no. DMI0327077, Nanoscale Science and Engineering Center for Scalable And Integrated Nanomanufacturing (SINAM).

**REFERENCES**


