A HIGH-SPEED AIR BEARING SPINDLE

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ABSTRACT
Spindles for micro cutting are characterised by high rotational speeds. An electric motor or an impulse air turbine is commonly used to drive spindles for micro cutting. In this paper a viscous turbine is proposed to drive the spindle. To determine the dimensions of the rotor a parameter study is carried out to achieve a maximum torque. The boundary conditions for the optimisation are a laminar and subsonic flow. The high-speed spindle is supported by aerodynamic journal bearings. To proof the concept of the viscous turbine a test-rig is designed, which is able to reach 500,000 rpm. For this working point the resonance frequencies are calculated.

INTRODUCTION
In robotics, the medical industry and many other fields there is a growing need for small parts with a complex 3D geometry. The materials used for these parts have specific properties and are complicated to manufacture. Conventionally, these parts are fabricated by cutting techniques. When maintaining conventional cutting speeds of at least 2 m/s and downscaling the tool tip to a diameter of 0.1 mm the rotational speed of the rotor must exceed 380,000 rpm. This is the reason that micro cutting is characterised by high rotational speeds. These speeds are commonly obtained by electric motors or impulse air turbines. The high-speed spindle discussed in this paper is driven by a viscous turbine and supported by aerodynamic bearings. The maximum attainable torque of the viscous turbine is very limited. Therefore, a parameter study is carried out to determine the maximum torque to be obtained. The slim rotor supported by air bearings with a relative small stiffness has lower resonance frequencies than the rotation frequency and as a consequence the spindle has to pass several critical speeds during start-up. With an analytical model the resonance frequencies of the proposed design are calculated.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>film thickness [m]</td>
</tr>
<tr>
<td>R</td>
<td>radius of the rotor [m]</td>
</tr>
<tr>
<td>ncell</td>
<td>number of cells</td>
</tr>
<tr>
<td>L</td>
<td>total length of one cell [m]</td>
</tr>
<tr>
<td>1/ncell</td>
<td>ratio length pocket / length cell</td>
</tr>
<tr>
<td>1/L</td>
<td>ratio film thickness / pocket height</td>
</tr>
<tr>
<td>1</td>
<td>ratio width of the cell / radius rotor</td>
</tr>
<tr>
<td>U</td>
<td>velocity of the rotor surface [m/s]</td>
</tr>
<tr>
<td>q</td>
<td>air flow per width in the turbine [m²/s]</td>
</tr>
<tr>
<td>p_sup</td>
<td>supply pressure [Pa]</td>
</tr>
<tr>
<td>p_amb</td>
<td>ambient pressure [Pa]</td>
</tr>
<tr>
<td>_p</td>
<td>p_sup − p_amb [Pa]</td>
</tr>
<tr>
<td>_</td>
<td>viscosity of air [Pa s]</td>
</tr>
<tr>
<td>_</td>
<td>density of air [kg/m³]</td>
</tr>
<tr>
<td>_</td>
<td>attitude angle [rad]</td>
</tr>
<tr>
<td>W</td>
<td>width of the journal air bearing [m]</td>
</tr>
<tr>
<td>_</td>
<td>rotational speed of the rotor [rad/s]</td>
</tr>
</tbody>
</table>

VISCOUS TURBINE MODELLING
The traction force of the viscous turbine is obtained by the shear stress of a laminar airflow along the surface of the rotor. The airflow is realized by inlets and outlets between the rotor and the housing. From one inlet air flows mainly across the pocket but some air flows across the ridge. A combination of a pocket and a ridge is called a cell. Figure 1 shows the cross section of one of the cells created along the shaft surface.

FIGURE 1. Sketch of the working principle of the viscous turbine
The pressure distribution in the pocket and over the ridge is assumed to be linear in the analytical model. The total airflow from the supply $q$ is equal to the sum of the air flow across the ridge and in the pocket. These are calculated using the one dimensional Reynolds equation valid for incompressible mediums (1) and laminar flows.

$$q = q_{\text{pocket}} + q_{\text{ridge}}$$

$$q = \frac{h^3}{12\eta \beta L} \Delta \rho \frac{U}{h} + \frac{h^3}{12\eta (1-\alpha) L} \frac{U}{2}$$  \hspace{1cm} (1)

The average of the traction is calculated with equation (2).

$$\tau = \tau_{\text{pocket}} + \tau_{\text{ridge}}$$

$$\tau = \frac{\alpha}{h} \left( \frac{\Delta p}{2 \beta \alpha L} - \frac{\beta \eta U}{h} \right) + (1-\alpha) \left( \frac{h}{2} \frac{\Delta p}{(1-\alpha) L} - \frac{\eta U}{h} \right)$$

$$\tau = \left( \frac{\beta}{1} - 1 \right) \frac{h}{2} \frac{\Delta p}{L} - \left( \alpha \beta + 1-\alpha \right) \frac{\eta U}{h}$$  \hspace{1cm} (2)

The Reynolds number across the pocket and the ridge are calculated following equation (3) and (4) respectively.

$$Re_{\text{pocket}} = \frac{\rho q_{\text{pocket}}}{\eta}$$  \hspace{1cm} (3)

$$Re_{\text{ridge}} = \frac{\rho q_{\text{ridge}}}{\eta}$$  \hspace{1cm} (4)

In case the Reynolds number is less than 1000, the flow is assumed to be laminar ($Re<1000$).

**PARAMETER STUDY VISCOUS TURBINE**

A parameter study is carried out to determine the dimensions of a cell that results in the maximum average shear stress on the rotor. Two boundary condition applied in the analytical model are 1) Reynolds number less than 1000 and 2) the maximum absolute velocity below the speed of sound.

The optimisation was carried out for a configuration with 3 cells, a film thickness ($h$) of 10 mm and a of 1/3. The results are listed in table 1.

**TABLE 1. Optimum geometry properties for maximum torque**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>$p$</td>
<td>2.2 bar</td>
</tr>
</tbody>
</table>

The influence of the ratio of the pocket length and the total cell length ($\gamma$) is small on the torque performance. Out of construction point of view is taken 0.9. Figure 2 shows the effect of the pocket depth where $\gamma=0.9$ and $\gamma=25$. To reach 500,000 rpm $\gamma$ needs to be 1/3.

![FIGURE 2. Torque of the turbine versus the rotational speed for different pocket depths](image)

To investigate whether the assumption of incompressibility by a rotational speed of 500,000 rpm is still valid, a FEM analysis is carried out. The compressible Reynolds equation was used to calculate the pressure distribution for a turbine running 500,000 rpm and with a pressure difference ($\Delta p$) of 2.2 bar. The pressure distribution is presented in figure 3 and shows an almost linear trend.

![FIGURE 3. Pressure distribution of a viscous turbine running 500,000 rpm with compressibility](image)

**VISCOUS TURBINE TEST-RIG DESIGN**

To proof the concept a prototype of the viscous turbine was designed with the dimensions derived from the optimisation. The housing is fabricated out of one piece. To study the effect of the pocket depth three compliant mechanisms are constructed in the housing. Screws between
the housing and the compliant mechanisms enable adjusting the depth of each individual pocket. In figure 4 the housing with the compliant mechanisms is shown.

**FIGURE 4.** Drawing of the housing of the viscous turbine with the three compliant mechanisms

At the top and bottom of the turbine aerodynamic journal bearings are placed to support the rotor in radial direction. In figure 5 an overall picture is shown.

**FIGURE 5.** Overall figure of the viscous turbine test-rig

On top of the radial journal bearing a hybrid magnetic/air thrust bearing is placed. This bearing has to support only the weight of the rotor. The axial bearing is not shown in figure 5.

**AIR BEARINGS**

As mentioned above the rotor is supported in radial direction by aerodynamic journal bearings. The characteristics of these journal bearings are dependent on the rotational velocity. Not only the stiffness but also the attitude angle of an aerodynamic journal bearing changes with the rotational speed. The angle between the load vector and the displacement vector defined as the attitude angle (\( \kappa \)) is depicted in figure 6.

**FIGURE 6.** Definition of attitude angle (\( \kappa \))

The attitude angle is 90° for aerodynamic plain bearings with concentric shaft position running at low velocity. For an infinite rotational speed this angle is 0°. In figure 7 the attitude angle versus the angular velocity is plotted for an aerodynamic plain journal bearing with a film thickness \( h \) of 10 \( \mu \)m, a radius \( R \) of 4.5 mm and a \( W/R \) ratio of 2. The stiffness has a coupling term caused by the attitude angle (\( \kappa \)). The stiffness is defined following (5) and (6).

\[
K_{xx} = \frac{F_{load}}{\Delta x} \cos(\kappa) = K \cos(\kappa) \tag{5}
\]

\[
K_{xy} = \frac{F_{load}}{\Delta x} \sin(\kappa) = K \sin(\kappa) \tag{6}
\]

\( K \) is the construction stiffness, which is plotted versus the angular velocity in figure 8.

**FIGURE 7.** Attitude angle \( \kappa \) versus the angular velocity

**FIGURE 8.** Construction stiffness \( K \) versus the angular velocity
RIGID ROTOR DYNAMIC MODEL
The dynamic behaviour of the viscous turbine with bearings has been investigated. The rotor is modelled as a rigid cylinder supported by air bearings. The bearing stiffness is represented by springs. A sketch of the model is presented in figure 9.

FIGURE 9. The model of the spindle

The resonance frequencies of this system are dependent on the stiffness of the bearings and the rotational speed of the rotor via the gyroscopic effect. The stiffness of the bearing varies with the rotational frequency. The resonance frequencies are calculated for the rotor of the test rig running at 500,000 rpm and supported by the aerodynamic bearings. The results are listed in table 2 and a Bode plot is presented in figure 10.

TABLE 2. Resonance frequencies

<table>
<thead>
<tr>
<th></th>
<th>x10^7 rad/s</th>
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</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.07</td>
</tr>
<tr>
<td>-2</td>
<td>0.11</td>
</tr>
<tr>
<td>-3</td>
<td>0.75</td>
</tr>
<tr>
<td>-4</td>
<td>1.10</td>
</tr>
</tbody>
</table>

FIGURE 10. The Bode plot of the rotor running at 500,000 rpm and supported by air bearings

CONCLUSIONS
A viscous turbine is proposed and the delivered torque of the turbine is optimised to reach 500,000 rpm. With the results of the optimisation a test-rig is designed to proof the concept. The resonance frequencies of the high-speed air bearing spindle are determined with a rigid rotor model. The rotor runs with 0.5 x 10^5 rad/s and has to cross 2 resonance frequencies.

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REFERENCES