# CROSS CALIBRATION FOR PRIMARY ANGLE STANDARDS BY A PRECISION GONIOMETER WITH A SMALL ANGLE INTERFEROMETER

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## INTRODUCTION

The traceability of angle standards like polygons, autocollimators and indexing tables can be accomplished by the circle closure principle or a primary artificial angular standard. The measuring system of primary standards with best positioning precision can be used to reduce the uncertainty for the calibration of polygons. Center for Measurement Standards (CMS) has built a primary standard by cross calibration method which combines a goniometer and a small angle interferometer, as shown in Fig. 1.

The goniometer was a high precision rotary air-bearing stage which was custom-made by Aerotech Company and provided a superior angular positioning performance. A Heidenhain RON905 incremental encoder was mounted on the shaft of the goniometer with positioning resolution of 0.035 arc second and positioning accuracy of 0.2 arc second. CMS designed and manufactured a sine bar mechanism upon the table of the goniometer. Two retro-reflectors were mounted on the each side of the rotation bar. A laser interferometer measured the relative displacement of those retro-reflectors. The length of the rotation bar was 400 mm, and the resolution of the laser interferometer was 10 nm. It could achieve an angular resolution of 0.035 arc second. The mechanism of the small angle interferometer was made of super Invar to reduce the deviation by temperature drift. All of the primary standard measuring system was supported by passive pneumatic isolators and covered in a black box to avoid the influences of temperature, airflow or light disturbance. It could get the best performance of the primary standard measuring system with minimum disturbance to reduce the measurement uncertainty.

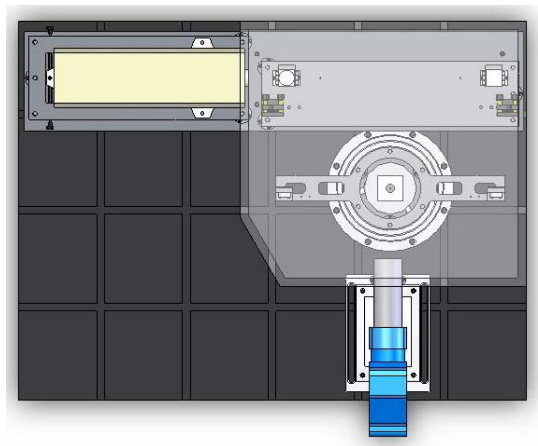

*FIGURE 1. Assembly diagram of the goniometer and the small angle interferometer system*

## CROSS CALIBRATION METHOD

The schematic diagram of the cross calibration system is shown in Fig. 2. The n-sided polygon to be calibrated is fixed at the center of the goniometer which is also to be calibrated.

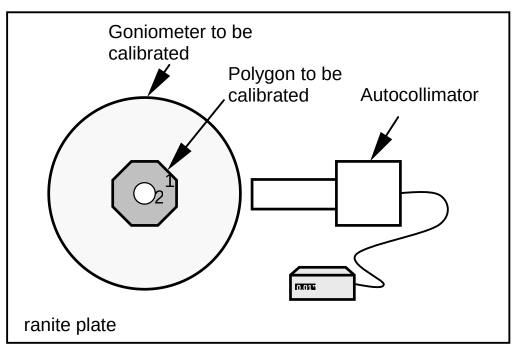


*FIGURE 2. Diagrammatic sketch of the cross calibration system of the polygon and the goniometer.*

Fig. 3 shows the measurement principle of the two adjacent faces from the polygon. In position 1, the face 1 of the polygon is located on the zero point of the goniometer and then aligned in order to bring it near the null of the autocollimator. At this moment, the readings of the goniometer and the autocollimator are  and , respectively. After rotating the goniometer with a nominal angle of the polygon  and aligned the face 2 of the polygon near the null of the autocollimator, the readings of the goniometer and the autocollimator are  and , respectively in position 2. The mathematical model could be represented by the following equation:

$$\begin{aligned} &\alpha_2 - \alpha_1 - \gamma - A_{1,2} = \beta_2 - \beta_1 - B_{1,2} \\ &B_{1,2} - A_{1,2} = -(\alpha_2 - \alpha_1 - \gamma + \beta_1 - \beta_2) = \alpha_{1,1} \end{aligned} \tag{1}$$

where  is the angular error between the face 1 and 2 of the polygon,  is the angular error between the position 1 and 2 of the goniometer, and  is the first measurement result of the face 1 of the polygon located on the zero point of goniometer. Then, rotating the goniometer with gradually until it brings the face 1 back to a position near the autocollimator null and repeats the measurement sequence. The complete set of measurement equations as following:

$$\begin{aligned} &B_{1,2} - A_{1,2} = \alpha_{1,1} \\ &B_{2,3} - A_{2,3} = \alpha_{1,2} \\ &\quad\vdots \\ &B_{n-1,n} - A_{n-1,n} = \alpha_{1,n-1} \\ &B_{n,1} - A_{n,1} = \alpha_{1,n} \end{aligned} \tag{2}$$

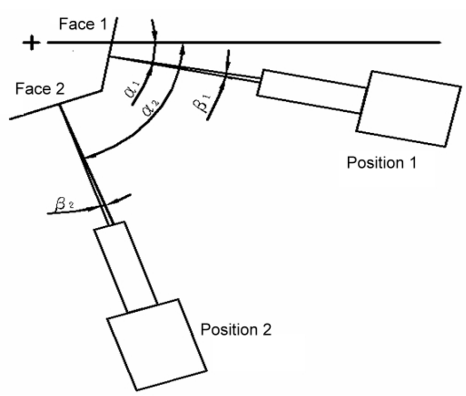


*FIGURE 3. Measurement principle of two adjacent faces of the polygon.*

Then, locating the face 2 of the polygon on the zero point of the goniometer and repeating the measurement mentioned above. We have linear equations:

$$\begin{aligned} &(B_{1,2} - A_{1,2}) = \alpha_{1,1} \\ &(B_{2,3} - A_{2,3}) = \alpha_{1,2} \\ &\quad\vdots \\ &(B_{n-1,1} - A_{n-3,n-2}) = \alpha_{n,n-1} \\ &(B_{n,1} - A_{n-2,n-1}) = \alpha_{n,n} \end{aligned} \tag{3}$$

There are two methods for analyzing the above equations. One is the direct summation method; another is the least square method [1].

On the basis of the direct summation method and the principle of circle closure:

$$\begin{aligned} &\sum_{i=1}^{n-1} A_{i,i+1} + A_{n,1} = 0 \\ &\sum_{i=1}^{n-1} B_{i,i+1} + B_{n,1} = 0 \end{aligned} \tag{4}$$

the deviations of the polygon and the goniometer can be separated directly, as shown in Tab. 1.

By dint of the least square method and the principle of circle closure, we combine the measurement relations, Eq. (3), with the two constraints, Eq. (4), and express the resultant set of linear equations in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \begin{bmatrix} B_{1,2} \\ B_{2,3} \\ B_{3,4} \\ \vdots \\ B_{n\ 1,n} \\ B_{n,1} \\ A_{1,2} \\ A_{2,3} \\ A_{3,4} \\ \vdots \\ A_{n\ 1,n} \\ A_{n,1} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{1,3} \\ \vdots \\ \alpha_{1,n-1} \\ \alpha_{1,n} \\ \alpha_{2,1} \\ \alpha_{2,2} \\ \alpha_{2,3} \\ \vdots \\ \alpha_{2,n-1} \\ \alpha_{2,n} \\ \vdots \\ \alpha_{n,1} \\ \alpha_{n,2} \\ \alpha_{n,3} \\ \vdots \\ \alpha_{n,n-1} \\ \alpha_{n,n} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

or AX=Y, where A is an coefficient matrix, X is a column vector of the deviations of the goniometer and the polygon and Y is an vector.

Hence, the least square solution is given by $X = (A^T A)^{-1} A^T Y$ .

*TABLE 1. Data sheet of cross calibration of a polygon and a goniometer.*

| Angle spread of polygon / Angle spread of indexing table | $0 \sim \gamma$ | $\gamma \sim 2\gamma$ | … | $(n-2)\gamma \sim (n-1)\gamma$ | $(n-1)\gamma \sim n\gamma$ | Accumulated value | Deviation of goniometer $B_{i,i+1}$ |
|---|---|---|---|---|---|---|---|
| $0 \sim \gamma$ | $B_{1,2} - A_{1,2}$ | $B_{1,2} - A_{2,3}$ | … | $B_{1,2} - A_{n-1,n}$ | $B_{1,2} - A_{n,1}$ | $nB_{1,2}$ | $B_{1,2}$ |
| $\gamma \sim 2\gamma$ | $B_{2,3} - A_{1,2}$ | $B_{2,3} - A_{2,3}$ | … | $B_{2,3} - A_{n-1,n}$ | $B_{2,3} - A_{n,1}$ | $nB_{2,3}$ | $B_{2,3}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $(n-2)\gamma \sim (n-1)\gamma$ | $B_{2,3} - A_{1,2}$ | $B_{2,3} - A_{2,3}$ | … | $B_{n-1,n} - A_{n-1,n}$ | $B_{2,3} - A_{n,1}$ | $nB_{n-1,n}$ | $B_{2,3}$ |
| $(n-1)\gamma \sim n\gamma$ | $B_{n,1} - A_{1,2}$ | $B_{n,1} - A_{2,3}$ | … | $B_{n,1} - A_{n-1,n}$ | $B_{n,1} - A_{n,1}$ | $nB_{n,1}$ | $B_{n,1}$ |
| Accumulated value | $-nA_{1,2}$ | $-nA_{2,3}$ | … | $-nA_{n-1,n}$ | $-nA_{n,1}$ | $\gamma = 360° / n$ | |
| Deviation of polygon $A_{i,i+1}$ | $A_{1,2}$ | $A_{2,3}$ | … | $A_{n-1,n}$ | $A_{n,1}$ | $\sum_{i=1}^{n-1} A_{i,i+1} + A_{n,1} = \sum_{i=1}^{n-1} B_{i,i+1} + B_{n,1} = 0$ | |

## SMALL ANGLE INTERFEROMETER

The optical principle of the small angle interferometer system is shown in Fig. 4 as an illustration. The optical source starts from the laser head and the polarizing beam splitter divides the laser beam into two components. The split beams are reflected at two retro-reflectors mounted on the each side of the rotation bar and it recombined at the polarizing beam splitter. At the end, the interference signal is received at the detector. The rotating angle can be determined as the following relationship:

$$\sin(\theta) = \frac{d}{L} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{d}{L}\right) \qquad (6)$$

where d is the optical path difference between two retro-reflectors, and L is the length of rotation bar, this value can be obtained by a given standard angle and Eq. (6).

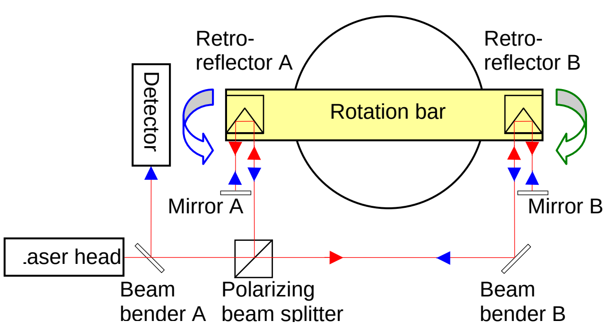


*FIGURE 4. Design principle of the small angle interferometer system.*

## EXPERIMENTAL RESULTS

In order to calibrate an angle standard of the goniometer, it could be done by the cross calibration. A polygon and the goniometer were calibrated by an autocollimator simultaneously, and the deviations of the polygon and the goniometer could be obtained at the same time. We used the 18-sided and 24-sided polygons to perform the cross calibration operation. Fig. 5 shows the measurement deviations of the goniometer which was determined by the cross calibrations with the 18-sided and 24-sided polygons. The maximum deviations were 0.09 and 0.12 arc second with uncertainty of 0.04 and 0.06 arc second, respectively. Each value of Y-axis meant the deviation between the relative value of X-axis and its foregoing value. Fig. 5 shows both of the deviation curves were agreed with each other, and the maximum difference was 0.02 arc second.

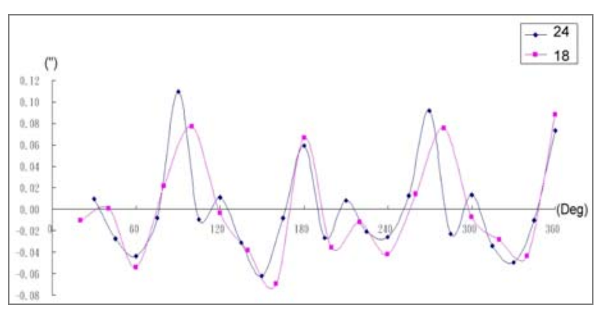

*FIGURE 5. Deviation curves of the goniometer determined by the cross calibrations with the 18-sided and 24-sided polygons.*

## CONCLUSIONS

The measurement range of the small angle interferometer on the goniometer is less than 16 degree. From the cross-calibration results of the 24-sided polygon, we got the angle deviation of 15 degree angle position of the goniometer. A self-calibration is used to determine the bar length with this angle deviation. For the future work, we will evaluate the uncertainty for the small angle interferometer.

## REFERENCES

1. Estler, W. T., 1998, Uncertainty Analysis for Angle Calibrations Using Circle Closure, Journal of the National Institute of Standards and Technology, 103(2): 141-151.