OPTIMAL TUNING OF A BIAXIAL SERVOMECHANISM USING A CROSS-COUPLED CONTROLLER AND DISTURBANCE OBSERVERS

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ABSTRACT
In order to improve the contouring accuracy of a biaxial servomechanism and the robustness against disturbance, a cross-coupled controller with a disturbance observer is studied, as well as the optimal tuning based on the integrated design methodology is proposed. Strict mathematical modeling and identification process of a servomechanism are performed first. An optimal tuning problem is formulated as a nonlinear constrained optimization problem including the relevant controller parameters of a servomechanism. The objective of the optimal tuning procedure is to minimize both contour error and settling time while satisfying constraints, such as relative stability and overshoots, etc. The effectiveness of the proposed optimal tuning procedure is verified through experimental results.

KEYWORDS
Contour error, Cross-couple control, Disturbance observer, integrated design, Kharitonov's theorem, Optimal tuning, Robustness, Servomechanism

INTRODUTION
Today a high-speed and high-precision servo system is one of the most important elements in assembly, information and telecommunication, automation, and aviation industries. As time passes, more complex and precision techniques, such as several-axis control, are required. Systems ought to have high contour precision and robustness against disturbance. Ohnishi and Umeno [1] proposed a two-degree of freedom controller to improve positioning accuracy and robustness against disturbance. Koren [2] and Srinivansan [3] defined the contour errors in multi-axis systems, and proposed methods to reduce contour errors using cross-coupled controllers. However, it is very difficult to select controller parameters because the cross-coupled control system is a multivariable, nonlinear, and time-varying control system. An optimization approach to the cross-coupled control system using genetic algorithms was studied before [4], but it does not include information about its mathematical modeling and stability.

In this paper, an optimal tuning method based on mathematical models is proposed for a biaxial servomechanism. To derive the mathematical model for optimization, an integrated design methodology [5] is applied as a systematic design tool. Experiments are performed to verify the performance of the developed system.

SYSTEM IDENTIFICATION
Models of a mechanical subsystem including a velocity controller are obtained through system identification processes. In order to identify xy-table shown in Fig.1, Simulink and toolbox of MATLAB are used for the identification. As an input signal Gaussian PRBS (Pseudo Random Binary Sequence) torque command is used and the velocity signal of a motor is used as an output. The first and second order models of x and y axes obtained through the identification processes are as follows:

$$G_{mx}(s)=\frac{16442}{s^2+360.7s+46580} \qquad G^n_{mx}(s)=\frac{45.96}{s+111.9}$$
$$G_{my}(s)=\frac{22700}{s^2+401.9+54090} \qquad G^n_{my}(s)=\frac{55.52}{s+114.9}$$

CONTOUR ERROR
The objective of the cross-coupled control is to reduce contour errors, defined as the shortest distance between the desired and actual contours, by coupling two axes and controlling the relative movements, rather than to reduce position errors by controlling each axis independently. In order to implement the cross-coupled controller, it is required that contour error should be calculated correctly. In case of

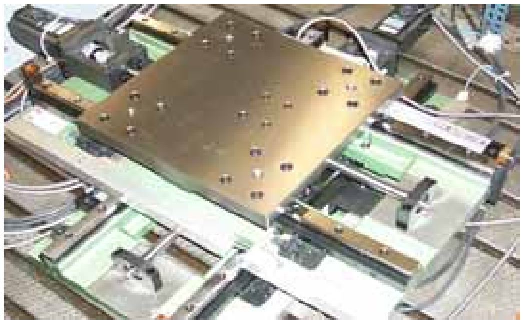

FIGURE 1. Experimental set-up (xy- table).

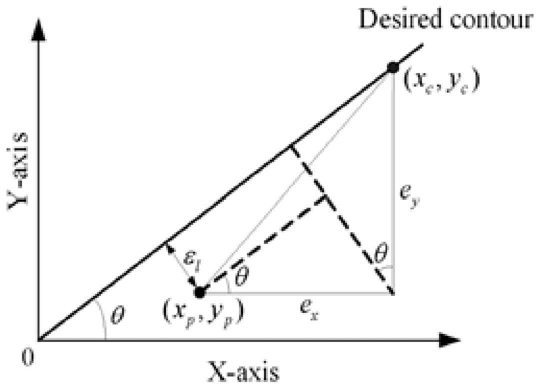

FIGURE 2. Contour error in linear motion.

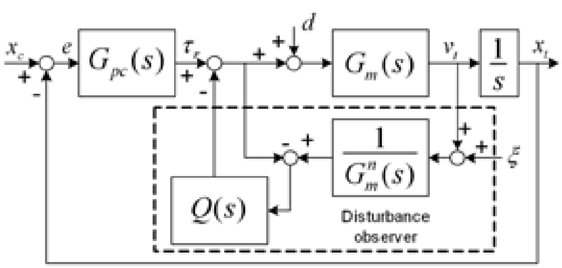

FIGURE 3. Block diagram of the disturbance observer.

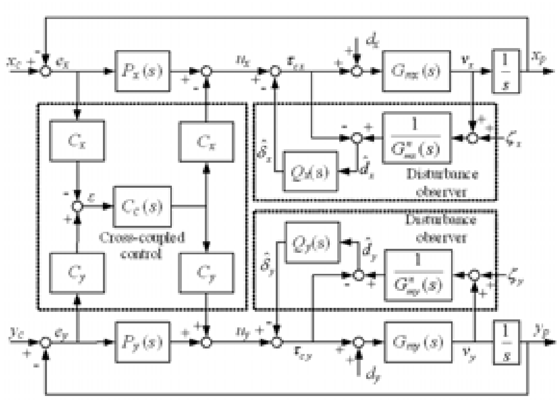

FIGURE 4. Block diagram of the overall system.

linear motion, the contour error is defined by ε_l as shown in Fig. 2. Using servo tracking errors, e_x and e_y, the contour error occurred in linear motion is given by

$$\varepsilon_l = -e_x \sin\theta + e_y \cos\theta \tag{1}$$

DISTURBANCE OBSERVER

In general, Q-filters have simple structure and good flexibility. They are appropriate to design a disturbance observer. Disturbance observer estimates disturbance using Q-filter as shown in Fig.3. Q-filters determine characteristics of disturbance observers. Disturbance elimination performance depends upon Q-filters. The velocity output v_t of the servo is represented with respect to the control input τ_r, disturbance d and measurement noise ξ as follows:

$$v_t = G_{\tau_r v_t}(s)\,\tau_r + G_{dv_t}(s)\,d + G_{\xi v_t}(s)\,\xi \tag{2}$$

where

$$G_{\tau_r v_t}(s) = \frac{G_m(s)G_m^n(s)}{G_m^n(s) + \{G_m(s) - G_m^n(s)\}Q(s)} \tag{3}$$

$$G_{dv_t}(s) = \frac{G_m(s)G_m^n(s)\{1 - Q(s)\}}{G_m^n(s) + \{G_m(s) - G_m^n(s)\}Q(s)} \tag{4}$$

$$G_{\xi v_t}(s) = -\frac{G_m(s)Q(s)}{G_m^n(s) + \{G_m(s) - G_m^n(s)\}Q(s)} \tag{5}$$

If $|Q(j\omega)|=1$, Eqs. (4) ~ (6) are given by

$$G_{\tau_r v_t}(s) \approx G_m^n(s) \tag{6}$$

$$G_{dv_t}(s) \approx 0 \tag{7}$$

$$G_{\xi v_t}(s) = -1 \tag{8}$$

Therefore, the real plant G_m acts like the nominal plant G_m^n and the effects of disturbance cancel each other. In this paper, the Q-filter should be selected to reduce disturbance in the low frequency region as $|Q(j\omega)|=1$, and eliminate measurement noise in the high frequency region as $|Q(j\omega)|=0$. Therefore, following type of the Q-filter is selected for the disturbance observer:

$$Q(s) = \frac{1 + \sum_{k=1}^{N-r} a_k (\tau s)^k}{1 + \sum_{k=1}^{N} a_k (\tau s)^k} \tag{9}$$

OVERALL SYSTEM

The structure of the overall system that combines the cross-coupled control system with a disturbance observer is constructed as shown in Fig. 4. In this paper, controllers, Q-filters and mechanical subsystems for the biaxial servo are defined as follows:

$$P_x(s) = K_{px} + \frac{K_{ix}}{S} \ , \ P_y(s) = K_{py} + \frac{K_{iy}}{S}$$

$$C_c(s) = K_{pc}$$

$$Q_x = \frac{1}{\tau_x S + 1} \ , \ Q_y = \frac{1}{\tau_y S + 1}$$

$$C_x = \sin\theta \ , \ C_y = \cos\theta$$

OPTIMAL TUNING PROCESS

Dynamic equations of the overall system including the mechanical subsystem are represented as a state space equation. Contrary to the cross-coupled control system for linear motions, the cross-coupling gains for circular motion are nonlinear and time-varying. It is difficult to represent this type of system in a state space equation. However, all curves can be converted into a sequence of micro straight lines by an interpolator. Therefore, the cross-coupled control system for an arbitrary curve is formulated by a state-space equation based on a series of linear motion trajectories.

In this paper, the objective of optimal tuning is to improve the response and contour accuracy. Optimal tuning of the multi-objective function is performed to minimize the settling time and the contour error as [7]

$$F(\mathbf{x}) = w_1 \varepsilon_l + w_2 t_s \qquad (10)$$

w_1 and w_2 are weighting factors of each objective function, ε_l is contour error, and t_s is settling time, respectively.

Modeling errors and uncertainties occur during system identification. Stability of the system has to be considered to tune the control gains. In order to secure robustness of the designed system in spite of the existence of uncertainties, relative stability criteria ought to be considered. Kharitonov theorem [6] is applied to consider the relative stability of the system which is formulated by the state-space equation. In order to secure the relative stability of the system with an uncertain parameter vector **q** in Eq. (11), following four polynomials called Kharitonov polynomials are obtained using the characteristic equation of the system.

$$\mathbf{q} = [n_{x1} \ m_{x1} \ m_{x2} \ n_{y1} \ m_{y1} \ m_{y2}]^{\mathbf{T}} \qquad (11)$$

where

$$n_{i1} \in [n_{i1}^-; n_{i1}^+], \ m_{ij} \in [m_{ij}^-; m_{ij}^+], \ i = x, y, \quad j = 1, 2$$

$$P^{++}(s,\mathbf{q}) = s^{10} + \alpha_9^+ s^9 + \alpha_8^+ s^8 + \alpha_7^- s^7 + \cdots + \alpha_1^- s + \alpha_0^-$$
$$P^{--}(s,\mathbf{q}) = s^{10} + \alpha_9^- s^9 + \alpha_8^- s^8 + \alpha_7^+ s^7 + \cdots + \alpha_1^+ s + \alpha_0^+$$
$$P^{+-}(s,\mathbf{q}) = s^{10} + \alpha_9^+ s^9 + \alpha_8^- s^8 + \alpha_7^+ s^7 + \cdots + \alpha_1^+ s + \alpha_0^-$$
$$P^{-+}(s,\mathbf{q}) = s^{10} + \alpha_9^- s^9 + \alpha_8^+ s^8 + \alpha_7^- s^7 + \cdots + \alpha_1^- s + \alpha_0^+$$

where

$$\alpha_i^+ = \max \alpha_i(\mathbf{q}), \ \alpha_i^- = \min \alpha_i(\mathbf{q}), \quad i = 1 \sim 9$$

If the roots of the Kharitonov polynomials have all negatives, the system is stable in spite of its uncertainty. If only the response of the system is considered, an excessive amount of overshoot occurs. To limit the overshoots, they should be considered as constraints. The Sequential Quadratic Programming (SQP) algorithm and optimum toolbox of MATLAB are applied to optimal tuning [7]. Plant uncertainty of ±1%, feedrate of 1*m/min* and weighting factors of $w_{1,2}$=0.5 are given for optimal tuning conditions. Table 1 shows constraints used in the optimal tuning procedure.

EXPERIMENTS

In order to verify the effectiveness of optimal tuning, experiments are performed in a precision x-y positioning system with a sampling period of 1*msec*. Instead of a circular motion 1024-polygon motion with the radius of 50*mm* is performed at the speed of 1*m/min*.

Fig. 5 shows the optimal tuning result when the xy-table is controlled only by the cross-coupled controller. Good control performance is obtained in areas where the direction of motion changes. On the other hand, the optimal tuning result obtained only through the disturbance observer shows better performance in areas where the direction of motion is constant as shown in Fig. 6. Fig. 7 shows the optimal tuning result obtained when the two controllers are combined together as shown in Fig. 4. Combining two controllers, good performance is obtained in various types of motion.

Table 1.Constarints for the optimal tuning.

Description	Equation
Relative stability	$g_1 : s_i^{++} < 0,\ s_i^{++} = \{s : P^{++}(s) = 0\},\ i = 1 \sim 10$ $g_2 : s_i^{--} < 0,\ s_i^{--} = \{s : P^{--}(s) = 0\},\ i = 1 \sim 10$ $g_3 : s_i^{+-} < 0,\ s_i^{+-} = \{s : P^{+-}(s) = 0\},\ i = 1 \sim 10$ $g_4 : s_i^{-+} < 0,\ s_i^{-+} = \{s : P^{-+}(s) = 0\},\ i = 1 \sim 10$
Maximum overshoot	$g_5 : M_x(\theta) < 5\%$, $g_6 : M_y(\theta) < 5\%$ $(0^\circ \le \theta \le 90^\circ)$ $M_x = \dfrac{\max(x_p(t)) - x_{ss}}{x_{ss}} \times 100\%$ $M_y = \dfrac{\max(y_p(t)) - y_{ss}}{y_{ss}} \times 100\%$

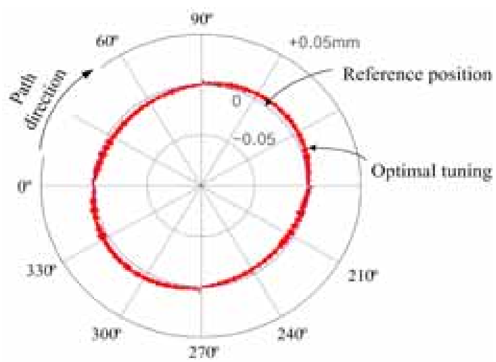

FIGURE 5. Cross-coupled control case.

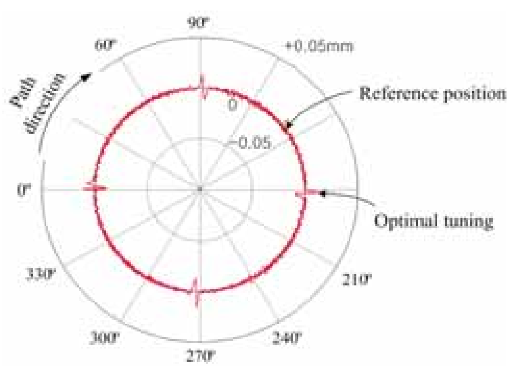

FIGURE 6. Disturbance Observer case.

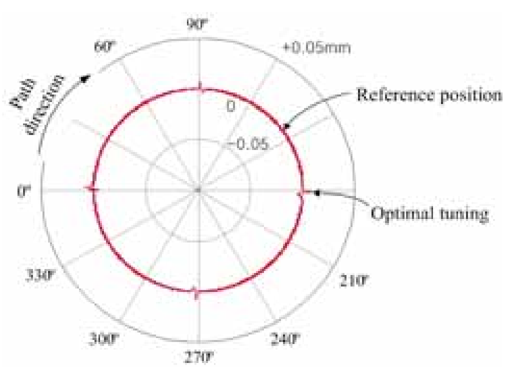

FIGURE 7. Cross-coupled control and Disturbance observer case.

CONCLUSIONS

The cross-coupled controller and disturbance observer are used to improve the response characteristics, contour accuracy and robustness against disturbance. The Optimal tuning method based on the integrated design method is proposed, and experiments are conducted on the xy-table. Experiments show that the contour error of the system with the optimal tuning results applied to it was smaller than any other systems. Combining the cross-coupled controller and the disturbance observer better performance is obtained in various types of motion.

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