CALIBRATION OF A PARALLEL MANIPULATOR WITH RESTRICTED DEGREES OF FREEDOM BY MEANS OF A REDUNDANCY MATRIX

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INTRODUCTION

The notions of forward and inverse coordinate transformation play an important role in the modelling and calibration of the kinematics and metrology in precision manipulators. Following an accepted convention, we shall define a Forward Transformation as a function mapping joint and/or sensor values into the spatial Degrees of Freedom (DoFs) at the manipulator's end-effector. Conversely, a function mapping the end-effector's DoFs into joint and/or sensor values shall be referred to as an Inverse Transformation. We treat both metrology and actuation aspects jointly, due to the similarity of the formalism.

One approach to dealing with machine errors in precision manipulators is to define parameters in the manipulator model, representing either deviations from the nominal layout or abstract coefficients in arbitrary expressions, such as polynomials. One can achieve this provided that the kinematics or metrology of the given manipulator can be modelled with closed-form analytical expressions. Based on the existence of such expressions in the presence of geometry parameters, manipulators can be divided into three types:

Forward Type

In this type, the forward coordinate transformation is a closed-form analytical function, while the inverse transformation is not. Examples of this type are manipulators with an open kinematic chain: certain Cartesian stages, single SCARA configurations, articulated Puma robots. Forward-type manipulators are usually applied to assembly and handling tasks, due to their relatively large workspace in comparison to size.

Inverse Type

Here, the inverse coordinate transformation is a closed-form analytical function, while the forward transformation is not. Examples of this type are parallel manipulators, i.e. ones possessing closed kinematic chains, which also allow for 6 DoF motion, such as the Stewart platform (Hexapod). Inverse-type manipulators are usually applied to precision measurement tasks, due to high motion reproducibility.

Mixed Type

In the mixed type, neither the inverse nor forward coordinate transformation can be expressed as a closed-form function, and both have to be computed with the use of numerical methods. Examples of this type are parallel manipulators, which allow for the motion of the end-effector in fewer than 6 DoFs. Some manipulators used in precision medical applications, as well as optical processing equipment are of the mixed type.

The issue of kinematics and metrology calibration for forward-type manipulators is well understood. A similar problem for the inverse type has been addressed by the Authors in [1]. <u>In this paper, the Authors show how to adapt the formalism developed in [1] to calibrating mixed-type manipulators.</u>

KINEMATIC AND METROLOGY MODELS

In a mixed-type manipulator, the kinematic model $\mathbf{f}_c$ consists of loop closure constraints formulated as implicit functions of the 6-DoF position $\mathbf{p}$ of the end-effector, as well as both the actuated ($\mathbf{q}_a$) and non-actuated joints ($\mathbf{q}_c$). Regarding the 6-DoF positions of all the other bodies in the manipulator, functionally and algebraically they are indistinguishable from the non-actuated joints and including them in $\mathbf{q}_c$ will

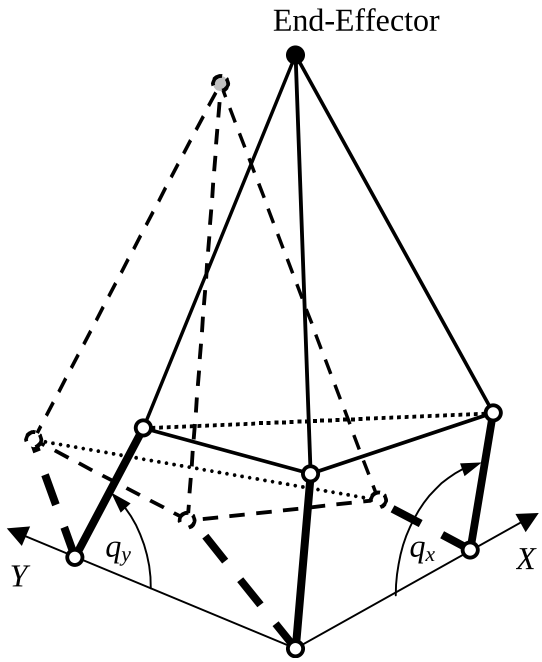

FIGURE 1. Example of a mixed-type mechanism, on which we chose to simulate our calibration method. The two actuated joints are q_x and q_y, they each have only 1 DoF, but together they fully determine the 6-DoF position of the tetrahedral end-effector. The linkage attached at the base frame origin adjusts itself in 2 DoFs as necessary. The subspace of positions reachable with the end-effector has only 2 DoFs

lead to coordinate transformations more useful in control. However, including the positions of the other bodies in **p** will also yield a correct calibration solution. The metrology model $\mathbf{f}_s$ describes the readings of some external sensors associated with the moving bodies to create redundancy beyond the limited number of DoFs in which the machine moves. In both the models, **g** is the vector of parameters representing the machine errors:

$$\begin{cases} \mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right)=\mathbf{0} \\ \mathbf{f}_s\left(\mathbf{p},\mathbf{g}\right)=\mathbf{q}_s \end{cases} \tag{1}$$

The advantage of such a formulation is to provide for joint values as function arguments and for sensors readings as function output, which allows analytical model expressions. For the purpose of a later exposition, we shall also define the following matrices as Jacobians of the model equations:

$$\begin{array}{lll} \mathbf{A}_c=\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{p}}, & \mathbf{B}_a=\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{q}_a}, & \mathbf{B}_c=\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{q}_c} \\ \mathbf{L}_c=\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{g}}, & \mathbf{A}_s=\dfrac{\partial\mathbf{f}_s}{\partial\mathbf{p}}, & \mathbf{L}_s=\dfrac{\partial\mathbf{f}_s}{\partial\mathbf{g}} \end{array} \tag{2}$$

Inverse Coordinate Transformation

Resolving the loop constraints leads to the actuated (and also non-actuated) joint values as functions of the end-effector position, hence an inverse coordinate transformation:

$$\mathbf{q}_a=\mathbf{f}_c^{-}\left(\mathbf{p},\mathbf{g}\right) \tag{3}$$

Since it is not guaranteed that the end position is reachable, we define the transformation as the minimum of a quadratic norm representing satisfaction of loop constraints ($\mathbf{M}_c$ can be used to establish a relative importance of the various DoFs):

$$E_c=\mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right)^T\mathbf{M}_c\mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right) \tag{4}$$

The sought minimum is attained when the gradient of criterion (4) is equal to zero (in the vicinity of the minimum, we are justified in neglecting the second term):

$$\min_{\mathbf{q}_a,\mathbf{q}_c} E_c \Rightarrow \begin{cases} \dfrac{\partial E_c}{\partial\mathbf{q}_a}=\left(\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{q}_a}\right)^T\mathbf{M}_c\mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right)=\mathbf{0} \\ \dfrac{\partial E_c}{\partial\mathbf{q}_c}=\left(\dfrac{\partial\mathbf{f}_c}{\partial\mathbf{q}_c}\right)^T\mathbf{M}_c\mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right)=\mathbf{0} \end{cases} \tag{5}$$

In general, the model equations are non-linear; applying the Newton-Raphson method to find a zero of (5) leads to the following iterative expression for the sought joint angles:

$$\begin{aligned} \begin{pmatrix}\mathbf{q}_a\\ \mathbf{q}_c\end{pmatrix}_{i+1} &= \begin{pmatrix}\mathbf{q}_a\\ \mathbf{q}_c\end{pmatrix}_i - \begin{pmatrix} \dfrac{\partial^2 E_c}{\partial\mathbf{q}_a^2} & \dfrac{\partial^2 E_c}{\partial\mathbf{q}_a\partial\mathbf{q}_c} \\ \dfrac{\partial^2 E_c}{\partial\mathbf{q}_c\partial\mathbf{q}_a} & \dfrac{\partial^2 E_c}{\partial\mathbf{q}_c^2} \end{pmatrix}_i^{-1} \begin{pmatrix}\dfrac{\partial E_c}{\partial\mathbf{q}_a}\\ \dfrac{\partial E_c}{\partial\mathbf{q}_c}\end{pmatrix}_i = \\ &= \begin{pmatrix}\mathbf{q}_a\\ \mathbf{q}_c\end{pmatrix}_i - \begin{pmatrix} \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_c \end{pmatrix}_i^{-1} \cdot \\ &\cdot \begin{pmatrix}\mathbf{B}_a^T\mathbf{M}_c\\ \mathbf{B}_c^T\mathbf{M}_c\end{pmatrix}_i \mathbf{f}_c\left(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}\right)_i \end{aligned} \tag{6}$$

Based on the inverse coordinate transformation, we shall also define two auxiliary matrices which will be necessary in further exposition:

$$\mathbf{A}_c^- = \frac{\partial \mathbf{f}_c^-}{\partial \mathbf{p}}, \quad \mathbf{L}_c^- = \frac{\partial \mathbf{f}_c^-}{\partial \mathbf{g}} \tag{7}$$

To obtain those matrices, we differentiate (5) over **p**:

$$\begin{aligned} \left(\frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_a}\right)^T \mathbf{M}_c \left(\frac{\partial \mathbf{f}_c}{\partial \mathbf{p}} + \frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_a}\frac{\partial \mathbf{q}_a}{\partial \mathbf{p}} + \frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_c}\frac{\partial \mathbf{q}_c}{\partial \mathbf{p}}\right) = \mathbf{0} \\ \left(\frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_c}\right)^T \mathbf{M}_c \left(\frac{\partial \mathbf{f}_c}{\partial \mathbf{p}} + \frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_a}\frac{\partial \mathbf{q}_a}{\partial \mathbf{p}} + \frac{\partial \mathbf{f}_c}{\partial \mathbf{q}_c}\frac{\partial \mathbf{q}_c}{\partial \mathbf{p}}\right) = \mathbf{0} \end{aligned} \tag{8}$$

and solve the resulting linear system for the sought matrix $\mathbf{A}_c^-$. Analogically, differentiating (5) over **g** leads to the other matrix, $\mathbf{L}_c^-$. Rewriting the results in terms of the previously defined closed-form Jacobians:

$$\begin{aligned} \begin{pmatrix} \mathbf{A}_c^- \\ \dfrac{\partial \mathbf{q}_c}{\partial \mathbf{p}} \end{pmatrix} = -\begin{pmatrix} \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_c \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{B}_a^T\mathbf{M}_c\mathbf{A}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{A}_c \end{pmatrix} \\ \begin{pmatrix} \mathbf{L}_c^- \\ \dfrac{\partial \mathbf{q}_c}{\partial \mathbf{g}} \end{pmatrix} = -\begin{pmatrix} \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_a^T\mathbf{M}_c\mathbf{B}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_a & \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_c \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{B}_a^T\mathbf{M}_c\mathbf{L}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{L}_c \end{pmatrix} \end{aligned} \tag{9}$$

We can now extend the notion of the inverse transformation and its Jacobians onto the sensor readings by appending the existing metrology model in a straightforward manner:

$$\begin{aligned} \begin{pmatrix} \mathbf{q}_a \\ \mathbf{q}_s \end{pmatrix} = \mathbf{f}_{cs}^-(\mathbf{p},\mathbf{g}) = \begin{pmatrix} \mathbf{f}_c^-(\mathbf{p},\mathbf{g}) \\ \mathbf{f}_s(\mathbf{p},\mathbf{g}) \end{pmatrix} \\ \mathbf{A}^- = \frac{\partial \mathbf{f}_{cs}^-}{\partial \mathbf{p}} = \begin{pmatrix} \mathbf{A}_c^- \\ \mathbf{A}_s \end{pmatrix}, \quad \mathbf{L}^- = \frac{\partial \mathbf{f}_{cs}^-}{\partial \mathbf{g}} = \begin{pmatrix} \mathbf{L}_c^- \\ \mathbf{L}_s \end{pmatrix} \end{aligned} \tag{10}$$

Forward Coordinate Transformation

In the presence of external sensors, the forward coordinate transformation provides the end-effector position (as well as the non-actuated joints and positions of bodies other than the end-effector) best fitting the given actuated joint and sensor values:

$$\mathbf{p} = \mathbf{f}_{cs}^+(\mathbf{q}_a, \mathbf{q}_s, \mathbf{g}) \tag{11}$$

By analogy to the inverse transformation, the best fit has to minimise a quadratic norm, which this time extends over both the loop constraints and the sensor readings:

$$\begin{aligned} E_{cs} = {} & \mathbf{f}_c(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g})^T \mathbf{M}_c \mathbf{f}_c(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}) \\ & + \left[\mathbf{f}_s(\mathbf{p},\mathbf{g}) - \mathbf{q}_s\right]^T \mathbf{M}_s \left[\mathbf{f}_s(\mathbf{p},\mathbf{g}) - \mathbf{q}_s\right] \end{aligned} \tag{12}$$

We demand again that the gradient of criterion (12) be equal to zero. In this case, the Newton-Raphson method yields the following iterative formula:

$$\begin{aligned} \begin{pmatrix} \mathbf{p} \\ \mathbf{q}_c \end{pmatrix}_{i+1} = \begin{pmatrix} \mathbf{p} \\ \mathbf{q}_c \end{pmatrix}_i - \\ \begin{pmatrix} \mathbf{A}_c^T\mathbf{M}_c\mathbf{A}_c + \mathbf{A}_s^T\mathbf{M}_s\mathbf{A}_s & \mathbf{A}_c^T\mathbf{M}_c\mathbf{B}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{A}_c & \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_c \end{pmatrix}_i^{-1} \cdot \\ \begin{pmatrix} \mathbf{A}_c^T\mathbf{M}_c & \mathbf{A}_s^T\mathbf{M}_s \\ \mathbf{B}_c^T\mathbf{M}_c & \mathbf{0} \end{pmatrix}_i \begin{pmatrix} \mathbf{f}_c(\mathbf{p},\mathbf{q}_a,\mathbf{q}_c,\mathbf{g}) \\ \mathbf{f}_s(\mathbf{p},\mathbf{g}) - \mathbf{q}_s \end{pmatrix}_i \end{aligned} \tag{13}$$

One auxiliary matrix we will need later is the Jacobian of the forward transformation over the geometry parameters. Differentiating (12) by analogy to (5) and (8), and solving the resulting linear system yields:

$$\begin{aligned} \mathbf{L}^+ = \frac{\partial \mathbf{f}_{cs}^+}{\partial \mathbf{g}}, \quad \begin{pmatrix} \mathbf{L}^+ \\ \dfrac{\partial \mathbf{q}_c}{\partial \mathbf{g}} \end{pmatrix} = \\ -\begin{pmatrix} \mathbf{A}_c^T\mathbf{M}_c\mathbf{A}_c + \mathbf{A}_s^T\mathbf{M}_s\mathbf{A}_s & \mathbf{A}_c^T\mathbf{M}_c\mathbf{B}_c \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{A}_c & \mathbf{B}_c^T\mathbf{M}_c\mathbf{B}_c \end{pmatrix}^{-1} \cdot \\ \begin{pmatrix} \mathbf{A}_c^T\mathbf{M}_c\mathbf{L}_c + \mathbf{A}_s^T\mathbf{M}_s\mathbf{L}_s \\ \mathbf{B}_c^T\mathbf{M}_c\mathbf{L}_c \end{pmatrix} \end{aligned} \tag{14}$$

CALIBRATION

During calibration, as the manipulator performs a sequence of movements, all the sensor readings and joint values are registered. By analogy to [1], we apply to them a composition of the forward and inverse transformations which we have defined above. This operation provides the anticipated sensor readings and joint values:

$$\mathbf{h}\left(\mathbf{q}_a,\mathbf{q}_s,\mathbf{g}\right)=\mathbf{f}_{cs}^{-}\left(\mathbf{f}_{cs}^{+}\left(\mathbf{q}_a,\mathbf{q}_s,\mathbf{g}\right),\mathbf{g}\right) \quad (15)$$

The discrepancy between the anticipated and the measured values depends on the model parameters effective in the above forward-inverse composition:

$$\mathbf{\Delta q}=\mathbf{h}\left(\mathbf{q}_a,\mathbf{q}_s,\mathbf{g}\right)-\begin{pmatrix}\mathbf{q}_a\\ \mathbf{q}_s\end{pmatrix} \quad (16)$$

The aim of calibration is to find the parameter values for which a quadratic norm of the above discrepancy (16) over all the measurement positions assumes a minimum:

$$E_{cal}\left(\mathbf{g}\right)=\sum_{k=1}^{K}\mathbf{\Delta q}_k^T\mathbf{M}_{as}\mathbf{\Delta q}_k \quad (17)$$

Matrix $\mathbf{M}_{as}$ in (17) reflects the relative accuracy of the various sensors and actuators. It can be shown that the calibration result will be optimal in the sense of Maximum Likelihood (ML) if $\mathbf{M}_{as}$ is the inverse of the sensor-actuator covariance matrix. We demand, once again, that the gradient of criterion (17) be equal to zero:

$$\min_{\mathbf{g}} E_{cal}\left(\mathbf{g}\right)\Rightarrow\frac{\partial E_{cal}}{\partial\mathbf{g}}=\sum_{k=1}^{K}\left(\frac{\partial\mathbf{h}}{\partial\mathbf{g}}\right)^T\mathbf{M}_{as}\mathbf{\Delta q}=\mathbf{0} \quad (18)$$

Applying the Newton-Raphson method to finding a zero of (18) leads to the following iterative formula:

$$\begin{aligned}\mathbf{g}_{i+1}&=\mathbf{g}_i-\left(\frac{\partial^2 E_{cal}}{\partial\mathbf{g}^2}\right)_i^{-1}\left(\frac{\partial E_{cal}}{\partial\mathbf{g}}\right)_i=\\&=\mathbf{g}_i-\left(\sum_{k=1}^{K}\mathbf{H}^T\mathbf{M}_{as}\mathbf{H}\right)^{-1}\cdot\sum_{k=1}^{K}\mathbf{H}^T\mathbf{M}_{as}\mathbf{\Delta q}_k\end{aligned} \quad (19)$$

where the matrix **H** is a generalised version of the redundancy matrix introduced by the Authors in [1] and can be computed with the previously introduced auxiliary matrices, cf. (7), (10) and (14):

$$\mathbf{H}=\frac{\partial\mathbf{h}}{\partial\mathbf{g}}=\frac{\partial\mathbf{f}_{cs}^{-}}{\partial\mathbf{p}}\frac{\partial\mathbf{f}_{cs}^{+}}{\partial\mathbf{g}}+\frac{\partial\mathbf{f}_{cs}^{-}}{\partial\mathbf{g}}=\mathbf{A}^{-}\mathbf{L}^{+}+\mathbf{L}^{-} \quad (20)$$

SIMULATION EXAMPLE

The operation of the method has been demonstrated in a Mathematica simulation on a simple example involving a 2-DoF manipulator, whose motions in the remaining 4 DoFs are restricted by a system of mechanical linkages, and whose spatial position is measured in 3 DoFs with external displacement sensors (Figure 1).

The model involves 3 moving bodies in addition to the tetrahedral end-efector (2 actuated bodies and 1 compliant linkage), 2 actuated joints, as well as 2 non-actuated joints (at the compliant linkage). The external sensors are idealised as providing absolute distances between 3 stationary surfaces (X, Y and Z planes) in the reference frame and a point on the moving end-effector (the top vertex of the tetrahedron). The geometry parameters include the lengths of all linkages and the dimensions of the tetrahedron.

In the course of the simulation, the stage is brought to $5^2 = 25$ different positions representing combinations of 5 values in each of the 2 available DoF. The simulation shows that all the geometry parameters can be calibrated from the sensor and actuator readings.

CONCLUSIONS

Our paper presents a methodology for calibrating parallel manipulators with restricted degrees of freedom. We propose a general form of an analytical kinematic model with joint values as arguments of loop closure constraints and an analytical metrology model with sensor readings as its output. We then derive iterative expressions for the inverse and forward coordinate transformations between the end-effector position and the joint and sensor values. Finally, we use the composition of both the transformations to calibrate the manipulator model by analogy to [1]. The advantage of the method is the constant, small size of the calibration matrix, irrespective of the number of measurement points.

REFERENCES

1. Meyer, P.J., van Eijk, J., 2004, In-Situ Calibration of a Redundant Measurement System for Manipulator Positioning, *Proceedings of the 2004 ASPE Topical Meeting, State College, PA, U.S.A.*