A QUANTITATIVE, CONSTRAINT-BASED DESIGN METHOD FOR MULTI-AXIS FLEXURE STAGES FOR PRECISION POSITIONING EQUIPMENT

Jonathan Hopkins and Martin L. Culpepper
Department of Mechanical Engineering
Laboratory for Manufacturing and Productivity
Massachusetts Institute of Technology
Cambridge, MA 02193 USA

SYNOPSIS
In this paper we introduce a new approach to the conceptual design of flexures and compliant mechanisms that draws upon the constraint-based design (CBD) principles of complimentary patterns and principles from projective geometry. We use projective geometric primitives to create generalized sets of 3D geometries (volumes, surfaces and lines) that represent either (1) a flexure mechanism's degrees-of-freedom or (2) the possible flexure constraints that result in these degrees of freedom. These shapes are intuitive for a designer to understand and the shapes may be represented mathematically. As such, this approach provides the necessary elements for a computer (mathematics) and a designer (shapes and visualization) to cooperate during design.

The final result of the work is to provide a set of topological shapes that represent the freedoms and constraints of parallel flexure systems. These freedom and constraint topologies (FACT) provide the foundation for creating software-based design tools for precision flexures. The explanation of FACT is somewhat abstract; therefore some hardware-focused designers may find that a bit of patience is required before the utility becomes apparent to precision engineering applications. This paper ends with a section discussing the practical utility of the abstract concepts.

INTRODUCTION
The essence of constraint-based design is the process of selecting and arranging machine elements (e.g. flexures) in a geometric layout that endows a device with desired positioning and alignment performance. Constraint-based design principles are central to precision engineering as the layout of a machine's constraints set limits on a device's degrees-of-freedom, stiffness, load capacity, repeatability, stability, etc… For over 100 years [1], CBD has been practiced by using a combination of linear visualization techniques (intersection of constraint and rotation lines), practical experience and rules of thumb. These conventional techniques are easily used to create precision flexure stages wherein the motions are planar (x, y and theta-z). A great deal of CBD experience, and sometimes trial-and-error, are required to design flexures that are simultaneously capable of planar and non-planar (theta-x, theta-y and z) motions.

The subject of this paper is based upon (1) the visualization of many different types of complex 3D volumes/surface and (2) the different interactions between these shapes. The unique aspect of this method is that the shapes and their well-known equations enable designers to approach constraint-based design using (1) the conventional semi-qualitative design rules of thumb and (2) new qualitative shapes and the quantitative equations that describe them. There are over 50 shapes-equations, that represent the full range of solutions to multi-axis flexures. As such, we are unable to discuss all of them in detail herein. We choose instead to pay special attention to the case where three constraints and three degrees of freedom exist.

BACKGROUND

Degrees of freedom and constraints
A first principle of constraint-based design is that a rigid body has six degrees of freedom — three orthogonal translations and three orthogonal rotations – and any non-redundant constraint upon a body removes one degree-of-freedom. The equation that expresses this principle is:

$$R = 6 - C \tag{1}$$

where C is the number of non-redundant constraints and R is the number of independent degrees of freedom. This is illustrated in Fig. 1.

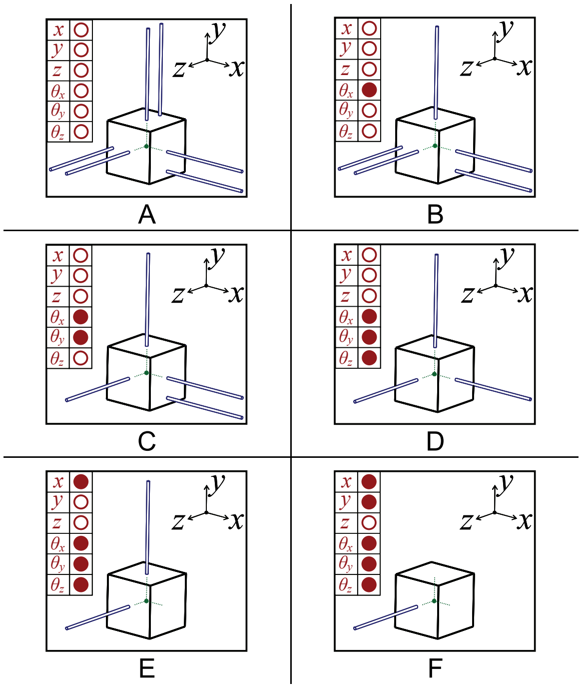

FIGURE 1. CONVENTIONAL DESCRIPTION SHOWING THE RELATIONSHIP BETWEEN CONSTRAINTS AND FREEDOMS. OPEN CIRCILES REPRESENT FREEDOMS.

It is customary in CBD to idealize flexure constraints as shown in the top of Fig. 2 [2,3]. Such constraints have zero compliance along their line of action and they have zero stiffness orthogonal to their line of action.

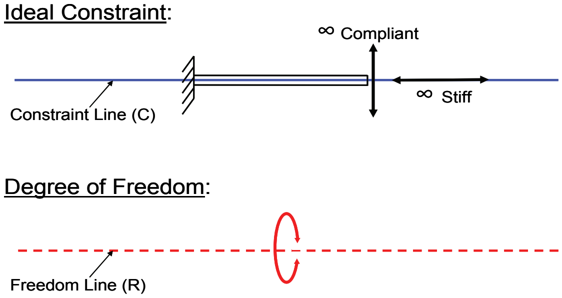

FIGURE 2. MODELING CONSTRAINTS AND FREEDOMS VIA CBD

This type of modeling approach is appropriate for finding the realistic directions of greatest compliance for a rigid stage constrained by flexible beams. Blanding observed that any stage's degrees-of-freedom could be represented by rotations about freedom lines [2]. This is shown at the bottom of Fig. 2. Blanding noted that even pure translational degrees of freedom could be modeled as a rotational freedom line that is perpendicular to the direction of translation located infinitely far away from the object translating.

<u>The principle of complimentary patterns</u>

Blanding's Rule of Complementary Patterns [2] defines the qualitative relationship between constraints and degrees of freedom. It states that every freedom line intersects every constraint line. Figure 3 shows an example of a block constrained by five non-redundant constraints. From Eq. 1 we would expect the block to move with only one degree-of-freedom. Blanding's Rule of Complementary Patterns finds this pure rotational freedom line to be the dotted red line. The constraints all intersect the freedom line (red dotted line). The principles of projective geometry are useful here to show that parallel lines intersect at infinity. The constraints at the bottom left therefore intersect the freedom line "at infinity". The constraints at the top and right intersect the freedom line in finite space, i.e. the top-middle-rear of the rigid block.

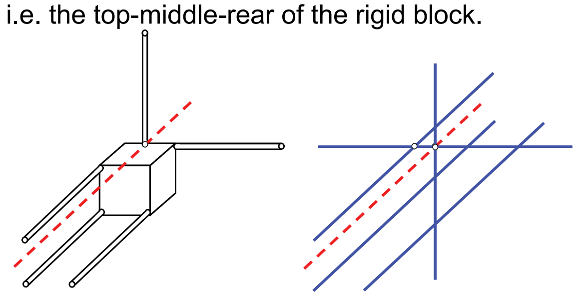

FIGURE 3. EXAMPLE OF ONE DEGREE-OF-FREEDOM FLEXURE SYSTEM

It is important to note that since the relationship between constraints and degrees of freedom is independent of the stage's shape, size, and location, the block could be removed from the picture entirely leaving only constraint lines (blue) and freedom (red) as shown in Fig. 3 to define the design problem.

THE IMPORTANCE OF DIFFERENT DESIGN CONCEPTS - FINDING ALL POSSIBLE FREEDOM AND CONSTRAINT TOPOLOGIES

The preceding principles help designers create functional designs, but they do not provide designers with a way to consider alternate concepts that possess the same functional characteristics. For instance, if one desires a flexure that has the degree-of-freedom as shown in Fig. 4, the design that is shown is necessary and sufficient – but what other designs could provide the same function? It is hard to describe the design in Fig. 4 as the best design without knowing what other designs could be used.

In many instances there are an infinite number of freedom lines that satisfy the Rule of Complementary Patterns. Consider the example shown in Fig. 4 of a block constrained by two non-redundant constraints whose constraint lines (blue) intersect inside the block. The degree-of-freedom solutions include every freedom line (red) that intersects both constraints (blue).

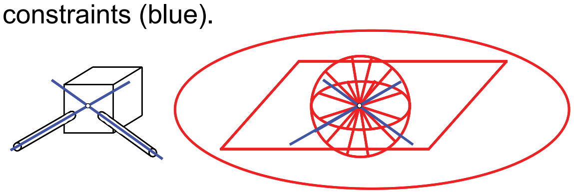

FIGURE 4. FINDING THE INFINITE PURE ROTATIONAL FREEDOM LINES (RED) FOR A BLOCK CONSTRAINED BY TWO CONSTRAINTS WHOSE CONSTRAINT LINES (BLUE) INTERSECT INSIDE THE BLOCK

It is also important to note that the constraints on the left of Fig. 4 are equivalent to any other constraints [2,3] in the plane of the first two constraints, which share the same intersection of the first two constraints. Thus a pencil can be used to represent the constraints in this design. From projective geometry, a pencil is a planar geometric shape that contains all possible coplanar lines that intersect in one point. The pencil in this case would share the intersection of the two constraints on the left of Fig. 4 and lie in the plane of the constraints shown in Fig. 4. We now have a geometric shape, which represents all arrangement of constraints that provide the same function as the design in Fig. 4.

We may do the same thing with the four degrees of freedom that must exist in this design. Considering the right side of Fig. 4 – the sphere represents an infinite number of freedom lines that satisfy this condition and intersect the constraint lines' intersection point. The plane drawn in Fig. 4 represents every freedom line that lies on the plane of the constraint lines and would therefore intersect the constraints. The hoop in Fig. 4 represents a single freedom line that is "infinitely" far from the center/intersection point within block. From projective geometry, a hoop of radius approaching infinity is equivalent to a straight line whose distance from the hoop's center is approaching infinity. A rotation about this line/hoop would yield a translation degree-of-freedom that is normal to the plane of the hoop and along the axis of symmetry of the hoop. Screw-like motions with non-zero pitch also exist as degree-of-freedom solutions for this example and could be found using screw theory. These shapes are omitted from Fig. 4 for clarity.

The preceding example shows that there is a multiplicity of freedoms that may be obtained with this design and they may be represented in general terms using the projective geometric shapes in Fig. 4. The shapes for freedoms and constraints are taken from projective geometry and equations/logical representations for them may be taken from regular geometry (e.g. equations of spheres, planes and vectors). This forms the quantitative basis for constraint-based design.

It can be shown that all flexure problems, 1 – 5 degrees of freedom, have general shapes that represent the freedoms and constraints and that these constraint and freedom shapes may be matched according to the rule of complimentary patterns. This is a powerful concept in that there are a finite number of shapes that represent all possible ways to constraint a flexure device. These shapes may then be mapped to desired motions, i.e. freedoms. As such, a designer can instantly relate desired motion, i.e. red freedom shapes, to a finite number of constraint shapes. In some instances, there are several constraint shape sets that can solve a given motion problem. In other cases, there are no solutions to a desire flexure motion problem. Either way, it is now possible for a designer to know and visualize what is possible and not possible in terms of different design concepts. The FACT shapes can be used to guide the designer in selecting the right lines from within those shapes to replace by physical flexure constraints in a real design.

Unfortunately, there are over 50 shapes and thus we are unable to discuss them all in this brief paper. Instead, we show the matches for one case, where three degrees-of-freedom are obtained with three constraints. Figure 5 shows the mapping of constraints (left) and freedoms (right). We refer to the matching sets from top to bottom as sets a – f. For instance, if the motion desired is represented by the red freedom lines in set a (the top set) – two rotations about skew lines in two different planes and a translation normal to those planes (e.g. the hoop), then these motions can be obtained by selecting three lines from the shapes on the left. The lines in these shapes are parallel within each plane and skew between each plane. If one examines these constraints and freedom topologies by overlaying them, one will find that this matching satisfies the principle of complementary patterns.

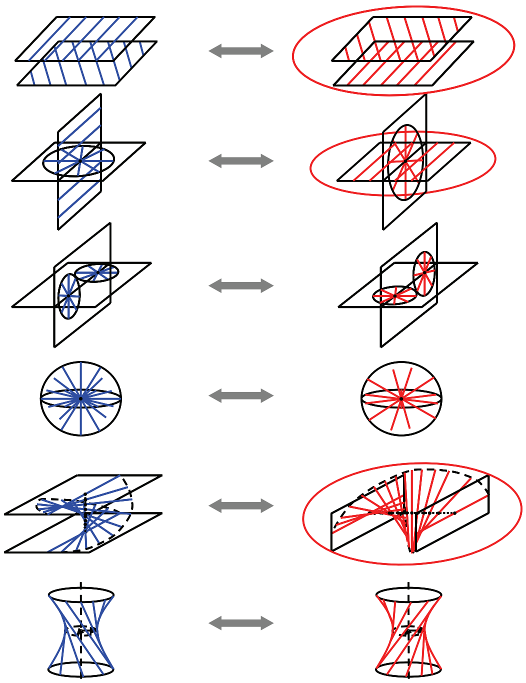

FIGURE 5. SOME EXAMPLES OF CONSTRAINT (LEFT) AND CORRESPONDING FREEDOMS (RIGHT)

Likewise set d (the spheres with radial lines) represents three rotational degrees of freedom about a central point. This function may be obtained by selecting three blue constraint lines from the left sphere. These three non-collinear constraints intersect at a common point and would therefore result in rotational degrees of freedom about the point of intersection. Again these matching satisfy the principle of complementary patterns.

THE PRACTICAL UTILITY OF THIS WORK

As noted previously, we are unable to show and fully describe all freedom and constraint topologies that are pertinent to precision flexure design. This information will come in a follow on paper. It is important however for us to describe how the full set of shapes would be useful to precision machine designers. The full set of shapes form an important design guide as they enable a designer to rapidly match a desired flexure motion performance with all possible flexure solutions. It is impractical to memorize all of the topologies and their equations and so the best way to take advantage of this knowledge is to integrate it within a software design tool that contains this information and makes it readily available to a designer in graphical and mathematic forms. We are presently working with the Virtual Reality Applications Center at Iowa State University to implement these shapes, their parametric equations and precision engineering principles to create an FEA-like design tool which uses virtual reality to display/overlap the topologies/shapes, enables a designer to select desired designs, and then performs optimization of the flexure kinematics, thermal stability and load capacity using the equations of the shapes and equations for stiffness, kinematics and thermal errors.

OTHER BENEFITS OF THE FACT METHOD

It should be noted that if one selects more than the desired number of constraints from an available constraint shape, the additional constraints do not remove additional degrees of freedom. That is, they are redundant constraints. This is an important observation as it enables flexure designers to see which constraints they may select to improve the symmetry of a flexure design (e.g. for thermal stability) or to improve the load capacity of a flexure without effecting the motions of the flexure.

ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation under Grant No. DMI-0500272: Constraint-based Compliant Mechanism Design Using Virtual Reality as a Design Interface.

REFERENCES

1. Maxwell JC. General Considerations Concerning Scientific Apparatus. The Scientific Papers of James Clerk Maxwell. Dover Press. 1890;2;505–21.
2. D. L. Blanding, “Exact Constraint: Machine Design Using Kinematic Principles” (1999)
3. Hale L. Ph.D. thesis. Principles and techniques for designing precision machines. Massachusetts Institute of Technology; 1999.