MODELING OF PIEZOELECTRIC ACTUATOR BASED NANO-POSITIONING SYSTEM UNDER SINUSOIDAL EXCITATION USING DUAL POLYNOMIAL REGRESSION FOR SURFACE GRINDING CONTROL

Y. Gao *, S. Tse
Department of Mechanical Engineering, Hong Kong University of Science and Technology
Clear Water Bay, Kowloon, Hong Kong, China
* E-mail: meygao@ust.hk

Introduction

In surface grinding, a high speed workpiece table motion is involved. In order to achieve active control for improved machining accuracy, the control commands will include substantial amount of high frequency signals. The signals may not be sufficiently implemented by a single actuating unit, typically the wheel infeed system, since it has a small frequency bandwidth for dynamic operations. A nano-positioning system utilizing a piezoelectric actuator was designed as an additional unit for providing small but fast control actions. The infeed system and the micropositioning system will work together under the composite control scheme to combine the advantages of both units [1-4].

In order to investigate the characteristics of the nano-positioning system that operates over a range of frequencies and to obtain the response of the system for performance analysis and assessment, in particular, for high frequency signals, sinusoidal signals are utilized to be the command signals for the experimental tests of the system.

For the system under the sinusoidal excitation, the hysteresis of the piezoelectric actuator has a significant effect. Based on our initial studies, sinusoidal signals would give additional problems in comparison with the commonly used step signals. This requires further studies of the hysteresis characteristics of the piezoelectric actuator.

In our previous investigation, a model for the hysteresis behavior and models for the transient response of the system under sinusoidal excitation were established [5-6]. The hysteresis was modeled based on a single polynomial regression approach and an exhaustive search method for minimizing the modeling errors was used. However, the single polynomial representation of the hysteresis behavior had an error up to approximately 15% [5-6]. In an attempt to reduce the modeling error, a new approach that is based on regression of a set of experimental data using two polynomials is proposed.

Nano-Positioning System Model

A model of the system response \( y(t) \) under a sinusoidal excitation \( v_d(t) \) [5-6] is given as

\[
y(t) = \frac{A}{\omega_n^2} \left[ 1 - e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi) \right] - \frac{GA}{(\omega RC)^2+1} e^{-\frac{t}{RC}} \\
+ \frac{(P+N)A}{(\omega RC)^2+1} \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2} t - \phi) - \frac{(Q+L)A}{\omega_n [(\omega RC)^2+1]} \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2} t),
\]

\[
- \frac{TA}{(\omega RC)^2+1} \cos \omega t - \frac{UA}{\omega [((\omega RC)^2+1)]} \sin \omega t
\]
where \( A = \frac{1}{m_{\text{m}} + m_{\text{n}}} k_{\text{pzt}} c_{33} V_0 \), \( \phi = \tan^{-1} \sqrt{1 - \zeta^2} \), \( T = \frac{(\omega_n^2 - \omega^2) - 2\zeta \omega_n \omega^2 RC}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2} \),
\[
L = \frac{\omega_n^2 R C (1 - 2\zeta \omega_n RC)}{\omega_n RC (\omega_n RC - 2\zeta)} + 1, \quad Q = RC - \frac{\omega_n^2 [2\zeta \omega_n + RC (\omega_n^2 - \omega^2)]}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2},
\]
\[
U = \frac{\omega_n^2 [2\zeta \omega_n + RC (\omega_n^2 - \omega^2)]}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2}, \quad G = \frac{\omega_n^2 R C^4}{\omega_n RC (\omega_n RC - 2\zeta)} + 1, \quad N = -G, \quad P = -T, \quad \omega = 2\pi f_c,
\]
\[
v_d(t) = \frac{V_0}{2} (1 - \cos t), \quad c_{33} = d_{33} n k_{\text{amp}}, \quad d_{33} = \frac{\partial y_{\text{pzt}}}{\partial v_d}, \quad m_{\text{pzt}}, \quad k_{\text{pzt}}, \quad R \quad \text{and} \quad C \text{ are the equivalent mass, stiffness, resistance, and capacitance of the actuator and drive system, respectively,} m_{\text{m}} \text{ is the mass of the moving part,} \quad \omega_n \text{ is the natural frequency,} V_0 \text{ and } \omega \text{ are the amplitude and frequency of the command signal } v_d(t), \text{ respectively,} \quad f_c \text{ is the excitation frequency,} \quad d_{33} \text{ is the piezoelectric constant,} \quad c_{33} \text{ is the piezoelectric coefficient related to } d_{33}, \text{ number of layers of the actuator } n, \text{ and the drive amplification factor } k_{\text{amp}}. \text{ and } y_{\text{pzt}}(t) \text{ is the actuator displacement.}

**Hysteresis Model**

**Single polynomial regression**

Figure 1 shows the hysteresis can be approximated by a single polynomial regression, noted as \([P_i(v_d(t)), i=1]\). However, a single curve cannot effectively represent the expansion and retraction in the hysteresis loop. It was found that the relative modeling error \( e_r \) between the theoretical response \( y(t) \) and the experimental response \( y_d(t) \) was \( e_r=13.4\% [5-6] \) in average.

**Dual polynomial regression**

Dual polynomial regression, noted as \([P_i(v_d(t)), i=1,2,3]\), includes the expansion curves \( P_1(v_d(t)) \), \( P_3(v_d(t)) \) and the retraction curve \( P_2(v_d(t)) \) (Figure 1). There exists two different derivative values for the actuator displacement \( y_{\text{pzt}}(t) \) with respect to the command voltage \( v_d(t) \) or two different values of the piezoelectric constant \( d_{33} \) at the ends of the hysteresis loop. Figure 1 shows the difference in \( d_{33} \) at one end. It should be noted that we use uncorrected \( c_{33}, \quad c_{33}=d_{33} n k_{\text{amp}}, \text{ for the case of two values in } c_{33} \) and corrected \( c_{33} \) for the case in which an average was used.

![Fig. 1 Single polynomial and dual polynomials](image)

**Results and Discussion**

**Theoretical and experimental results**

Theoretical response \( y(t) \) to a sinusoidal input signal \( v_d(t) \) can be determined by using Eq. (1) [5-6]. In the experimental tests for verifying the established model, the position was measured using a laser interferometer capable of dynamic measurement for up to 5kHz. Figures 2-9 show the
results of the piezoelectric coefficient $c_{33}$, the experimental response $y_e(t)$ and the theoretical response $y(t)$ for the excitation frequencies $f_e=408.9\text{Hz}$, $334.5\text{Hz}$, $322.3\text{Hz}$ and $247.8\text{Hz}$, respectively. It is noted that $n_c$ is the cycle count and the values of $f_e$ were estimated by using a signal processing technique.

**Discussion**

To verify the established hysteresis models for reduced modeling error $e_r$, the peak values of the experimental response $y(t)_{\text{max}}$ and that of the theoretical response $y(t)_{\text{max}}$ were examined (Figures 2-9 and Table 1). Compared with C1-C2, dual polynomials $[P_i(v_d(t)), i=1,2,3]$ or C3 (Table 1) reduced the relative modeling error $e_r$ by 51.6%, and 30.5%, respectively. C3 has the smallest error of $e_r=6.47\%$ on average. Using this approach, the modeling error is smaller than the ones using the models based on single and dual polynomials with uncorrected $c_{33}$ for a number of typical input voltages and frequencies. This demonstrates the effectiveness of the proposed method. It is noted that the relative error $e_r$ for C3 with $v_0=9\text{V}$ and $n_c=5$ was large (Table 1) due to a detachment similar to the one by a large step input command signal [2-4].

**Fig. 2**  Result of corrected $c_{33}$ for $n_c=1$

**Fig. 3**  Results of $y_e(t)$ and $y(t)$ for $n_c=1$

**Fig. 4**  Result of corrected $c_{33}$ for $n_c=2$

**Fig. 5**  Results of $y_e(t)$ and $y(t)$ for $n_c=2$

**Fig. 6**  Result of corrected $c_{33}$ for $v_0=5\text{V}$

**Fig. 7**  Results of $y_e(t)$ and $y(t)$ for $v_0=5\text{V}$

**Conclusions**
Sinusoidal input signals would give additional problems in comparison with the commonly used step signals due to the hysteresis effects thus require a good modeling approach. Using dual polynomials with corrected $c_{33}$, a relative modeling error of 6.47% was obtained. It is significantly smaller than the one of 13.4% obtained using the approach of single polynomial.

![Fig. 8 Result of corrected $c_{33}$ for $v_0=9\text{V}$](image)

![Fig. 9 Results of $y_e(t)$ and $y(t)$ for $v_0=9\text{V}$](image)

<table>
<thead>
<tr>
<th>Case $^a$</th>
<th>$v_0$ (V)</th>
<th>$n_c$</th>
<th>$f_c$ (Hz)</th>
<th>$y(t)_{\text{max}}$ (µm)</th>
<th>$y(t)_{\text{max}}$ (µm)</th>
<th>$e$ (µm)</th>
<th>$e_r$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>5</td>
<td>1</td>
<td>408.9</td>
<td>31.2607</td>
<td>34.6706</td>
<td>3.41</td>
<td>10.91</td>
</tr>
<tr>
<td>C2</td>
<td>5</td>
<td>1</td>
<td>408.9</td>
<td>31.1749</td>
<td>28.2542</td>
<td>2.92</td>
<td>9.37</td>
</tr>
<tr>
<td>C3</td>
<td>5</td>
<td>1</td>
<td>408.9</td>
<td>31.1749</td>
<td>28.2542</td>
<td>2.92</td>
<td>9.37</td>
</tr>
<tr>
<td>C1</td>
<td>5</td>
<td>2</td>
<td>334.5</td>
<td>27.4849</td>
<td>31.9400</td>
<td>4.46</td>
<td>16.21</td>
</tr>
<tr>
<td>C2</td>
<td>5</td>
<td>2</td>
<td>334.5</td>
<td>27.0796</td>
<td>27.9736</td>
<td>0.89</td>
<td>3.30</td>
</tr>
<tr>
<td>C3</td>
<td>5</td>
<td>2</td>
<td>334.5</td>
<td>27.2274</td>
<td>24.7493</td>
<td>2.48</td>
<td>9.10</td>
</tr>
<tr>
<td>C1</td>
<td>5</td>
<td>5</td>
<td>322.3</td>
<td>27.5795</td>
<td>31.1582</td>
<td>3.58</td>
<td>12.98</td>
</tr>
<tr>
<td>C2</td>
<td>5</td>
<td>5</td>
<td>322.3</td>
<td>25.4072</td>
<td>23.2276</td>
<td>4.18</td>
<td>15.25</td>
</tr>
<tr>
<td>C3</td>
<td>5</td>
<td>5</td>
<td>322.3</td>
<td>27.5451</td>
<td>27.2901</td>
<td>0.26</td>
<td>0.93</td>
</tr>
<tr>
<td>C1</td>
<td>9</td>
<td>5</td>
<td>247.8</td>
<td>163.1977</td>
<td>37.9030</td>
<td>125</td>
<td>76.77</td>
</tr>
<tr>
<td>C2</td>
<td>9</td>
<td>5</td>
<td>247.8</td>
<td>163.0409</td>
<td>42.3907</td>
<td>122</td>
<td>74.66</td>
</tr>
<tr>
<td>C3</td>
<td>9</td>
<td>5</td>
<td>247.8</td>
<td>163.0409</td>
<td>41.3143</td>
<td>121</td>
<td>74.00</td>
</tr>
</tbody>
</table>

$^a$ C1 - Single polynomial $[P_i(v_d(t)),i=1]$ $^b$ C2 - Dual polynomials with uncorrected $c_{33}$ $[P_i(v_d(t)),i=1-3]$ $^c$ C3 - Dual polynomials with corrected $c_{33}$ $[P_i(v_d(t)),i=1-3]$

References