1.0 Introduction
Limitations to milling efficiency imposed by the process dynamics include: 1) chatter, or self-excited vibrations that lead to large forces, displacements, and poor surface quality; and 2) surface location error, or workpiece geometric inaccuracies that result from dynamic displacements of the tool (and/or workpiece) during stable machining. Other limiting factors include machine tool quasi-static positioning errors, thermal errors, contouring errors, and tool wear, for example.

Stability and surface location error in peripheral end milling may be determined using time domain simulation, where the tool displacements are determined by numerical integration of the equations of motion, typically in two orthogonal directions within the plane of the cut (i.e., perpendicular to the tool axis) [e.g., 1-4]. Generally, the cutting force is computed using the uncut chip area, which depends on both the current and time-delayed tool deflections, and appropriate cutting force coefficients; the system dynamics are represented by modal parameters determined from a fit to a finite number of vibration modes. Time-domain simulation can conveniently incorporate such effects as the cutting edge helix angle, non-proportional teeth spacing, and runout, for example. When searching large parameter spaces, however, an attractive alternative to time-domain simulation is analytical solution of the milling problem. In this paper a frequency domain solution to surface location error is presented.

2.0 Frequency domain solution
Early work led to a fundamental understanding of regeneration of waviness, or the overcutting of the machined surface by the vibrating cutter, as a primary feedback mechanism for the growth of self-excited vibrations due to the modulation of the instantaneous chip thickness, cutting force variation, and subsequent tool vibration [5-6]. This work led to the development of frequency domain solutions for process stability, where the information was presented in the form of stability lobe diagrams that identify stable and unstable cutting zones as a function of axial depth and spindle speed [6-7]. Clearly, a frequency domain surface location error solution would complement the frequency domain stability diagrams and provide a more complete picture of the role of milling dynamics on process efficiency.

Surface location error can be described using Fig. 1. Even under stable cutting conditions, the tool experiences periodic (forced) vibrations which depend on the system dynamic stiffness and excitation frequency. The position of the tool in its periodic vibration cycle as it exits (down milling) or enters (up milling) the cut determines the actual location of the machined surface. Depending on the excitation frequency, which is governed by the spindle speed and number of teeth on the cutter, the surface may either be undercut (less material removed than commanded) or overcut (more material removed).

In order to determine surface location error using a frequency domain (or steady-state) approach, two basic assertions are made. First, although vibrations of the cutter occur in both the x and y directions, y-direction vibrations dominate the final surface location. Second, regeneration can be neglected in stable machining. Based on these assumptions, the concept is to: 1) express the y-direction cutting force in the frequency domain, $F_y(\omega)$ using a Fourier series; 2) determine the frequency-domain y displacement, $Y(\omega)$, by multiplying $F_y(\omega)$ by the tool point frequency response function, or FRF, in the y-direction, $Y(\omega) = \frac{Y}{F_y}F_y$; and 3) inverse Fourier transform this result and sample at the cut entry (up milling) or exit (down milling) to find the surface location error. The reader may note that, unlike time domain simulation, the measured FRF can be used directly without the requirement for a modal fit.
The equivalent Fourier series for the cutting force model provided in Eq. 1 that relates the tangential, \( F_t \), and normal, \( F_n \), cutting force components to the chip width (or axial depth of cut in peripheral end milling), \( b \), and chip thickness, \( h \), is applied, \( F_s(\phi) \) can be expressed as shown in Eq. 2, where the summations account for all possible teeth within the cut, the circular tool path approximation has been applied, \( c \) is the chip load (or feed per tooth), \( g(\phi) \), is a switching function that is equal to 1 when tooth \( i \) is engaged in the cut and 0 otherwise, the angle of each tooth \( (N \) total) is \( \phi_i = \omega t + \frac{2\pi}{N}(i-1) \) for proportional teeth spacing, and \( \omega \) is the spindle rotating frequency (in rad/s). See Fig. 2, where \( x \) is the tool feed direction.

\[
F_t(\phi) = k_t bh(\phi) + k_{tb}b
\]

\[
F_n(\phi) = k_n bh(\phi) + k_{nb}b
\]

\[
F_s(\phi) = \sum_{i=1}^{N} g(\phi) \left( 1 - \cos 2\phi_i \right) + \sum_{i=1}^{N} g(\phi) \sin 2\phi_i - \sum_{i=1}^{N} g(\phi) \sin \phi_i + k_{tb} \sum_{i=1}^{N} g(\phi) \cos \phi_i
\]

The equivalent Fourier series for the \( y \)-direction cutting force, \( F_y(\phi) = \sum_{j=1}^{N} a_j + \sum_{j=1}^{N} (a_j \cos n\phi_i + b_j \sin n\phi_i) \), can be written once the Fourier coefficients are determined. The \( a_j \) term, for example, can be found using Eq. 3, where the integral for a full revolution of the selected tooth may be divided into three parts. These integrals are delineated by \( \phi \), which represents the cut entry angle in down milling or cut exit angle in up milling, and \( \pi \), which defines the maximum angle that a tooth can be engaged in the cut (if \( \phi \) is defined positive in a clockwise sense from the positive \( y \)-axis). Considering a down milling cut, for example, only the middle of the three integrals in Eq. 3 is nonzero due to the switching function embedded in \( F_s(\phi) \). For up milling, only the first integral in Eq. 3 is nonzero, but the procedure remains the same.

\[
a_0 = \frac{1}{2\pi} \int_{0}^{2\pi} F_s(\phi) d\phi = \frac{1}{2\pi} \int_{0}^{\pi} F_s(\phi) d\phi + \frac{1}{2\pi} \int_{\pi}^{2\pi} F_s(\phi) d\phi
\]

The \( a_n \) coefficients are computed using Eq. 4 and the \( b_n \) coefficients using Eq. 5. Again, the integral can be partitioned using \( \phi_i \) and \( \pi \) as shown in Eq. 3. Closed-form equations for \( n = 3, 4, 5, \ldots \) can be determined by observing the recursive patterns after integration.

\[
a_n = \frac{1}{\pi} \int_{0}^{\pi} F_s(\phi) \cos(n\phi) d\phi
\]

\[
b_n = \frac{1}{\pi} \int_{0}^{\pi} F_s(\phi) \sin(n\phi) d\phi
\]

To accurately represent milling forces, however, it is also necessary to account for the influence of the teeth helix angle, \( \beta \). This can be accomplished by separating the tool into \( A \) axial slices; each slice is assumed to have a zero helix angle and the slices are rotated relative to one another by the angle \( \psi = \frac{2\tan(\beta) dB}{d_2} \), where \( dB \) is the slice height and \( d \) is the cutter diameter. The Fourier series is now written as

\[
F_y(\phi) = \sum_{j=1}^{N} a_j + \sum_{j=1}^{N} (a_j \cos n\phi_i + b_j \sin n\phi_i)
\]

where \( \phi_i = \omega t + \frac{2\pi}{N}(i-1) - \psi(j-1) \).

2.2 Numerical demonstrations

To numerically verify the approach, comparisons between a comprehensive time domain simulation [8] and the analytical approach have been completed. Figure 3 shows the results for 50% radial immersion up milling cuts carried out at spindle speeds from 6900 rpm to 7700 rpm \( (N = 4, \beta = 30 \text{ deg}, d = 12.7 \text{ mm diameter}, c = 0.1 \text{ mm/tooth}, b = 1 \text{ mm and } 2.5 \text{ mm}, k_t = 700 \text{ N/mm}^2, k_n = 210 \text{ N/mm}^2, k_{tb} = k_{nb} = 0 \text{ N/mm, symmetric structural dynamics with a stiffness of } 1 \times 10^7 \text{ N/m, } 1\% \text{ viscous damping, and } 500 \text{ Hz natural frequency, } f_s). \) Good agreement is seen between the surface location error predicted by time domain (open circles) and the frequency
domain approach (solid line). To interpret the figure, a positive error indicates an overcut surface for up milling (the opposite is true for down milling).

2.3 Experimental results
To obtain initial experimental verification of the analytical approach described in the previous sections, cutting tests were completed on a single degree-of-freedom flexure in order to provide a simple dynamic system. The flexure and tool were selected so that the flexure response (in the flexible $y$-direction) was 40 times more flexible than either the tool point response or the flexure response in the orthogonal direction. See Fig. 5.

Next, analytical simulations were completed for the same system over a range of axial depths and spindle speeds to determine the variation in surface location error within a single stability lobe. The surface location error contours (now represented as a height map for the control variables $b$ and spindle speed) are provided in Fig. 4. It is observed that at the traditional 'best' spindle speed, $\Omega = 60 \cdot f_k = 7500$ rpm ($k = 1$) near the lobe peak, the surface location error is highly sensitive to variations in spindle speed (for fixed dynamics) or the system natural frequency (for a fixed spindle speed).

Surface location error tests were completed by first preparing workpieces with 5.4 mm wide by 10 mm tall ribs that were then finish machined on each face to reduce the rib width for the top 5 mm (the bottom 5 mm was left untouched to provide a control for the experiments). The $x$-direction down milling cutting conditions for the finishing passes on each side of the ribs were 1 mm radial depth of cut (4% radial immersion for
the 25.4 mm diameter tool), \( b = 5 \text{ mm} \), and \( c = 0.1 \text{ mm/tooth} \). See Fig. 6.

Six ribs were machined at spindle speeds ranging from 13425 rpm to 13675 rpm in increments of 50 rpm. This approximately centered the tests on the ‘best’ spindle speed for the flexure-workpiece natural frequency of 451.7 Hz (5.35\( \times 10^6 \) N/m stiffness and 0.35% damping). After machining, the rib widths were measured at three locations along the rib height using a touch-probe coordinate measuring machine (2 mm diameter ruby probe). At each height, \( A \) (7.5 mm from the rib top), \( B \) (3 mm) and \( C \) (1 mm), 23 points were recorded on each side of the ribs and the rib width was determined from straight-line fits to these points. These points were obtained at 2 mm increments and the total measurement range was located near the center of the ribs (25 mm from each end) to avoid transient contributions to the rib width. The measurements at point \( A \) were used to verify that each rib started with the same width, while the measurements at \( B \) and \( C \) served to demonstrate the dependence of surface location error on the axial location along the helical cutting edge.

The experimental surface location error results and frequency domain approach predictions are shown in Fig. 7. Additionally, the two standard deviation error bars for the coordinate measuring machine rib width measurements are provided (12 total measurements were completed for each rib). These are included to show that the measurement repeatability was a small fraction of the overall surface location variation (1-2 \( \mu \text{m} \)). The trend of undercut to overcut as the tooth passing frequency passes through the flexure natural frequency is seen with a total experimental variation of approximately 250 \( \mu \text{m} \) for a 200 rpm spindle speed change. The agreement with analytical results is reasonable, including the trend of increasing difference between the \( B \) and \( C \) surface location error values near resonance.

3.0 Conclusions

This paper provided numerical and experimental validation of a new frequency domain solution to surface location error in milling. The primary benefits of this approach are: 1) the ability to use measured or predicted frequency response functions directly; and 2) its compatibility with frequency domain stability solutions available in the literature.

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5.0 References


![Figure 7: Comparison of experimental and analytical surface location error results. Data points were recorded at locations B and C along the rib height.](image-url)