MODEL FOR 3D STRUCTURE OF VITRIFIED-BONDED WHEEL

Masahiro Higuchia, Tomomi Yamaguchia, Shoichi Shimadab, Noboru Matsumoric, Isao Yoshizawac, Hidekazu Ogurac

a Kansai University, 3-3-35 Yamate-cho, Suita, Osaka 564-8680, Japan
b Osaka Electro-Comunication University, 18-8 Hatsu-cho, Neyagawa, Osaka 572-8530, Japan
c Mizuho Co. Ltd., 82 Fukurojiri, Terada, Jyoyo, Kyoto 610-0221, Japan

1. Introduction
In the field of machine design, geometric modeling is generally done in order to decide on a structure to fulfill intended functions. However, in the case of designing a vitrified-bonded wheel, a geometric description of the structure is hardly ever done. The reason is the 3D structures of vitrified-bonded wheels are too complex to be described with traditional modeling techniques using spheres, lines, circles, etc. Therefore, development of a new vitrified-bonded wheel depends on the trial and error method. It is obvious that such a method is costly and inefficient. Consequently, the development of a modeling method that represents the complex vitrified wheel structure is required.

At first glance, the structure of the vitrified wheel looks as if grains are held firmly together with bonds and pores of various sizes distributed randomly. But, closer observation of the vitrified wheel surface revealed that the spatial distribution of abrasive grains is not disorderly but fractal [1,2]. This made us realize that the vitrified wheel structure can be described using fractal modeling techniques. On this view we have discussed the method for describing a geometric model that serves as a drawing for a wheel structure. The full detail of the model building process is given as follows.

2. Methods and approaches
The vitrified wheels are manufactured by the mixing of abrasive grains with vitreous bonds, pressing, drying and burning. Therefore, in the development of the modeling method, we have considered two points; one point is that grain and bond particles agglomerate in the mixing process; the other point is that the grains are spanned with bond bridges in the burning process. To describe the former, a variant of the diffusion-limited aggregation algorithm [3] has been used. On the other hand, to describe the latter, a variant of the sandpile algorithm [4] has been used. Figure 1 shows the process of a wheel structure built from these algorithms.

2.1 Model for aggregate made up by mixing
A very porous vitrified-bonded wheel is manufactured from raw materials as an abrasive grain, a vitreous bond and a pore agent made of polystyrene. If these particles smaller than a several microns are mixed together with water containing a dextrin, they form a plastic mass. In Fig.2, a scanning-electron micrograph of the plastic mixture is shown. It looks as if the particles of various sizes attach randomly. Therefore, it is thought that the structure of the plastic mixture depends on the sequence of random attachments of the different-sized particles. Under this view, agglomerate simulations of the grain, bond and pore agent have been performed by applying the diffusion-limited aggregation algorithm.

First of all, we assume that each of the particles of the grain, bond and pore agent is the sphere having an

---

Fig. 1. Processes for modeling vitrified-bonded wheels
average diameter and variance, and that the number of each particle forming the plastic mixture is proportional to the weight percentage of the particle. The model for the plastic mixture is defined as follows.

**Step 1.** A seed particle is placed at the origin of a three dimensional lattice whose spacing is shorter than the particle size.

**Step 2.** Particles are selected one at a time with a probability proportional to the number of particles. And the diameter of the selected particle is decided as a sample of a Gaussian random variable with variance.

**Step 3.** The particle is allowed to execute a random walk from far away as shown in Fig.3. That is, it steps in the direction that is specified by selecting one of fourteen numbers using a random number table.

**Step 4.** If the wandering particle touches the seed particle or the growing agglomerate, then it sticks at their surfaces.

**Step 5.** Return to **Step 2.** This is repeated until the agglomerate of 3000 particles is formed.

### 2.2 Model for vitrified wheel structure made up in burning process

Figure 4 shows a scanning-electron micrograph of the burnt vitrified wheel. It is totally different from the plastic mixture in structure. This indicates that the burning causes a structural change of the mixture. In the early stage of burning process, the large pores are formed where the pore agent particles were burnt down. Then, the bond particles melt at the high firing temperature. The molten bonding material penetrates into a gap between grain particles due to the capillary attraction, partly wets and pulls them together by surface tension. On subsequent cooling, glassy necks of solidified bonding material, so called bond bridges, develop between them. The vitrification process of the bond has been described by means of a variant of the sandpile algorithm.

The original sandpile algorithm is very simple. If the slope is too large, the pile equilibrium is off, and the pile will collapse until the average slope reaches a critical value where the system is barely stable with respect to small perturbations. We used a slightly modified version in order to reproduce the wheel structure from the agglomerate discussed in section 2.1. The reproducing procedure is as follows.

**Step 1.** The pore agent particles are removed from the agglomerate.

**Step 2.** The total volume $V$ of the bond particles, which exist in the gap between two grain particles, is calculated.

**Step 3.** A gap is filled with a number of disks of which volume is $V$.  

![Fig.2 SEM photograph of agglomerate made up grain, vitreous bond and pore agent particles](image1)

![Fig.3 Technique for describing a random motion of a particle in three dimensional space](image2)

![Fig.4 SEM photograph of burnt vitrified wheel](image3)
Step 4. The bond bridge is characterized by the volume $v(x)$ of each disk as shown in Fig.5.

Step 5. A disk $x$ of the bond bridge is unstable, if the local difference of volume $\Delta v(x) = |v(x) - v(x+1)|$ is smaller than the critical value $kv(x)$, $0 < k < 1$.

Step 6. If the disk $x$ is unstable, its volume decreases by the $kv(x)$ and the volume of the disk $x+1$ increases by the $kv(x)$. Algorithmically speaking, if $\Delta v(x) < kv(x)$, then $v(x) \rightarrow v(x) - kv(x)$ and $v(x+1) \rightarrow v(x+1) + kv(x)$.

Step 7. The bond bridge simply changes its shape until all $\Delta v(x)$'s are larger than $kv(x)$.

3. Simulation results

Firstly, we have simulated what mixture is made up from the grain, bond and pore agent particles. The result of the simulation is given in Fig.6. In order to investigate the cohesive state of particles, total mass $M(r)$ of particles within a radius $r$ of a typical point of the agglomerate was calculated. In Fig.7, the mass of the agglomerate is plotted against its radius. It is clear from the fitted line that its mass scales with its radius like $M(r) \propto r^D$, where $D$ is the fractal exponent.

Figure 8 illustrates the fractal exponents of various agglomerates. In our simulations, a very compact agglomerate that has a fractal exponent ranging from 2.80 to 2.86 is obtained, when varying the weight percentage of wheel components. The values of fractal exponents less than the spatial dimension of 3 signify that the agglomerate includes voids. As can be seen from the figure, the larger the weight percentage of pore agent, the lower the fractal exponent of the agglomerate becomes, since the pore agent of such a large size and low density tends to cohere on the outside of agglomerates. Thus, it is presumed that the weight percentages of the wheel components have an important effect upon the structure of the mixture.

Secondly, we have performed simulations of the vitrification process of the bond particle using the agglomerates mentioned above. Figure 9 shows the obtained models that correspond to the structure of the mixture.
vitrified wheel. Our model is quite different from the usual model [5,6] where grains are located at the tops of a tetrahedron or hexahedron and mounted by bond bridges in the shape of a cylinder. Our model visually seems to resemble real structures of vitrified wheels.

If one divides the total weight of grains and bond bridges by the volume of the sphere required to cover a model, then one can measure numerically the bulk density of the model. On the other hand, one can measure experimentally the bulk density of vitrified wheels. Figure 10 shows the comparison of the numerical values with experimental ones. It was found that close agreement between numerically and experimentally measured values of bulk density is obtained. Therefore, our model is very useful as a drawing for a wheel structure.

In further work, a method for describing the topography of the wheel surface will be devised based on the wheel structure model. Because, the spatial distribution of abrasive grains over the wheel surface and their morphology are more important to the wheel cutting performance than the wheel structure.

4. Conclusions
This paper proposes a method for describing morphological models of vitrified-bonded wheels. Since the vitrified wheel structure is considered as a fractal object, computer simulations using recursive algorithms, i.e. the diffusion-limited aggregation algorithm and the sandpile algorithm, have been applied to modeling. The structure model obtained bears resemblances not only in morphology but also in bulk density to an actual wheel structure. Consequently, it is considered that our modeling method plays an important role in the development of new wheels.

References