SHAPE MEMORY ALLOY WIRES FOR ACTUATING POSITIONING SYSTEMS WITH ELASTIC BEARINGS

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Introduction
Shape Memory Alloys (SMAs) produce a significant amount of force and displacement when they undergo a thermal cycle. This is due to phase changes from austenite to martensite during cooling, and martensite to austenite during heating. SMAs are commercially available in many forms such as wires, tubes, thin sheets, ribbons, springs and even custom components. Due to its unique characteristics and simple actuation principle, SMAs are used as inexpensive actuators in many fields, such as mechanical, civil, biomedical and aerospace engineering [1]. Bias spring actuators, which preload the SMA specimen with a spring, are the most common form. The contraction produced in SMA wires during heating is about 3% to 6% of the wire’s original length, and they generate a pull force that is directly proportional to the cross sectional area of the wire. If the spring is sufficiently stiff, it pulls the wire to its original length during cooling period due to convection and conduction through the ends of the wire.

A precision instrument was recently developed by Chikkamaranahalli et.al [2,3] for characterizing shape memory alloy wires used in bias spring actuators. The design is based on the principles of precision engineering, which results in eliminating the errors in characterizing shape memory alloy wires used for actuation. The instrument uses electric current to resistively heat a SMA wire mounted between a load cell and an externally pressurized linear air bearing. A tension spring, stretched between the air bearing and a non-rotating spindle micrometer, preloads the SMA wire. A displacement sensor, mounted collinear with the SMA wire, load cell, and air bearing, measures the displacement that occurs when the SMA wire is heated or cooled.

In this paper, we consider the feasibility of using SMA wires as an actuator for binary micro positioning systems with nanometer resolution. Such a system would employ elastic bearings and an attenuation mechanism for reducing the contraction of the SMA wire. We first present experimental data from characterizing a Flexinol™ (Dynalloy Inc., Costa Mesa CA) wire that should be suitable for this application. Then we consider one conceptual design of the elastic bearings and attenuation mechanism.

Thermo-mechanical behavior of SMA
Figure 1 shows the time history of temperature and displacement for a ∅ 0.381 mm × 50.8 mm Flexinol™ wire that was resistively heated by ramping the current quasistatically from 0 mA to 1300 mA. The wire was cooled by ramping down the current in the same manner from 1300 mA to 0 mA. The SMA wire shortened by 2.515 mm as its temperature increased from about 28°C to about 98°C. The cycle was repeated ten times over 350 minutes to demonstrate the stability and repeatability of the cycle. The rate of contraction is much higher than elongation since cooling occurs slowly due to natural convection and conductive heat loss through the ends of the SMA. The repeatability and reproducibility of this actuation cycle suggest that SMA wires might be an inexpensive actuation means for a micro positioning systems based on compliant mechanisms and elastic bearings.

Design of a positioning system
Shape memory alloys can only pull therefore an adequate amount of preload force is essential in designing a positioning system actuated by SMA. Figure 2 shows data from a multi-cycle experiment with a Flexinol™ wire (Ø 0.381 mm × 50.8 mm). The wire was preloaded to about 5.2 N, exerting a tensile stress of about 45 MPa. The temperature of the wire...
increased from 28°C to 87°C, and the wire contracted by 2.382 mm during the first cycle. However, it did not return to its original length upon cooling. This shows that the preload on the SMA wire was insufficient to stretch the wire back to its original length, which results in a residual stress in the wire. The amount of elongation progressively decreased with each subsequent cycle.

The conceptual design of a one DOF positioning system is shown in Figure 3. This concept is derived from a novel lever mechanism described by Woody and Smith [4]. A pull bar is attached to two symmetric lever mechanisms. Each lever mechanism consists of a lever arm attached to two cartwheel hinges. The hinges are composed of a pair of leaf springs at right angles to each other that intersect at the pivot point. In the planar mechanism considered by Woody and Smith, the transmission ratio is limited since the output link cannot be positioned near the pivot point due to the size of the cartwheel. This concept overcomes this limitation by positioning the cartwheels above and below the translating the carriage and the output link of the lever mechanism. Unfortunately, the mechanism becomes more complex to fabricate and assemble since it is challenging to produce monolithically.

Each lever is attached through a link to the carriage that is guided by a double compound flexural bearing. One end of the SMA wire is attached to the pull bar, while the other end is attached to a non-rotating, spindle micrometer mounted on a surrounding frame. A capacitive displacement sensor is to be placed collinear with the axis of symmetry of the entire mechanism. The displacement sensor will be used for characterizing the repeatability of the positioning system.

The system should be compliant enough that the SMA wire can actuate it, though, if needed, multiple SMA wires can be used to displace the system. Multiple wires, however complicates the preloading system. The necessary equivalent stiffness \( k_{eq} \) of the system is determined from the amount of force and strain that can be produced during heating of the SMA. As shown in Eq. (1), the equivalent stiffness can be calculated knowing the maximum displacement of the wire \( \Delta_{max} \), the maximum pull
force $F_{\text{max}}$ and the preload force $F_p$, is the preload force, then the equivalent stiffness, $k_{eq}$, of the system that can be driven by this actuator is given by Eq. (1)

$$k_{eq} = \frac{F_{\text{max}} - F_p}{\Delta_{\text{max}}}$$

Eq.(2) is then used to determine the necessary transmission ratio to attenuate the wire contraction to the desired displacement range.

Transmission Ratio = $N = \frac{\Delta_{\text{max}}}{\Delta_{\text{desired}}} = \frac{L_i}{L_s}$ (2)

Figure 4 illustrates a simple stiffness model of the lever mechanism. The pulling force $F$ of the SMA wire causes the pull bar to displace by $x_f$. As a result, the carriage is displaced by distance $x_g$. The longitudinal stiffness of the links joining the levers to the pull bar and carriage is represented by $k_p$ with an elongation $e_p$ in the direction of the force.

The rotational stiffness corresponding to an angle of rotation $\theta$ within a cartwheel hinge is represented by $k_t$ and is given by Eq. (3), where $I_t$ is the second moment of area and $R_t$ is the radial length of each spoke of the cartwheel flexure.

$$k_t = \frac{4E I_t}{R_t^2}$$

If the stiffness of the carriage is represented by $k_g$, and its displacement is denoted by $x_g$. The stiffness of the carriage can be calculated with Eq. (4) where $k_b$ is the stiffness of a fixed-guided beam or with Eq. (5) which is expressed in terms of the size of the beam. $E_g$ is the elastic modulus, $I_g$ is area moment of inertia, and $L_g$ is the length of the individual beams.

$$\frac{1}{k_g} = \frac{1}{4k_b} + \frac{1}{4k_b}$$

$$k_g = \frac{2E_g I_g}{L_g^2}$$

**Equivalent stiffness model for the system**

The equivalent stiffness of the mechanism is determined by considering equating the potential energy stored in the equivalent spring $k_{eq}$ with the potential energy stored in the four cartwheel hinges and the double compound flexure supporting the carriage. This is shown in Eq. (6).

$$\frac{1}{2} k_{eq} x_f^2 = \frac{1}{2} (k_t e_p^2) + \frac{1}{2} (4k_t \theta^2)$$

Equations (7) and (9) relate the displacement of the guide bar $x_f$ and carriage displacement $x_g$ to the rotation $\theta$ of the cartwheel and the lengths of the lever.

$$x_f = L_t \theta$$

$$x_g = L_s \theta$$
Substituting these expression and that from Eq. (2) into Eq. (6) gives $k_{eq}$ in terms of the parameters $N$, $k_g$, $k_t$, and $L_s$. Equation (9) shows that the equivalent stiffness of the system is highly dependent on the transmission ratio $N$ and shorter lever arm $L_s$.

$$k_{eq} = \frac{1}{N^2} \left( k_g + \frac{4}{L_s^2} k_t \right)$$  (9)

Figure 5 shows a contour plot of the necessary cartwheel stiffness $k_t$ calculated with Eq. (9) for various values of $L_s$ and stiffness of the guide $k_g$. The vertical lines indicate that there is little dependence on the guide stiffness (due to the high transmission ratio), so a stiff guide mechanism can be selected without adverse consequences.

![Figure 5 Contour plot relating the carriage stiffness $k_t$, the shorter lever arm $L_s$ and the cartwheel stiffness $k_t$.](image)

**Detailed design and analysis**

Based on the results of the equivalent stiffness model given in the previous section, the dimensions listed in Table 1, for $L_s=0.004$ m and $L_f=0.488$ m, were selected for prototyping the mechanism with ABS polymer ($E=1700$ MPa). Ideally, these dimensions should result in a transmission ratio of $N=121.89$ and an equivalent stiffness of $k_{eq}=7.067 \times 10^3$ N/m.[5] However, finite element analysis of the mechanism indicates that this ideal value can not be achieved due to longitudinal compliance of the cartwheel hinge. As shown in the inset, substantial displacement of the pivot point occurs, and this displacement is directly transferred to the carriage. This increases the output motion of the carriage, reducing the transmission ratio.

![Figure 6. Results of finite element analysis](image)

**Conclusion**

We conclude that the compliant mechanism described in this paper may not be well suited for SMA actuated positioning systems due to the finite compliance of the cartwheel hinges. Some additional consideration of other reduction lever mechanisms is necessary.

**References**


