# Identification of Friction Elements through Limit Cycle Analysis

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**Abstract:** A friction identification method in a frequency domain is proposed for a precision servomechanism. A nonlinear friction model including static, Coulomb, and viscous friction as well as Stribeck effect is considered in this paper. Friction models are formulated by using describing functions. Friction elements are estimated through the limit cycle analysis in a velocity control loop. A Butterworth filter is incorporated into the feedback loop to increase the accuracy of the friction identification process. A model-based friction compensator is applied as a feedforward controller to compensate for the nonlinear characteristics and to verify the effectiveness of the proposed identification method.

**Keywords:** Describing function, Friction identification, Limit cycle, Nonlinearity, Nyquist criterion, Servomechanism, Stribeck effect

## 1. Introduction

The need for high-density magnetic storage devices, assembly machines for semiconductors, and high-speed machine tools has increased the demand of high-performance servomechanisms. Friction is a dominant nonlinear factor that seriously deteriorates positioning accuracy of the servomechanism(1). A friction compensator is indispensable to fabricate high-performance servomechanisms.

A lot of study concerning the identification and compensation of friction has been proposed so far(1-5). Generally, friction elements in the servomechanism have been estimated from motor torque or current through recursive identification processes(2). However, these methods require additional equipments such as torque meters and current sensors as well as time-consuming identification process.

This paper proposes a simple identification method to estimate friction elements in servomechanisms without additional equipment. A nonlinear friction model including static, Coulomb and viscous friction, as well as Stribeck effect is considered in this study. In order to identify friction coefficients of the servomechanism, an accurate linear element model of the system should be obtained in advance. A nonlinear element composed of two relays is devised to extract various limit cycle conditions in a velocity control loop(3). The velocity control loop contains a linear element and two nonlinear elements. One nonlinear element refers to the friction model on the velocity feedback part and the other nonlinear element means the proposed nonlinear element to extract limit cycles in the velocity control loop of the servomechanism. Equivalent nonlinearity acting on the linear element is derived from the combination of two nonlinear elements in the form of a describing function(6-7). Using this method, the friction identification becomes the estimation of describing function parameters.

Frequencies and magnitudes of stable limit cycles in the velocity control loop are obtained from several experiments. A Butterworth filter is incorporated intentionally into the velocity loop to satisfy the requirements of the describing function approximation and to increase accuracy of friction estimation. The identified accurate friction model is applied for a model-based friction compensator with a feedforward controller. Circular motion experiments are conducted to verify the effectiveness of the proposed identification method.

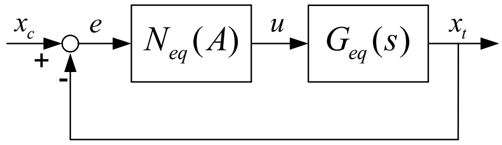



**Fig. 1** Block diagram of a nonlinear system.

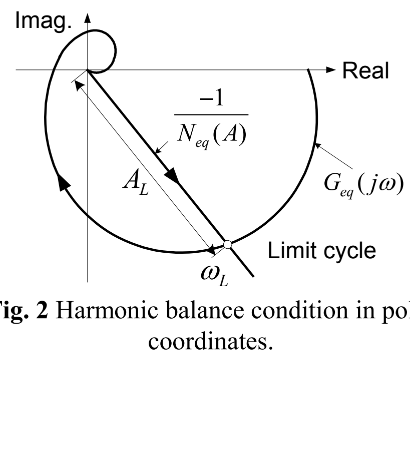



**Fig. 2** Harmonic balance condition in polar coordinates.

## 2. Limit cycle analysis

Consider a nonlinear system composed of an equivalent linear element $G_{eq}(s)$ and an equivalent nonlinear element $N_{eq}(A)$ as shown in Fig. 1. The first step of the friction identification through the limit cycle analysis is an approximation of the nonlinear element to a quasi-linear element by using a describing function approximation. Following assumptions are required[6-7]: ①An input signal, $x_c$, fed to the system is zero. ②The nonlinear element has only the sinusoidal input. ③The linear element has low-pass property to filter out high-frequency harmonics of the nonlinear element.

The next step is to formulate the equation of a harmonic balance condition[6-7] used to predict the existence of limit cycles. The harmonic balance condition is derived from Nyquist criterion as follows:

$$1 + G_{eq}(j\omega) \cdot N_{eq}(A) = 0 \qquad (1)$$

It is possible to plot both the frequency response function $G_{eq}(j\omega)$ (varying $\omega$) and the negative inverse describing function (varying $A$) in polar coordinates as shown in Fig. 2. If the two curves intersect, then there exist limit cycles.

When some parameters (i.e. friction coefficients) of the describing function are unknown and characteristics of both the linear element and limit cycles are known, it could be used the harmonic balance condition to estimate the unknown parameters of the describing function.

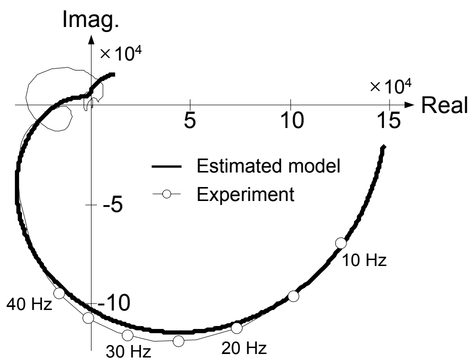



**Fig. 3** Polar plot of the identified linear element.

## 3. Identification of a linear element

In order to identify friction elements of a servomechanism through the proposed method, an accurate linear element model should be obtained in advance. In order to obtain the accurate model of the linear element, a pair of biased square signals are applied for the excitation[3]. This method does not suffer from the problems due to the nonlinear effects and, therefore, is able to provide accurate results.

Fig. 3 shows a polar plot of the identified linear element of the servomechanism shown in Fig. 4. The linear element is a type 0 system in which $G_m(j\omega)$ locus starts at positive real axis and ends at the origin of polar coordinates as $\omega$ increases from zero to infinity.

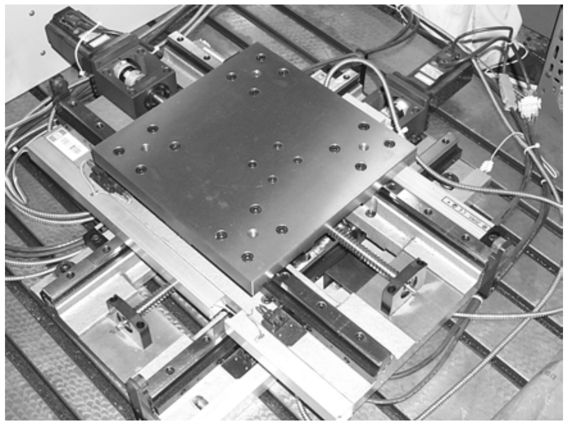


**Fig. 4** Servomechanism for the case study.

## 4. Modeling of nonlinear elements

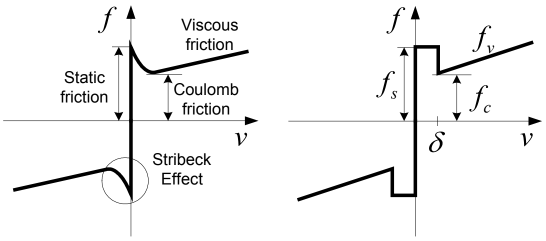
Friction models have been discussed extensively in the literature[1]. It is generally known that friction force or torque varies as a function of velocity. In this paper, a general nonlinear friction model including static, Coulomb and viscous friction, as well as Stribeck effect is proposed as shown in Fig. 5. Several basic nonlinear elements composed of a relay and a dead-zone are used to construct the friction model. By using describing functions of the corresponding elements, a quasi-linear element of the friction model is obtained as follows:

$$N_f(A) = \begin{cases} N_A(A) & , A \le \delta \\ N_A(A) - N_B(A) + N_C(A) & , A > \delta \end{cases}$$

$$\begin{cases} N_A(A) = 4f_s/\pi A & , \forall A \\ N_B(A) = 4(f_s - f_c)\sqrt{1-(\delta/A)^2}\Big/\pi A & , A > \delta \\ N_C(A) = f_v\{1 - N_d(\delta/A)\} & , A > \delta \\ N_d(x) = 2\left(\sin^{-1}x + x\sqrt{1-x^2}\right)\Big/\pi \end{cases} \qquad (2)$$

where $\delta$ is a boundary lubrication velocity, $f_s$, $f_c$, and $f_v$ are static, Coulomb, and viscous friction element, respectively.

A velocity control loop composed of a linear and two nonlinear elements is devised to identify the friction model as shown in Fig. 6.




(a) original model (b) simplified model

**Fig. 5** Proposed friction model.

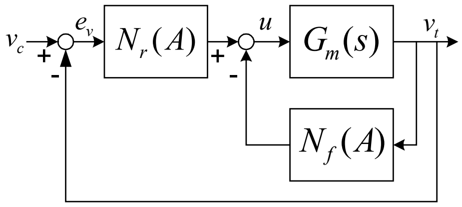



**Fig. 6** System structure for friction identification.

In Fig. 6, $G_m(s)$ and $N_f(A)$ imply the linear element and the friction model, respectively. And $N_r(A)$ is the proposed nonlinear element to extract various limit cycle conditions in the servomechanism.

As described in section 3, the identified linear element is a type 0 system. Therefore, the frequency at which $G_m(j\omega)$ locus intersects the imaginary axis is the natural frequency of the system. In such a case, frequencies of limit cycles extracted to identify friction elements must exist in the fourth quadrant of polar coordinates. According to this condition, the nonlinear element, $N_r(A)$, is devised as a combination of two relays having different amplitudes given by

$$N_r(A) = -\frac{4d_1}{\pi A} - j\frac{4d_2}{\pi A} \tag{3}$$

where $d_1$ and $d_2$ are the magnitudes of two relays.

According to the assumptions described in section 2, nonlinear elements in the servomechanism must be represented as an equivalent nonlinear element. The describing function of the equivalent nonlinear element is determined as the parallel combination of the describing functions as follows:

$$N_{eq}(A) = N_r(A) + N_f(A) \tag{4}$$

When the limit cycle analysis is applied to a position control loop, the linear element has sufficient low-pass characteristics due to an integral element[4-5]. However, higher harmonics cannot be sufficiently filtered out in the velocity control loop. Therefore, in order to satisfy the assumptions of the describing function approximation and to increase the accuracy of friction estimation, a Butterworth filter $G_{lp}(s)$ is incorporated into the velocity control loop as shown in Fig. 7.

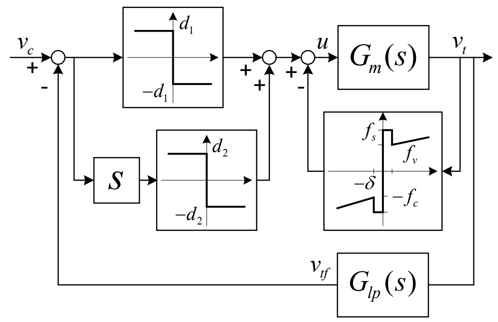
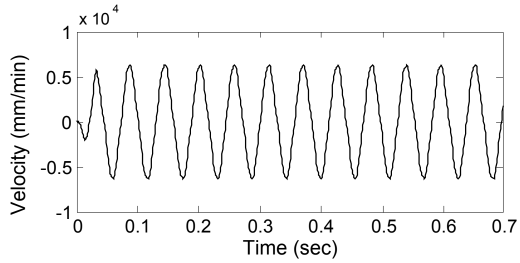
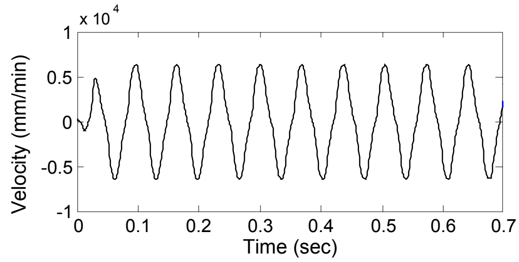



**Fig. 7** System configuration for friction identification.




(b) case A : $d_1$=23, $d_2$=51




(b) case B : $d_1$=30, $d_2$=45

**Fig. 8** Time responses of limit cycles.

## 5. Experiments

Experiments for friction identification are conducted in the precision x-y positioning system shown in Fig. 4. By varying amplitudes of the nonlinear element $N_r(A)$, various limit cycle conditions are intentionally extracted in the velocity loop. Fig. 8 shows limit cycle oscillations obtained from the experiments. Using amplitudes and frequencies of the extracted limit cycles, unknown parameters of the equivalent nonlinear element are obtained from the harmonic balance condition. Table 1 shows estimated friction coefficients.

A two-degree-of-freedom controller including a PID feedback controller and a model-based feedforward friction compensator shown in Fig. 9 is introduced to verify the identified friction model. The model-based friction compensator estimates actual velocities of the table from the position command, and compensates for friction torque by adding voltage signal corresponding to the friction to torque command fed to the motor. Circular motion experiments are conducted for performance evaluation of the friction compensator. In the circular motion experiments, frictions, especially static friction and Stribeck effect, cause large position errors that appear every 90° interval around the circle, referred to as quadrant glitches. Fig. 10 shows the position error in a circular motion with and without the friction compensator. Clearly, position errors in the vicinity of zero-velocity are reduced through the friction compensator. Especially, quadrant glitches decrease from 20 μm to less than 5 μm.

Experimental results confirm that the friction elements in a servomechanism are effectively identified by using the proposed method.

**Table 1** Identified friction coefficients.

| Friction elements | Value | Unit |
|---|---|---|
| Static element | 0.28 | (N·m) |
| Coulomb element | 0.17 | (N·m) |
| Viscous element | $2.56\times10^{-2}$ | (N·min) |
| Sliding velocity (Stribeck) | 122 | (mm/min) |

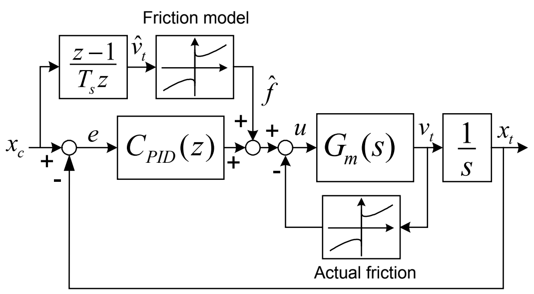



**Fig. 9** block diagram for friction compensation.

## 6. Conclusions

This paper proposes a new application of the limit cycle analysis to identify friction elements in a servomechanism. A friction model including static, Coulomb, and viscous friction as well as Stribeck effect is formulated by using the describing function approximation. A nonlinear element composed of two relay elements is devised to extract various limit cycles. To improve the accuracy of friction estimation, a Butterworth filter is incorporated in the feedback loop to fulfill the requirements of the describing function approximation. Using the accurate model of a linear element and characteristics of the extracted limit cycles, friction elements in a servomechanism are simply identified. Another advantage of the proposed method is that existing control hardware is sufficient to identify friction coefficients and implement its compensation without excessive equipment. The estimated friction elements are applied for a model-based friction compensator. Circular motion experiments confirm that quadrant glitches are reduced to less than 5μm through the friction compensator.

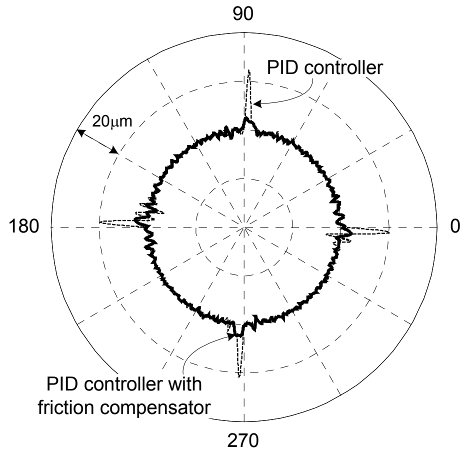



**Fig. 10** Circular motion profiles.

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