

# Shape Control in Ultra-precision Finishing for the Femoral Head of the Artificial Hip Joint by Means of Abrasive Waterjet

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## 1. Introduction

A hip joint replacement is provided to a patient with difficulty in walking caused by articular rheumatism or coxarthrosis, which is very important in social rehabilitation. Since the life of an artificial hip joint is restricted to about 20 years, it is necessary to replace it again. The main factor in which the life is restricted is wear destruction of a socket made from ultra high molecular weight polyethylene (UHMWPE) which plays the role of a cartilage. It is known that wear destruction of a socket not only brings about functional deterioration as a joint, but the wear powder discharged further stimulates an osteoclast [1]. In order to extend the life of an artificial hip joint, it is necessary to reduce wear on the sliding surface of an artificial hip joint.

One of the remedy to reduce the wear is to improve surface roughness and roundness of a femoral head, a sphere shape made of metal such as Co-Cr-Mo alloy. It is said that less than 10 nm Ra of surface roughness and 100 nm of roundness are required. In the last paper, a novel machining method of a sphere-shaped femoral head of an artificial hip joint by means of abrasive waterjet was proposed [2]. The proposed method can realize up to 7 nm Ra of surface roughness, however its roundness got worse.

In this paper, in order to improve roundness of a femoral head, a machining model is analyzed. Machined workpiece shape can be calculated by means of a discrete machining model. Furthermore, a nozzle feed speed planning method to improve the roundness is proposed. The feed speed planning can be conducted by feed-forward calculation of machining volume according to the machining model.

## 2. Finishing method for a femoral head

In the last paper, the machining system which immerses a workpiece into slurry and irradiates a waterjet to the workpiece under the slurry was proposed [2]. In the proposed method, abrasive particles in the slurry involved and accelerated in a waterjet collide with a workpiece. The proposed method is advantageous from the viewpoints of nozzle wear and abrasive reuse as well as ultra fine particle use.

A waterjet technology is applied to finish processing of an artificial hip joint femoral head as shown in Fig.1. The proposed finishing method is as follows: First, abrasive

particles are mixed in the water of the machining chamber beforehand. Then, high pressured waterjet shoots out under the water to the workpiece in a tangential manner. Abrasive particles gather around the machining point. The workpiece rotates under the abrasive mixed water. The abrasive nozzle is controlled to circulate around the workpiece.

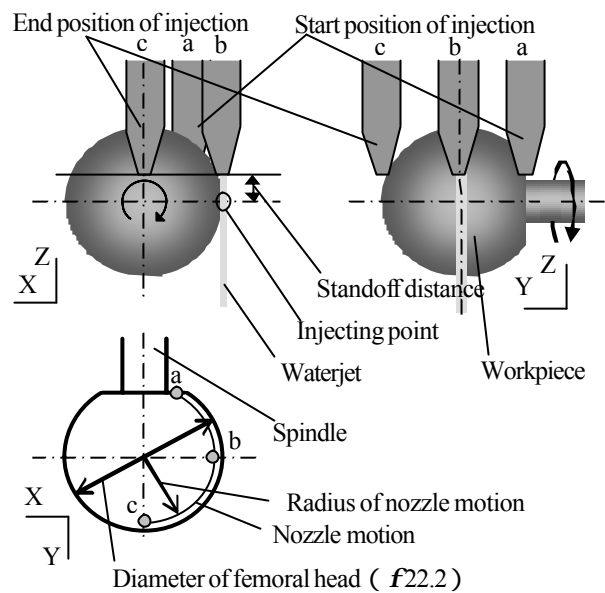


Fig.1 Finishing for the femoral head of an artificial hip joint

## 3. Analysis of machining volume

### 3.1 Machining model

Fig.2 explains a machining model. When a point on a workpiece comes near the outer of a waterjet (point A), some abrasives erode the point on a workpiece with the stand-off distance of  $s_A$ , the impact angle of  $\alpha_A$  and the distance from center of jet of  $r_A$ . Then, as the workpiece rotates, the stand-off distance, the impact angle and the distance from center of jet are varied. Until the point reaches point B where the impact angle decreases to 0 degree, the point on the workpiece is eroded by abrasives. The volume of erosion during a rotation is equal to the volume of erosion during the period mentioned above.

By the way, the density of abrasive is reduced during machining. Therefore, for the analysis of a machining volume, it is necessary to analyze the relationship of the volume of erosion with the stand-off distance  $s$ , the impact angle  $\alpha$ , the

distance from center of jet  $r$  and the density of abrasive  $d$ .

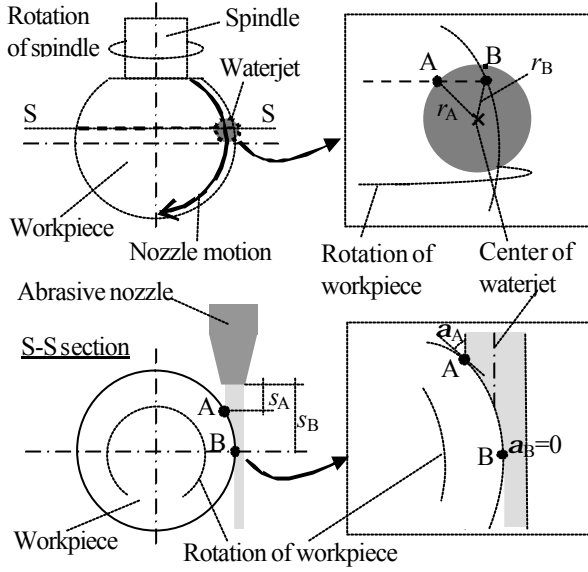


Fig.2 Machining model of finishing for a femoral head

### 3.2 Machining ability distribution within a waterjet

Machining ability distribution within a waterjet can be calculated from the property of abrasives and a workpiece, the velocity and mass distribution of abrasives. However, it is difficult to measure the velocity and mass distribution of abrasives. Therefore, in this paper, machining ability distribution is experimentally determined.  $W(\mathbf{a}, r)$  is defined as the erosion ratio per unit time and area, at impact angle  $\mathbf{a}$  and distance from center of jet  $r$ . Based on Bitter's equation, the erosion ratio is described as follows:

in case of  $\mathbf{a} < \mathbf{a}_0$

$$W(\mathbf{a}, r) = \frac{J(r)\cos^2 \mathbf{a} \sin n\mathbf{a}}{\mathbf{b}} + \frac{J(r)\sin^2(\mathbf{a})}{\mathbf{e}} \quad (1)$$

in case of  $\mathbf{a} > \mathbf{a}_0$

$$W(\mathbf{a}, r) = \frac{J(r)\cos^2 \mathbf{a}}{\mathbf{b}} + \frac{J(r)\sin^2(\mathbf{a})}{\mathbf{e}} \quad (2)$$

where  $J(r)$  is the kinetic energy of abrasives per unit time,  $\mathbf{b}$  is the coefficient of cutting wear,  $\mathbf{e}$  is the coefficient of deformation wear and  $n = p/2\mathbf{a}_0$ .  $\mathbf{a}_0$  is called as a critical angle which represents a reflection angle of an abrasive when the vertical velocity of an abrasive is zero.

When  $\mathbf{a} = p/2$ , from Equation (2), the coefficient of deformation wear is obtained by the following equation.

$$\mathbf{e} = \frac{J(r)}{W(p/2, r)} \quad (3)$$

When the waterjet shoots out to a workpiece with the impact angle of  $\mathbf{q}$ , from Equations (1) and (3), the coefficient of cutting wear is obtained by the following equation.

$$\mathbf{b} = \frac{J(r)\cos^2 \mathbf{q} \cdot \sin n\mathbf{q}}{W(\mathbf{q}, r) - W(p/2, r)\sin^2 \mathbf{q}} \quad (4)$$

Then, from Equations (3) and (4), Equations (1) and (2) can be transformed as follows:

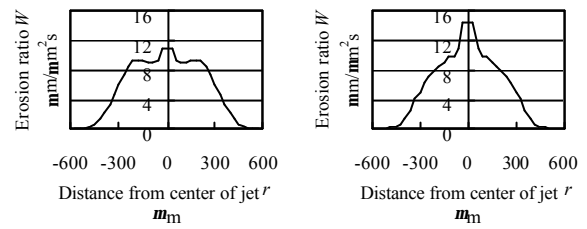
in case of  $\mathbf{a} < \mathbf{a}_0$

$$W(\mathbf{a}, r) = \frac{(W(\mathbf{q}, r) - W(p/2, r)\sin^2 \mathbf{q})\cos^2 \mathbf{a} \cdot \sin n\mathbf{a}}{\cos^2 \mathbf{q} \cdot \sin n\mathbf{q}} + W(p/2, r) \cdot \sin^2 \mathbf{a} \quad (5)$$

in case of  $\mathbf{a} > \mathbf{a}_0$

$$W(\mathbf{a}, r) = \frac{(W(\mathbf{q}, r) - W(p/2, r)\sin^2 \mathbf{q})\cos^2 \mathbf{a}}{\cos^2 \mathbf{q} \cdot \sin n\mathbf{q}} + W(p/2, r) \cdot \sin^2 \mathbf{a} \quad (6)$$

Therefore, if  $W(p/2, r)$ ,  $W(\mathbf{q}, r)$  and  $\mathbf{a}_0$  are obtained, the volume of erosion with any the impact angles can be calculated using equation (5) and (6). The erosion ratio with the impact angle of  $p/2$  and  $p/4$  are obtained experimentally as shown in Fig.3. Furthermore,  $\mathbf{a}_0$  is determined as 80 degrees experimentally.



(a)  $\mathbf{a} = p/2$  (b)  $\mathbf{a} = p/4$

Fig.3 Erosion ratio of waterjet

### 3.3 Relationship of stand-off distance and density of abrasive with erosion ratio

Taking into consideration an effect of stand-off distance  $s$ , the erosion ratio is described as  $f(s)W(\mathbf{a}, r)$ , where  $f(s)$  is the coefficient of reduction. Furthermore, the erosion ratio is directly proportional to the density of abrasive  $d$ . Therefore the erosion ratio with stand-off distance  $s$  and density of abrasive  $d$  is obtained by

$$W(\mathbf{a}, r) \cdot f(s) \cdot k_d d \quad (7)$$

where  $k_d$  is the coefficient of proportion. The erosion ratio is estimated by the depth of groove. Relationships of stand-off distance and density of abrasive with the depth of a groove are obtained by experiments as shown in Figs. 4 and 5 respectively.

From these experimental results,  $f(s)$  and  $k_d$  are obtained as

$$f(s) = -0.0352(s - 8.5) + 1 \quad (8)$$

$$k_d = 0.667 \quad (9)$$

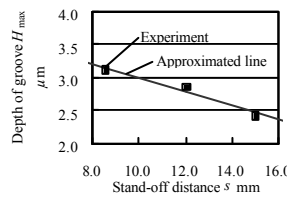


Fig.4 Relationship between the stand-off distance and the depth of groove

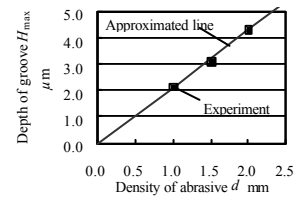


Fig.5 Relationship between the density of abrasive and the depth of groove

## 4. Simulation method

In this chapter, a simulation method to estimate the shape of a workpiece after injection is proposed. The surface of a femoral head is divided into minute elements with the width of  $d$  as shown in Fig.6. Then the machining volume of each element is calculated. Furthermore, the range of waterjet is divided into minute concentric circular elements. Elements on the inner side are defined as from  $B_{-1}$  to  $B_{-N}$ . Elements on the outer side are defined as from  $B_1$  to  $B_N$ .

Machining volume  $h_j$  in minute elements during a rotation is obtained by

$$h_j = \frac{60}{p} \cdot \frac{1}{2pR_j d} \left( \sum_{i=9,N}^{-j} f(s_{ij}) \cdot k_a d \cdot W(\mathbf{a}_{ij}, r_i) \cdot S_{ij} + \sum_{i=j}^N f(s_{ij}) \cdot k_a d \cdot W(\mathbf{a}_{ij}, r_i) \cdot S_{ij} \right) \quad (10)$$

where  $p$  is the rotational speed of a workpiece,  $R_j$  is the radius of the minute element  $A_j$  and  $S_{ij}$  is the size of the area where the minute element of workpiece  $A_j$  and the minute element of waterjet  $B_i$  intersect each other.

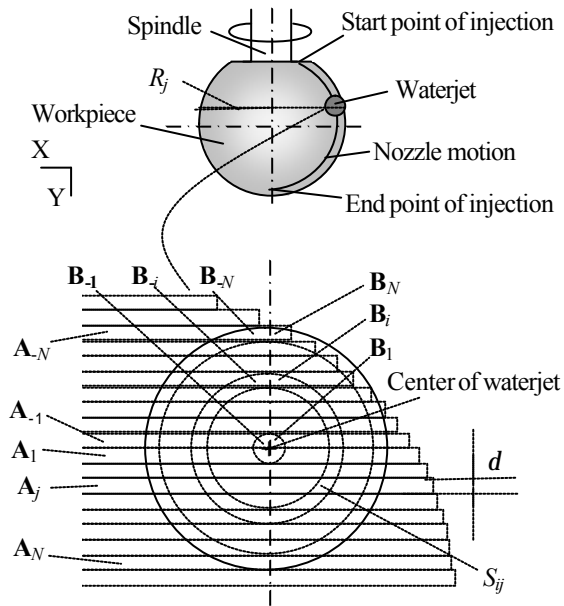


Fig.6 Machining model for simulation

## 5. Shape control method

### 5.1 Nozzle feed speed planning method

Relationship between the reciprocal of feed speed of a nozzle and the depth of cut is obtained by simulation as shown in Fig.7, assuming that the density of abrasive is constant during machining, where the width of each minute element is 30  $\mu\text{m}$ . Therefore, the relationship between the feed speed of nozzle and the depth of cut at the angle of  $\mathbf{b}$  is described as

$$h(\mathbf{b}) = \frac{1}{v(\mathbf{b})} k_v(\mathbf{b}) \quad (11)$$

where  $k_v(\mathbf{b})$  is the coefficient of proportion.

The nozzle feed speed planning method is as follows: First,  $k_v(\mathbf{b})$  in Equation (11) is calculated by the simulation. Then, by

using  $k_v(\mathbf{b})$ , the feed speed to achieve the desired depth of cut is obtained by

$$v(\mathbf{b}) = k_v(\mathbf{b}) \frac{1}{h_d(\mathbf{b})} \quad (12)$$

The feed speed of a nozzle is planned as shown in Fig.9.

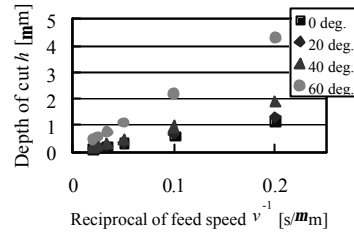


Fig.7 Relationship between the feed speed and the depth of cut

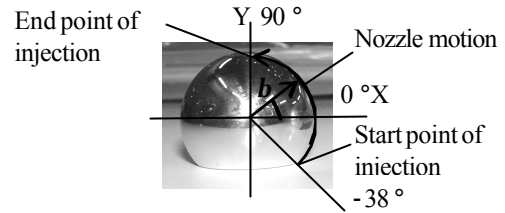


Fig.8 Definition of the angle of a femoral head

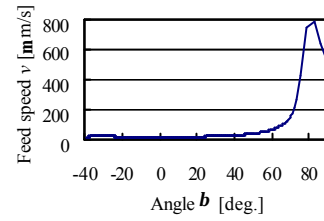


Fig.9 Feed speed to achieve the desired depth of cut

## 5.2 Compensation of density of abrasive

It is necessary to consider the abrasive density change, because the density of abrasive decreases during machining. In section 3.3, it is clarified that the erosion ratio is proportional to the density of abrasive  $d$ . Therefore, it is possible to compensate for the machining volume error caused by the density change of abrasive, by multiplying the feed speed in Equation (12) by  $k_a d$ .

## 6. Simulations and experiments

### 6.1 Conditions of experiments and simulations

In the simulations and experiments, the femoral head of an artificial hip joint with the diameter of 22.2mm made from Co-Cr-Mo alloy was used as a workpiece. The femoral head is in the condition after grinding and before giving hand lapping. Its roundness before the experiments is about 300 nm. Table 1 shows conditions of the experiments and simulations. The following three cases are investigated by the experiments and simulations.

- (1) In case of constant feed speed of 0.01mm/s.
- (2) In case of variable controlled feed speed without abrasive density compensation.
- (3) In case of variable controlled feed speed with abrasive density compensation.

Table 1 Conditions of experiments and simulations

Feed speed of nozzle [mm/s]	0.01, Controlled
Offset distance [mm]	-0.1
Particle size of abrasive [ $\mu\text{m}$ ]	1.0
Density of abrasive [wt%]	3.2
Standoff distance [mm]	8.5
Water pressure [MPa]	200
Rotational speed of workpiece [rpm]	3000

## 6.2 Investigation of roundness

The geometry error of a workpiece after injection with the constant feed speed of 0.01mm/s is shown in Fig.10. The geometry error of a workpiece after injection with the variable controlled feed speed without abrasive density compensation is shown in Fig.11. The geometry error of a workpiece after injection with the variable controlled feed speed with abrasive density compensation is shown in Fig.12, respectively.

In the case of injection with the constant feed speed, the depression was formed around 90 degrees of a workpiece. It is because around 90 degrees, the time of shooting waterjet per unit length of a circumference increases as the circular velocity by the rotation of a workpiece decreases. For the reason, the roundness of a workpiece after injection got worse to 576 nm.

By using the feed speed planning method, the roundness of a workpiece after injection was improved to 304 nm. However, the radius of a workpiece after injection increases around 90 degrees because of the reduction in the density of abrasive.

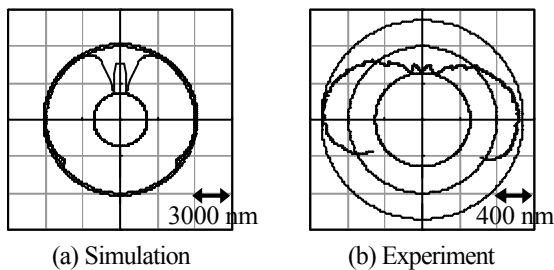


Fig.10 Geometry error (with the constant feed speed)

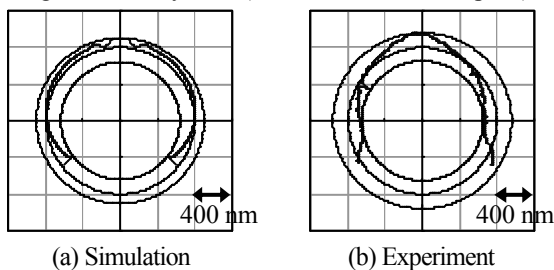


Fig.11 Geometry error (with the feed speed planning method)

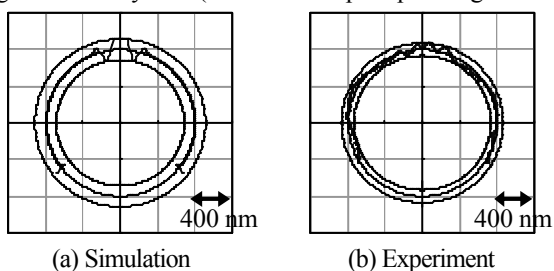


Fig.12 Geometry error (with the correcting method)

By using the compensation method, the roundness of a workpiece after injection was successfully improved to 136 nm. It means that the compensation method can reduce the influence of the reduction in the density of abrasive, and thus the machining volume can be constant on the whole surface of a workpiece.

Furthermore, it was verified that the experimental and simulation results of the machined workpiece shape have good correspondence qualitatively. Therefore, the proposed simulation method is verified to be feasible to estimate the shape of workpiece after injection.

## 6.3 Discussions

By using the compensation method, the roundness of a workpiece after injection was successfully improved to 136 nm. However, the required roundness of 100 nm was not achieved. By detail observation, there is found the uneven shape around 90 degrees as shown in Fig.12 (b), and thus the roundness of a workpiece gets worse. The cause of this uneven shape is considered that the mesh is divided along Y axis. For the reason, the mesh along the angle is larger around 90 degrees. Therefore, it is necessary to change the width of mesh according to the angle of a workpiece.

## 7. Conclusions

In this paper, a new machining method for the femoral head of an artificial hip joint by means of abrasive waterjet is presented. Especially, in order to estimate the shape of a workpiece after injection, simulation method was proposed. Furthermore, in order to improve the roundness of a femoral head, the shape control method was proposed. By using the proposed method in finishing for the femoral head of an artificial hip joint, the roundness of a workpiece could be successfully improved to 136 nm.

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