Viscoelastic Stress Analysis of Precision Aspherical Glass Lens Forming Process using Finite Element Method

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1. Introduction

The conventional production of lenses is a rather complicated process. Glass raw material which has been pressed to a rough form has to be processed on both sides in order to obtain a finished precision optical component. There have been recent advances in the optical fabrication techniques such as magnetorheological polishing (MRF), ion beam polishing and single point diamond turning (SPDT), but the overall complexity and cost involved in these processes are very high for medium to high volume production of aspherical glass optics. The conventional fabrication methods are more suitable for manufacturing spherical glass lenses. Aspherical lenses on the other hand have one or both surfaces that do not conform to a sphere and provide greater advantage over spherical lenses due to reduced light losses and aberrations, better image quality, and compact lens assemblies. In order to fulfill the high demands on refractive and reflective optical components, it is necessary to develop accurate fabrication techniques that generate aspherical optical surfaces in glass with a figure accuracy of $\lambda/10$ (λ = 633 nm), and a surface roughness of 1-5 nm rms.

For over 30 years, work has been done on developing a process for precision molding of aspherical glass lenses [1, 2]. In a lens molding process (see Figure 1) glass blank or gob is initially heated to a temperature between its transition and softening temperature (heating stage), and subsequently pressed between the two mold halves into a lens shape (forming/molding stage). Controlled cooling of the formed lens is then carried out with the molds in the closed position to remove forming stresses and avoid heat sink marks on the lens surface (annealing stage). Lens is eventually released at a temperature close to room temperature and finally allowed to cool to the ambient temperature. This is a net shape and an environment friendly process and if designed correctly can be suitably adapted for high volume production of precision aspherical glass lenses. Some of the challenges associated with this process include precise mold fabrication, careful temperature control, lens shrinkage, curve conformance and other process issues such as mold wear, mold life and sticking of glass to molds at high temperatures.

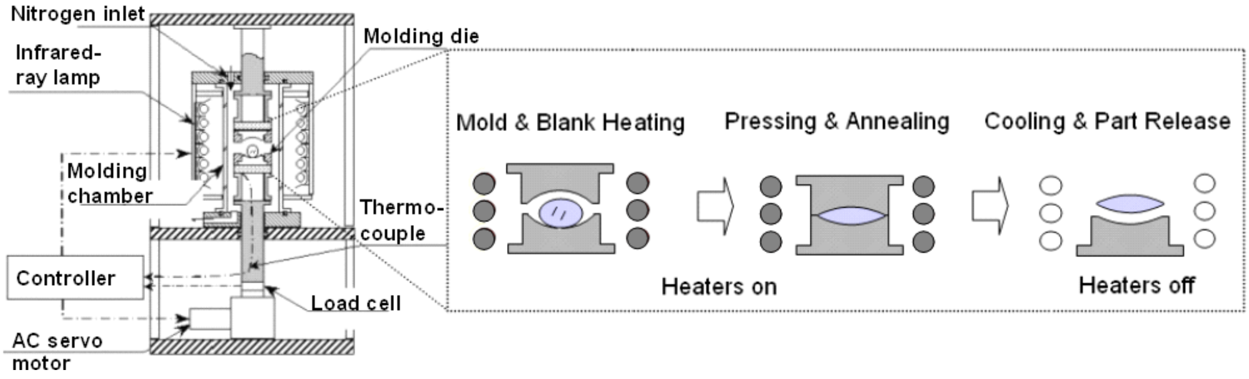


Figure 1: Schematic illustration of a lens molding process

The primary objective of our research is to gain a fundamental understanding of the lens molding process by developing a finite element method (FEM) based simulation model of the process. Specifically in this work it is aimed to incorporate the viscoelastic response of glass, into the numerical calculation and observe its influence on the resultant stress distribution inside a molded lens at the end of the molding stage by independently varying the forming velocity and temperatures. The knowledge of stress distribution at the end of the molding stage is important to design the subsequent annealing step in order to obtain a stress free lens with a homogenous refractive index.

2. Previous Work

Most of the research done in the area of glass forming has been performed in the field related to container forming, television and automobile windscreen screen manufacture etc. All these processes involve bulk glass forming in which demands for precision, irregularity and surface finish are not as stringent as in case of lens molding. A limited amount of work has been done in the past to apply FEM to a glass molding process in general, and in most of these works glass was modeled as a Newtonian fluid and viscoelastic effects were neglected. In the past, the authors have investigated the performance of a lens molding process by conducting actual aspherical lens forming experiments [3]. FEM simulations were also performed by modeling glass as an elastic-plastic [3] and a viscoelastic material [4]. The authors had concluded from the experimental studies that precision aspherical lenses can be molded with a surface variation less than 5 fringes and irregularity less than $\lambda/10$. Qualitative and semi-quantitative agreement between the experimental and the predicted results for curve conformance and stress distribution were obtained. The authors had demonstrated a need for a sophisticated numerical model incorporating accurate high temperature glass properties for more reliable results.

3. Theory of Viscoelasticity as Applied to Glass Forming

Viscoelasticity [5] describes a time dependent response of liquids and solids to either mechanical stress or strain. In the class of viscoelastic materials such as glass, the application of a constant load is followed by a deformation, which can be made up of instantaneous deformation (elastic effect) followed by continual deformation with time (viscous effect), that results in the decay of the applied load and is termed as relaxation. This can best be illustrated with the help of mechanical models – combinations of springs and dashpots, where the spring represents the elastic behavior and dashpot represents the viscous behavior. A single Maxwell model consisting of a spring and a dashpot in series or a generalized Maxwell model (several Maxwell models placed in parallel), shown in Figure 2, can be used to represent a real viscoelastic material behavior. The stress relaxation modulus and the stress relaxation function for this model can be represented by Equations (1) and (2) respectively:

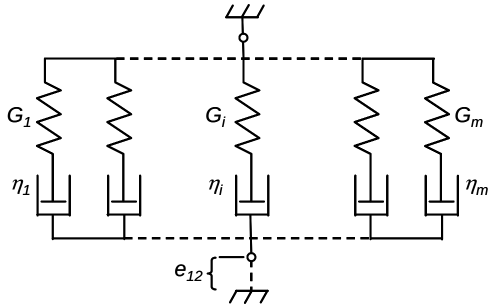


Figure 2: Generalized Maxwell model for modeling viscoelastic stress relaxation

$$G_1(t) = 2G\sum_{i=1}^{n} w_i e^{-t/\tau_i} \qquad (1)$$

$$\psi_1(t) = \frac{G_1(t)}{G_1(0)} = \sum_{i=1}^{n} w_i e^{-t/\tau_i} \qquad (2)$$

where, $G_1(t)$ is the shear stress relaxation modulus, a time dependent analog of its elastic counterpart G (shear modulus), τ_i is the shear stress relaxation time (sec) given by η_i/G_i, (η_i is the viscosity of the dashpot and G_i is the shear modulus of the spring) and w_i is the weighing factor such that $\sum w_i = 1$. In an FEM calculation, the change in stress due to relaxation during forming at high temperature between increments $t\text{-}\Delta t$ and t is calculated and converted to nodal forces and subtracted from external forces [6].

4. Numerical Modeling

2D-Axisymmetric simulations of the lens molding process were conducted using a commercial general purpose, non-linear FEM program MARC, which is suitable for viscoelastic modeling of materials. Figure 3 shows the FEM model of a lens molding process that was used to study the effect of viscoelasticity on residual stress distribution. Only the forming stage of the lens molding process was simulated to observe the effect of process conditions on residual stress distribution inside a molded lens. It should be noted that the simulation conditions were chosen corresponding to the experimental conditions used by the authors in [3].

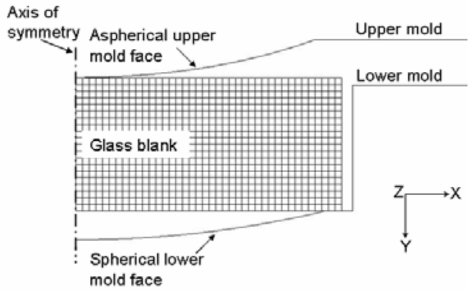


Figure 3: FEM model of lens molding

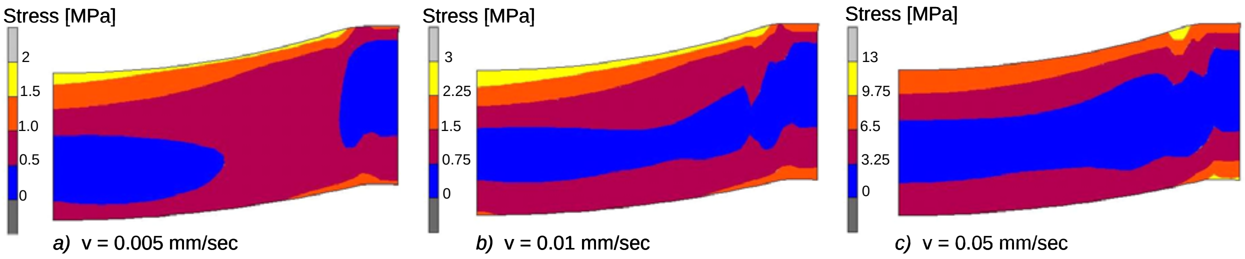
Four-node axisymmetric, quadrilateral elements were used to model the glass disk blank shown in Figure 3. Upper and lower molds were modeled as rigid bodies. Friction and heat transfer at the glass-mold interface was neglected. The simulations were carried out at three different molding velocities: *(i)* 0.05 mm/sec *(ii)* 0.01 mm/sec and *(iii)* 0.005 mm/sec for a constant glass forming temperature of 685 °C; and also at three different temperatures: *(i)* 670 °C *(ii)* 685 °C and *(iii)* 690 °C for a constant forming velocity of 0.01 mm/sec. Schott grade BK-7 optical glass was used in this study. The diameter of the glass blank used in the simulations was 12 mm and height was 6 mm. Elastic and viscous properties input to the simulation at different temperatures are shown in Table 1 [7-9]. Viscosity measurements at different temperatures for BK 7 glass were performed by the authors using the parallel plate viscometer technique [10].

	Temperature = 670 °C	Temperature = 685 °C	Temperature = 690 °C
Elastic modulus, [MPa]	27737	23626	19515
Shear modulus, [MPa]	10668	9086	7505
Poisson's ratio	0.3	0.3	0.3
Viscosity, [MPa-sec]	158	37	25
Relaxation time, [sec]	0.0148	0.0040	0.0033

Table 1: High temperature glass properties used for the viscoelastic forming analysis

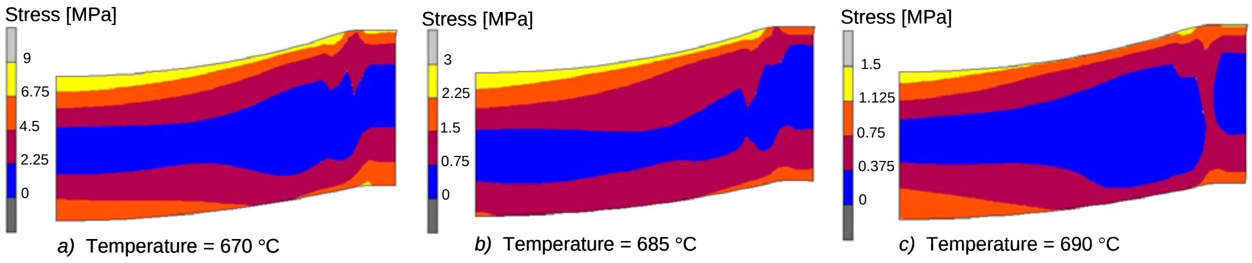
5. Results and Discussion

Figure 4 shows the predicted residual Von Mises stress distribution inside a molded lens for three different forming velocities at the end of the molding stage. It can be observed from the Figure that stresses continue to rise as the molding velocity is increased. The stresses inside the formed lens almost increase by a factor of five as the velocity is increased ten times. This is primarily because at higher velocities the stresses inside the lens do not have enough time to relax causing the building up stress by the end of the molding stage. At lower velocities however, the stresses continue to relax as the glass is being formed resulting in lower stress. The magnitude of the overall stress is not only important for carefully designing the subsequent annealing stage but also from the point of view of lens shrinkage during the cooling stage. Higher stresses result in greater elastic deflection of the glass lens as it cools down to room temperature.



a) v = 0.005 mm/sec *b)* v = 0.01 mm/sec *c)* v = 0.05 mm/sec

Figure 4: Predicted residual Von Mises stress distribution inside a formed lens for different molding velocities *a)* 0.005 mm/sec *b)* 0.01 mm/sec and *c)* 0.05mm/sec at a constant molding temperature of 685 °C



a) Temperature = 670 °C *b)* Temperature = 685 °C *c)* Temperature = 690 °C

Figure 5: Predicted residual Von Mises stress distribution inside a formed lens for different molding temperatures *a)* 670 °C *b)* 685 °C and *c)* 690 °C at a constant molding velocity of 0.01 mm/sec

Figure 5 shows the predicted residual Von Mises stress distribution inside a molded lens for three different molding temperatures of glass. As the temperature is increased the stresses inside the molded lens continue to decrease. Viscosity has a strong dependence on temperature and as the temperature is increased beyond the transition temperature of glass, its viscosity continues to fall exponentially making the material more fluid and easier to deform resulting in lower stresses. The other reason for lower molding stresses at elevated temperature is as explained above, the lower relaxation time, because of which stresses relax faster at higher temperature (see Table 1 for relaxation times at different temperatures).

Figure 6 shows the predicted load time plot for a glass lens molding simulation at a forming temperature of 690 °C and velocity of 0.01 mm/sec. The predicted results are in qualitative agreement with the experimental results in Reference [3]. Due to the unavailability of experimental data for these simulation conditions the authors were unable to compare the predicted and experimental observations, but the results validate the viscoelastic methodology implemented in the simulation. Minimal loads are required to form glass blank during most part of the molding stage. During the later part as the molds are approaching the closed position the load begins to rise rapidly. This is because the flow of glass is constrained by the upper and lower molds (see Figure 3) that adds resistance to material flow and thus requiring higher force to further deform glass into a lens shape.

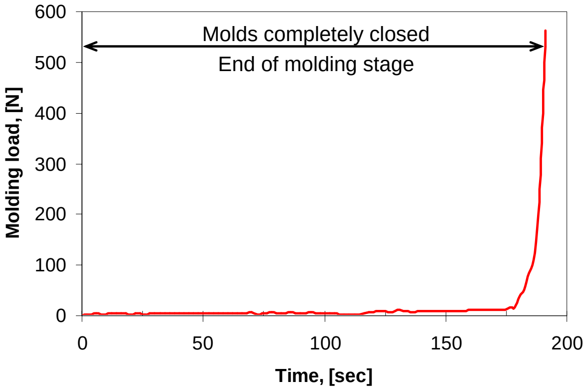


Figure 6: Predicted load time plot during glass lens molding at a forming temperature of 690 °C and velocity of 0.01 mm/sec

6. Conclusions

A numerical FEM model of an aspherical lens molding process incorporating the viscoelastic characteristic of glass has been presented. The molding stage has been simulated for different process conditions. The predicted results of load and stress distribution are in qualitative agreement with the viscoelastic theory and the experimental trends. Precise information of high temperature glass properties and description of heat transfer and friction mechanism at the glass-mold interface would be needed to make accurate numerical predictions.

7. References

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