The Effects of Asymmetrical Radial Stiffness in Precision Rotating Machines: The Duality of Fixed and Rotating Sensitive Directions

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Abstract

When radial forces act on the structural loop of a machine tool or gauge, errors arise due to finite, asymmetrical structural loop stiffness. For all fixed and rotating sensitive direction applications, the possible errors are as follows: size, nonconcentricity, and form (two—lobe). The type of the error depends on the state of the radial force and the state of the asymmetrically stiff structural loop element. The state of the force or the structural loop element may be either fixed or rotating relative to the fixed coordinate frame of the machine base.

Several authors and standards describe these errors for specific examples of fixed and rotating sensitive direction machine tools and gauges. This article extends this analysis and attempts to show the duality that exists between the fixed and rotating sensitive directions for machine tools and gauges. The purpose of this effort is to provide a systematic accounting of these errors in the uncertainty analysis as well as to provide a useful troubleshooting guide. The results are presented in the form of a structured table which covers all cases. Additionally, simple mathematical descriptions of the errors are presented in matrix and traditional form. Due to space considerations in this abstract, these results are presented separately as indicated.

The analysis presented here distinguishes between the following machine states: fixed or rotating sensitive direction; fixed or rotating asymmetrically stiff structural loop element; and fixed or rotating radial force. For each combination of sensitive direction, force, and structural loop element, the errors are categorized. Superposition may be used to combine individual states in order to analyze real machines with multiple forces and structural loop elements.

Experimental verification is presented for the two most interesting cases. Specifically, the effect of rotating unbalance and asymmetrical stiffness in the support structure for an air-bearing diamond boring machine is presented. Additionally, the effect of a radial gauge force on rotating asymmetrically stiff workpiece is shown for a roundness measurement. Both of these experiments show excellent agreement with the analysis presented.

Finally, the duality that exists between rotating and fixed sensitive direction applications for applied radial loads and asymmetrical stiffness. Through this duality, it is possible to show that the rotating and fixed sensitive direction states indicate a change in the location of an observer and thus the analysis of error is the same in both cases. Again, experimental results are presented to illustrate the duality of the fixed and rotations sensitive directions.

Keywords: asymmetrical stiffness, compliance, sensitive direction, axis of rotation, structural loop, precision machine design, dynamic stiffness.

Nomenclature:

Principal stiffnesses of rotating element k_{1_R}, k_{2_R} and of fixed element k_{1_F}, k_{2_F}

Components of fixed force in fixed frame W_{X_F} W_{Y_F}

Components of rotating force in rotating frame W_{X_R} W_{Y_R}

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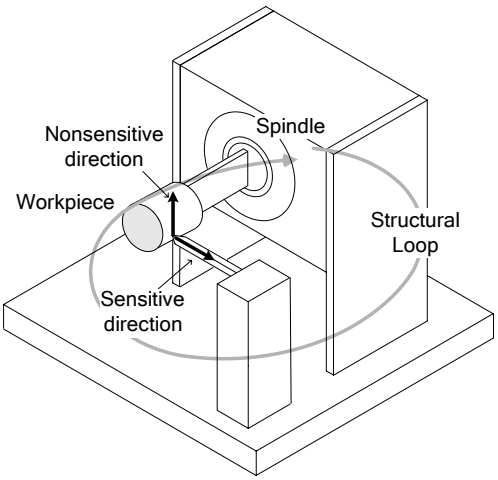

Figure 1 Fixed sensitive direction

Fixed sensitive direction

An example of a fixed sensitive direction application is shown Figure 1. This figure shows a lathe schematically and conceptually illustrates the possibilities for asymmetrical stiffness in elements in the structural loop. Specifically, the workpiece is shown to be attached to the spindle rotor by an element with a rectangular cross-section. For the angle of rotation shown, the rotating structural loop is stiffer vertically than horizontally. As the workpiece rotates, the orientations of the elements rotate. It can be shown that any linear structural loop element, such as the workpiece, chuck, or rotor, can be modeled as in this way regardless of the actual cross section of the material.

Additionally, this figure shows the possibility of asymmetrical stiffness in the spindle support. This is illustrated by a flexure where the vertical stiffness is higher than the horizontal stiffness. These orientations remain fixed as the spindle rotates.

The errors for all four possible cases are summarized graphically in Table 1. These cases are described in detail in the oral presentation and the forthcoming paper [1] where experimental confirmation of results for a precision roundness gauge.

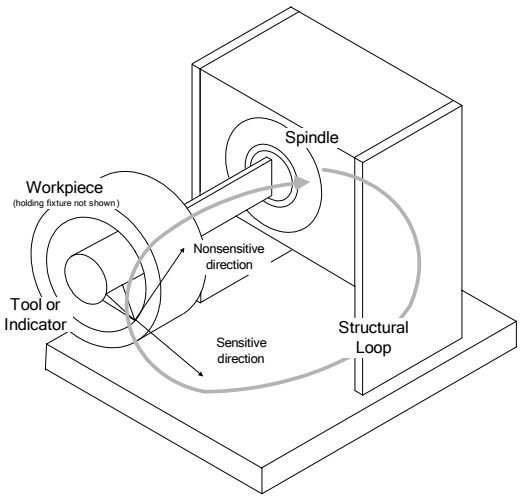

Figure 2 Rotating sensitive direction (work holder is removed

Rotating Sensitive Direction

Figure 2 shows a schematic example of a rotating sensitive direction machines such as a boring machine (or bore roundness gauge). This figure conceptually illustrates asymmetrically stiff in elements—both rotating and fixed—in the structural loop. Specifically, the tool or indicator is shown to be attached to the spindle rotor by a shaft with a rectangular cross-section. For the angle of rotation shown, the rotating structural loop is stiffer vertically than horizontally. As the tool or indicator rotates, the orientation of this element rotates.

Additionally, this figure shows the possibility of asymmetrical stiffness in the spindle support. This is illustrated by a flexure where the vertical stiffness is higher than the horizontal stiffness. These orientations remain fixed as the spindle rotates.

The results for all four possible cases are summarized graphically in Table 2. These cases are described in detail in the oral presentation and the forthcoming paper [2] where experimental results are presented for a diamond boring machine.

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[1] Dalrymple, T. M. The Effects of Asymmetrical Radial Stiffness in Precision Rotating Machines: Part 1 The Fixed Sensitive Direction, to be submitted to Precision Engineering at 2004 ASPE annual Meeting

[2] Dalrymple, T. M. The Effects of Asymmetrical Radial Stiffness in Precision Rotating Machines: Part 2 The Rotating Sensitive Direction, to be submitted to Precision Engineering at 2004 ASPE annual meeting

Table 1 Fixed sensitive direction first order effects. Principal axes align with respective coordinate systems

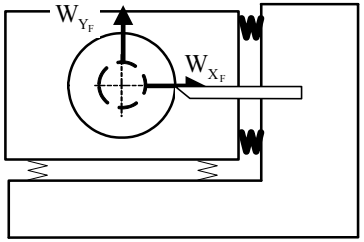
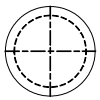
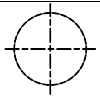
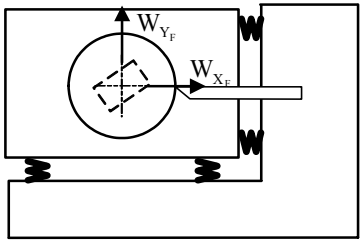
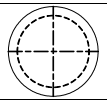
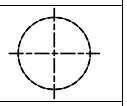
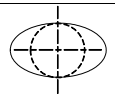
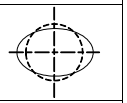
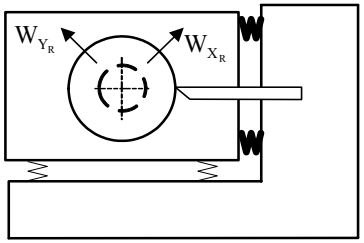
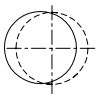
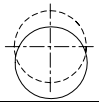
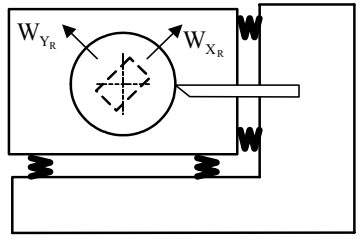
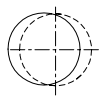
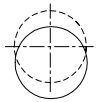
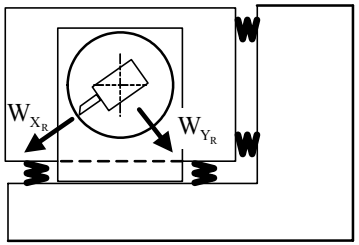
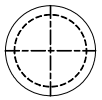
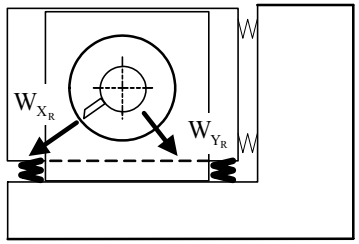
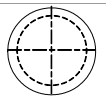
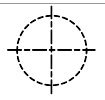
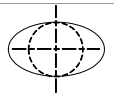
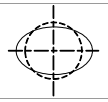
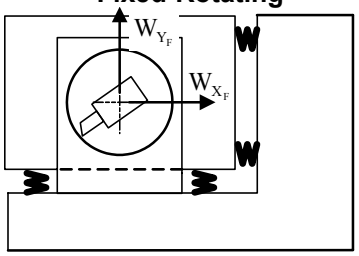
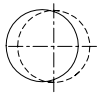
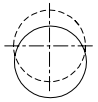
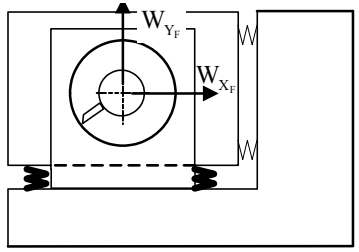
Schematic illustration States of the Force(W)--Structural loop	Errors due to indicated applied forces	Machine Tool Example (negative sign on force applies)	Metrology Examples (positive sign on force applies)
Fixed-Fixed  	$k_{1_F} = k_{2_F}$ or $k_{1_F} \neq k_{2_F}$ $\mp W_{X_F}$:  size $\mp W_{Y_F}$:  none	Lathe with symmetrical or asymmetrical spindle support • W_{X_F} Normal cutting force • W_{Y_F} Tangential cutting force	Roundness gauge with symmetrical or asymmetrical spindle support • W_{X_F} Normal gauge force • W_{Y_F} Frictional gauge force
Fixed-Rotating  	$k_{1_R} = k_{2_R}$ $\mp W_{X_F}$:  size $\mp W_{Y_F}$:  none	Lathe with symmetrical rotating elements • W_{X_F} Normal cutting force • W_{Y_F} Tangential cutting force	Roundness gauge no rotating asymmetries • W_{X_F} Normal gauge force • W_{Y_F} Frictional gauge force
	$k_{1_R} \neq k_{2_R}$ e.g. $k_{1_R} > k_{2_R}$ $\mp W_{X_F}$:  size & form $\mp W_{Y_F}$:  form	Lathe with asymmetrical spindle rotor or workpiece • W_{X_F} Normal cutting force • W_{Y_F} Tangential cutting force	Roundness gauge with asymmetrical spindle rotor or workpiece • W_{X_F} Normal gauge force • W_{Y_F} Frictional gauge force
Rotating-Fixed  	$k_{1_F} = k_{2_F}$ or $k_{1_F} \neq k_{2_F}$ $\mp W_{X_R}$:  non-concentric $\mp W_{Y_R}$:  non-concentric	Lathe with symmetrical or asymmetrical spindle support • W_{X_R} Rotating unbalance • W_{Y_R} Rotating unbalance	Cylindricity gauge with symmetrical or asymmetrical spindle support • W_{X_R} Rotating unbalance • W_{Y_R} Rotating unbalance
Rotating-Rotating  	$k_{1_R} = k_{2_R}$ or $k_{1_R} \neq k_{2_R}$ $\mp W_{X_R}$:  non-concentric $\mp W_{Y_R}$:  non-concentric	Lathe with symmetrical or asymmetrical spindle rotor or workpiece • W_{X_R} Rotating unbalance • W_{Y_R} Rotating unbalance	Cylindricity gauge with symmetrical or asymmetrical rotor or workpiece • W_{X_R} Rotating unbalance • W_{Y_R} Rotating unbalance

Table 2 Rotating sensitive direction first order effects. Assume principal axes align with respective coordinate systems

Schematic illustration States of the Force(W)--Structural loop	Errors due to indicated applied forces		Machine Tool Example (positive sign on force applies	Metrology Examples (negative sign on force applies
Rotating-Rotating 	$k_{1_R} = k_{2_R}$ or $k_{1_R} \neq k_{2_R}$		Boring Machine with symmetrical or asymmetrical boring bar • W_{X_R} unbalance or normal gauge force • W_{Y_R} unbalance or tangential gauge force (friction)	Roundness bore gauge with symmetrical or asymmetrical probe extension • W_{X_R} unbalance or normal gauge force • W_{Y_R} unbalance or tangential gauge force (friction)
	$\pm W_{X_R}$	$\pm W_{Y_R}$		
				
	Size	none		
Rotating- Fixed 	$k_{1_F} = k_{2_F}$		Boring machine with symmetrical spindle support • W_{X_R} unbalance or normal cutting force • W_{Y_R} unbalance or tangential cutting force	Roundness bore gauge with symmetrical workhead support • W_{X_R} unbalance or normal gauge force • W_{Y_R} unbalance or tangential gauge force (friction)
	$\pm W_{X_R}$	$\pm W_{Y_R}$		
				
	Size	none		
	$k_{1_F} \neq k_{2_F}$ e.g. $k_{1_F} > k_{2_F}$		Boring machine with asymmetrical spindle support • W_{X_R} unbalance or normal cutting force • W_{Y_R} unbalance or tangential cutting force	Roundness bore gauge with asymmetrical workhead support • W_{X_R} unbalance or normal gauge force • W_{Y_R} unbalance or tangential gauge force (friction)
	$\pm W_{X_R}$	$\pm W_{Y_R}$		
				
	size & form	form		
Fixed-Rotating 	$k_{1_R} = k_{2_R}$ or $k_{1_R} \neq k_{2_R}$		Horizontal boring machine with symmetrical or asymmetrical boring bar • W_{X_F} none • W_{Y_F} gravity	Horizontal bore gauge with symmetrical or asymmetrical probe extension • W_{X_F} • W_{Y_F} gravity
	$\pm W_{X_F}$	$\pm W_{Y_F}$		
				
	non-concentric	non-concentric		
Fixed-Fixed 	$k_{1_F} = k_{2_F}$ or $k_{1_F} \neq k_{2_F}$		Horizontal boring machine with symmetrical or asymmetrical spindle support • W_{X_F} • W_{Y_F} gravity	Horizontal roundness bore gauge with symmetrical or asymmetrical spindle support • W_{X_F} • W_{Y_F} gravity
	$\pm W_{X_F}$	$\pm W_{Y_F}$		