An Investigation of the Dynamic Absorber Effect in High-Speed Machining

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Introduction

Recent improvements in machine and spindle designs have led to the increased use of high-speed machining (HSM) in the manufacture of discrete parts, especially in the aerospace industry [1]. HSM seeks to increase depth of cut, and the corresponding material removal rate (MRR), through the selection of optimum cutting parameters. It is recognized that a major practical limitation on the productivity of HSM systems is regenerative chatter [2]. One method of pre-process chatter prediction and avoidance is the well-known stability lobe diagram. Stability lobe diagrams, which predict system stability as a function of selected machining parameters, are can used to select the best available spindle speed for maximized MRR. Stable and unstable regions (separated by the stability “lobes”) are seen in these diagrams, depending on the selected spindle speed and axial depth of cut, b, for peripheral end milling.

The equation for the limiting depth of cut, \( b_{\text{lim}} \), at each spindle speed is shown in Eq. 1, where \( K_s \) is the specific cutting energy, \( m^* \) is the average number of teeth in the cut (which depends on the number of teeth on the cutter and the selected radial immersion), and \( \text{Re}[G_{zz}(w)]_{\text{Oriented}} \) is the negative portion(s) of the real part of the oriented tool point frequency response function (FRF) [2]. As seen in Eq. 1, \( b_{\text{lim}} \) can be increased by minimizing the value of \( \text{Re}[G_{zz}(w)]_{\text{Oriented}} \), resulting in increased depths of cut and higher MRR.

This work describes a method to shift the stability lobes up to increased depth of cut levels by decreasing the minimum value of the negative real part of the tool point FRF. This is achieved by matching the fundamental frequency of the cantilevered tool to a natural frequency of the holder/spindle, similar to the changed assembly response observed when implementing a damped dynamic absorber. In this work, the ‘dynamic absorber effect’ is verified experimentally with simple flexures. A model of a lumped-mass, two degree-of-freedom (DOF) system, created through receptance coupling techniques applied to two single DOF systems, is used to predict the response at the top of the flexure assembly as the ratio between the top flexure and bottom flexure mass and stiffness is modified.

Flexure Designs

The 2DOF flexure shown in Figure 1, produced by assembling two single DOF flexures, was used to experimentally validate the ‘dynamic absorber effect’. Three flexures were produced for testing purposes, a single top flexure, a large base flexure, and a small base flexure. Based on available tooling and material, the three flexures were designed for particular natural frequency values [3]; results are shown in Table 1. For the case when it was required to match the natural frequencies of the single DOF modes, additional mass was added to the base flexures to lower the natural frequency to approximately 300 Hz, the cantilevered natural frequency of the top flexure.

![Figure 1: 2DOF flexure.](image)

<table>
<thead>
<tr>
<th>Flexure Design</th>
<th>( m ) (kg)</th>
<th>( k ) (N/m)</th>
<th>( c ) (kg/s)</th>
<th>( W_n ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Base Flexure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexure Theory Prediction</td>
<td>1.54</td>
<td>( 1 \times 10^7 )</td>
<td>-</td>
<td>405.6</td>
</tr>
<tr>
<td>Modal Testing Results</td>
<td>1.43</td>
<td>( 9.41 \times 10^6 )</td>
<td>46.9</td>
<td>408.3</td>
</tr>
<tr>
<td>Modal Testing Results With Mass</td>
<td>2.46</td>
<td>( 8.85 \times 10^6 )</td>
<td>70.6</td>
<td>301.9</td>
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<tr>
<td><strong>Small Base Flexure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexure Theory Prediction</td>
<td>0.146</td>
<td>( 1 \times 10^6 )</td>
<td>-</td>
<td>416.5</td>
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<tr>
<td>Modal Testing Results</td>
<td>0.108</td>
<td>( 6.34 \times 10^5 )</td>
<td>4.18</td>
<td>385.0</td>
</tr>
<tr>
<td>Modal Testing Results With Mass</td>
<td>0.155</td>
<td>( 5.68 \times 10^5 )</td>
<td>2.23</td>
<td>304.7</td>
</tr>
<tr>
<td><strong>Top Flexure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexure Theory Prediction</td>
<td>0.223</td>
<td>( 1 \times 10^6 )</td>
<td>-</td>
<td>337.0</td>
</tr>
<tr>
<td>Modal Testing Results</td>
<td>0.1454</td>
<td>( 5.04 \times 10^5 )</td>
<td>1.17</td>
<td>296.3</td>
</tr>
</tbody>
</table>

After manufacture, the two single DOF base flexures and one single DOF top flexure were modally tested to extract the natural frequency \( W_n \), mass \( m \), stiffness \( k \), and damping \( c \) values. All flexures, including the top flexure, were adhered to ground with cyanoacrylate and instrumented with an accelerometer. The modal mass, damping, and
stiffness values were extracted from the measured FRFs by fitting the data using a peak picking method [4]. The modal testing flexure theory results are compared in Table 1 for the three flexures.

**Flexure Results**

Modal testing was performed on the coupled flexures to verify the ‘dynamic absorber effect’ that takes place when the cantilevered natural frequencies of the base and top flexures are matched. For all testing, ground was defined as a machining tombstone with a large mass and stiffness and all connections were made with cyanoacrylate. Figure 2 displays the real part of the FRF for the large base flexure (connected to ground), the real FRF of the top flexure (connected to ground), and the real FRF of the combined flexures with the large base flexure connected to ground and the top flexure connected to the top of the base flexure. From Figure 2, it can be seen that the natural frequency of the cantilevered base flexure is 408 hertz and the natural frequency of the top flexure cantilevered to ground is 296 hertz. The modes are separated. When the top flexure is connected to the base flexure, the negative real magnitude of the dominant mode of the combined system is 38.5 % less than the negative real magnitude of the top flexure cantilevered to ground, even in the absence of a direct match in the cantilevered SDOF natural frequencies.

For the next set of modal testing, additional weight was added to the base flexure so that the cantilevered natural frequency of the base flexure was lowered to approach the cantilevered natural frequency of the top flexure. Figure 3 displays the real FRF of the modified large base flexure (connected to ground), the real FRF of the top flexure (connected to ground), and the real FRF of the combined flexures with the modified base flexure connected to ground and the top flexure connected to the top of the modified base flexure. From Figure 3, it can be seen that the dominant natural frequency of the cantilevered base flexure is now 302 Hz and the dominant natural frequency of the top flexure cantilevered to ground is again 296 Hz. The modes of the individual cantilevered flexures are close and now interact. *When the top flexure is connected to the base flexure, the negative real value of the dominant mode of the combined system is 69 % less than the negative real value of the top flexure attached directly to ground.* By matching the natural frequencies of the individual cantilevered systems, the dominant mode of the base flexure interacts with the dominant mode of the top flexure. The two modes of the combined system split around the original cantilevered natural frequency and the minimum value of the negative real response of the dominant combined system mode is significantly smaller than the cantilevered response of the top flexure attached directly to ground. The base flexure has acted as a ‘dynamic absorber’ to decrease the response of the top flexure.

Figure 4 compares the real FRFs for the combined systems when 1) the top and bottom flexure cantilevered modes are separated; and 2) the top and bottom flexure dominant modes have similar natural frequencies. As shown, when the modes have similar natural frequencies, the combined system response is significantly reduced.

**Model Development**

For the coupled flexure assembly, Figure 5 shows the lumped parameter model that was used to describe the system. The base flexure, system A, was modeled as a single degree-of-freedom (DOF) substructure, defined as a mass, \( m_1 \),...
connected to ground through a spring, $k_1$, and a viscous damper, $c_1$. System B was modeled as a single DOF substructure with free-free boundary conditions; it consisted of a mass, $m_2$, connected to a massless coordinate, $x_3$, through a spring, $k_2$, and a viscous damper, $c_2$. The response of the assembly, system C, at coordinate $X_2$ (representing the uppermost point on the top flexure) for a harmonic force, $F_2$ (applied at coordinate $X_2$) is computed using receptance coupling substructure analysis (RCSA) [5-7] based upon the receptances of the two single DOF substructures. It is assumed that the substructure receptances for the rotational degrees-of-freedom are negligible (by design for flexures) and that the substructures are rigidly connected.

The direct FRF of the assembled system at the top flexure free end, $G_{22}(w)$, shown in Figure 5 was derived using receptance coupling [8]. The resulting equation, expressed in terms of the system A and B mass, damper, and spring constants, is shown in Eq.2, where the frequency, $w$, has units of radians per second.

$$G_{22}(w) = \frac{X_2}{F_2} = -\frac{1}{w^2 m_2} - \left(\frac{1}{w^2 + iwc_1 + k_1}\right) + \left(\frac{m_2 w^2 - iwc_2 - k_2}{w^2(iwm_2 c_2 + m_2 k_2)}\right)^{-1} \left(\frac{1}{w^2 m_2}\right)$$

(2)

Predicted Responses

To test the model, the parameters for the mass, dampers, and springs produced from modal testing (Table 1) were substituted in Eq. 2 to solve for $G_{22}(w)$. Figure 6 displays measured and predicted results for Re($G_{22}(w)$) for the coupled flexure system with the large base. For this case, the individual flexures had approximately the same natural frequency. Two additional coupled flexure responses were also compared to predicted results to further validate the model. Figure 7 displays results for Re($G_{22}(w)$) for the coupled flexure system with the large base and separated individual flexure modes. Figure 8 displays the results for the coupled flexure system with the small base. In this case, the mass and spring values, as determined by modal testing, of the small base were approximately 16 times smaller than the values for the large base, and the individual flexure modes were matched with approximately the same natural frequency. In all instances, the model results approximate the actual results produced from modally testing the coupled, 2DOF flexure system. Discrepancies between actual and predicted results can be attributed to the decision to neglect rotational degrees of freedom and unmodeled flexibility/damping in the connection between flexures.

Once the model was validated, the model parameters could then be manipulated to predict Re($G_{22}(w)$) coupled flexure results for other cases. Of interest was the $G_{22}(w)$ response when the natural frequencies of the individual base and top flexures remain matched, as shown in Eq. 3, but the ratio between the base and top flexure mass and spring values was increased by a multiplier value $MV$ as shown in Eq. 4, i.e., as $MV$ increases, the base flexure becomes stiffer and more massive relative to the top flexure.
As shown in Figure 9, which displays the real response of the coupled flexure system, when MV is equal to 1 (the base flexure and top flexure are identical), the coupled flexure system has two modes that split around the individual flexure natural frequency of 300 Hz. As MV increases, the two modes of the coupled flexure system approach each other and, when MV is large enough, the combined response approaches a single DOF. Of particular interest is the fact that the minimum value of the negative real part of the dominant mode decreases with MV until the two modes begin to converge to a single DOF and the value of the negative real part begins to increase again. In other words, in terms of the value of the negative real part of the dominant coupled system mode, the optimum point is not found by increasing the mass and stiffness of the base flexure ad infinitum.

Discussion and Conclusions

The results of the flexure testing and modeling can be extended to the case of the coupled spindle/holder and tool in high-speed machining operations. The fundamental tool mode can be considered analogous to the top flexure mode and each holder/spindle mode is analogous to the base flexure mode. As shown in the flexure results, if the fundamental cantilevered tool mode can be matched to a holder/spindle mode, the minimum value of the negative real part of the FRF for the coupled holder/spindle/tool system, as measured at the tool point, can be reduced. The ‘dynamic absorber effect’ that occurs due to the interaction between the modes reduces the tool point negative real minimum value relative to the response that would occur if the tool was cantilevered directly to ground (i.e., an infinitely stiff holder/spindle). From Eq. 1, by taking advantage of the ‘dynamic absorber effect’, the value for \( b_{\text{lim}} \) can therefore be increased. The tool natural frequency can be varied through tool overhang length and/or diameter adjustment, changes in tool material (modulus), or manipulation of the connection between the tool and the holder.

A second conclusion that can be drawn from this testing is that the largest multiplier value, or greatest mass and stiffness of the base flexure, does not lead to a minimized real part of the coupled flexure system at the free end of the top flexure, \( \text{Re}(G_{22}(w)) \). In terms of a holder/spindle/tool assembly, it can be surmised that during the design of spindles for certain tool geometry ranges, if the interaction between tool and holder/spindle modes is taken into consideration, the stiffest, largest mass spindle may not be the optimum selection for stable machining.

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References