

# New Heterodyne Interferometric Method of Displacement Measure and Control with a Sub-nanometric Precision Over a Range of Several Millimeters

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**Abstract:** We present a new type of interferometric position control method with sub-nanometric accuracy. Our method is based on the use of a heterodyne Michelson interferometer and a high frequency clock. With this system, one can control the displacement of target mirror by a quantified step of value  $\lambda/128$ . As the relative uncertainty on the laser wavelength in vacuum is  $1.6 \times 10^{-9}$  (for  $\lambda = 632.991\,528$  nm), each step corresponds to a displacement of 4.495 246 313(8) nm. This system is dedicated to the watt balance project of the national institute of metrology of France, the BNM. The aim of this project is to give a new definition to the mass unit by linking it to an invariant quantity – the Planck's constant.

**Keywords:** position control, laser interferometry, watt balance, metrology

## 1. Introduction

In the international system of unit SI, the kilogram is the only remaining base unit which still relies on a material artifact. This definition is unsatisfactory, especially because not much is known about the stability of the kilogram prototype. A new definition of the unit of mass is needed which is based either directly or indirectly on an invariant atomic process or fundamental constants. Various experiments are pursued worldwide which, in a first step, allow the monitoring of the kilogram and, later on, could serve as a basis for its replacement. At present, one of the most promising approaches is an experiment called the watt balance in which mechanical and electrical power are equated [1]. If the electrical power is measured in terms of the two quantum effect, Josephson effect and quantum Hall effect, the unit of mass can be linked to the Planck constant. A new type of watt balance is currently developed at the Bureau National de Métrologie (BNM, France) with the collaboration of several laboratories.

The comparison between both powers needs two steps. In the first step, the weight of a mass unit  $m$  is counterbalanced by a Laplace force  $F_z$  induced by a current  $i$  passing through a coil of length  $l$ :

$$F_z = Bil = mg \quad (1)$$

To obtain a relative accuracy of  $10^{-9}$  on the mass measurement, it is necessary to have the same level of accuracy for each physical quantity represented on Eq. (1). Now, this is not possible to measure the product  $Bil$  with such accuracy. That's why a second stage is needed. In this second step, the coil is moving with a velocity  $v$  and an electromotive force appears inducing a voltage  $\varepsilon$  between each end of the coil.

$$mgv = \varepsilon i \quad (2)$$

The current  $i$  can be measured by comparison to a voltage standard based on Josephson effect and to a resistance standard based on quantum Hall effect.

$$mgv = \frac{A}{K_J^2 R_K} \quad A = \frac{f_1 f_2 i}{k} \quad (3)$$

where  $K_J$  and  $R_K$  are respectively the Josephson and the von Klitzing constants. Eq. (3) leads to a relation between the mass unit  $m$  and the Planck constant  $h$ :

$$m = h \frac{A}{4gv} \quad (4)$$

Hence, to measure  $m$  we need to measure two frequencies  $f_1$  and  $f_2$ , the acceleration of earth gravity  $g$  and the speed  $v$  of the moving coil. This speed is planned to be equal to 2 mm/s with an accuracy of 2 pm/s. In this paper, we present the method used to achieve such accuracy on the speed measurement. It combines the use of a heterodyne Michelson interferometer and a high frequency electronic circuit that allows us to lock in the optical phase at the output of the interferometer (*i.e.* the position of a mobile mirror) on the electronic phase of the high frequency circuit.

## 2. Description of the Phase-Locked Interferometric Method

Neglecting refractive index of air fluctuations, a displacement of a movable mirror given by a one-pass Michelson's interferometer by an amount of  $\lambda/2$  corresponds to a phase shift equal to  $2\pi$ . The method we proposed here consists in reversing this property of Michelson's interferometers. Consider the sketch on Fig. 1. An electronic board generates two synchronized signals  $s_1$  and  $s_2$  at the same frequency  $f$ . Electronic circuit allows us to make phase-jumps of quantified value on either signals. Signals  $s_1$  and  $s_2$  are respectively sent to a mixer and to a laser head which transpose the signal from ultrasonic range to optical frequency range thanks to a Bragg cell. This allows to perform the two optical components of the heterodyne interferometer. The optical beam passes through the interferometer. The two components are recombined at the output of the interferometer, resulting on a signal  $s_3$  at a same frequency that  $s_1$  and  $s_2$ . Signal  $s_3$  contains position information. Then  $s_3$  is phase-compared with  $s_2$  and a signal error is sent to a lock in electronic that pilot an actuator supporting the movable mirror.

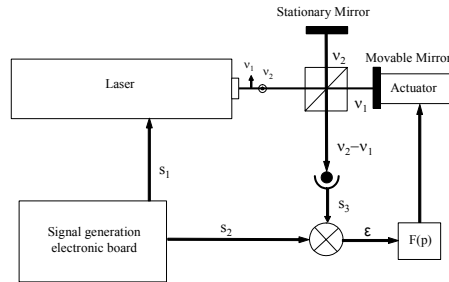


Fig. 1: Principle

## 3. Experimental Setup

Experimental setup is detailed on Fig. 2. The laser head allows us synchronizing the reference frequency  $f$  to an external 20 MHz signal through a fibered input. The internal 20 MHz signal is hence replaced by a 20 MHz frequency signal coming from our electronic board. Phase jump of  $2\pi/32$  is induced by a home-made specific electronic circuit. This corresponds to a step of  $\lambda/128$ . The uncertainty on the step position value is directly related to the uncertainty onto the laser wavelength. The wavelength in air of the laser is given by  $\lambda = c/nf$  where  $f$  is calibrated by beat frequency technique with regard to a national reference. The wavelength of the laser has been calibrated by Zygo and the nominal value is equal to  $\lambda = 632.991\,528(1)$  nm [2]. Hence we expect a step value of 4.945 246 313(8) nm. This value is the limit on accuracy that can be achieved with our method. One part of the beam issue from the interferometer is used to lock-in a piezoelectric actuator supporting the

movable mirror while other part is sent into electronic board of the commercial system to measure the displacement. Simultaneously, a weather station measures the room temperature ( $\sigma_T = 5 \text{ mK}$ ), pressure ( $\sigma_P = 3 \text{ Pa}$ ), humidity content ( $\sigma_H = 1\%$ ) and  $\text{CO}_2$  content ( $\sigma_{\text{CO}_2} = 10 \text{ ppm}$ ). These values allow us to measure the fluctuations of the refractive index of air using Edlén equations [3].

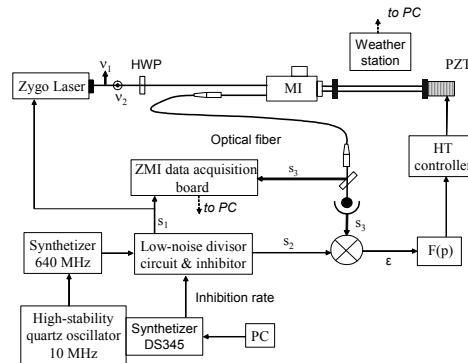


Fig. 2: Experimental Setup

#### 4. Experimental Results

Step by step displacement of the target mirror is depicted on Fig. 3. We can see steps corresponding to each phase jumps at the mixer. Noise level on each step is equal to  $0.22 \text{ nm}$  (at  $1\sigma$ ). Fluctuations of the room temperature, pressure, humidity content and  $\text{CO}_2$  content are respectively,  $0.1^\circ\text{C}$ ,  $100 \text{ Pa}$ ,  $<1\%$  and  $60 \text{ ppm}$ . Using Edlén equations, refractive index of air variation is  $17 \times 10^{-8}$ . This corresponds to a negligible correction factor. Uncertainty on the speed of the movable mirror is inversely proportional to the number of steps. The displacement range of the PZT actuator used in our experiment is limited to  $3 \mu\text{m}$  for an applied voltage of  $100 \text{ V}$ . Note that our method do not have displacement range limit. This displacement is represented on Fig. 4. The slope of this curve is equal to the speed of the movable mirror. This speed  $\theta$  is imposed by the period of repetition of the phase jumps equal to  $20 \text{ Hz}$ . As each step is equal to  $4.945 \text{ nm}$ ,  $\theta$  is equal to  $98.904(1) \text{ nm/s}$ . Uncertainty on  $\theta$  is calculated with a step number  $\eta$  equal to  $565$  and a noise level on each step  $\sigma_p$  equal to  $0.22 \text{ nm}$  at  $1\sigma$ . Relative uncertainty on the speed already reaches  $1 \times 10^{-3}$ . To reach a relative uncertainty of  $10^{-9}$ , we need to make  $3 \times 10^5$  steps that is to say  $1.5 \text{ mm}$  of displacement. Displacement range in watt balance project is planned to be  $60 \text{ mm}$ , so such uncertainty could be achieved with our method.

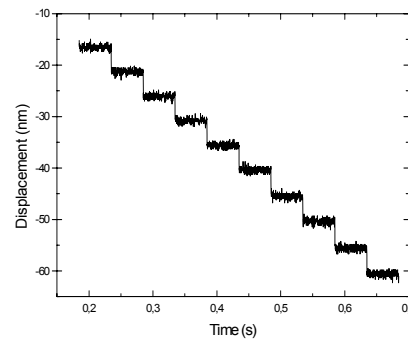


Fig. 3: Displacement measurement

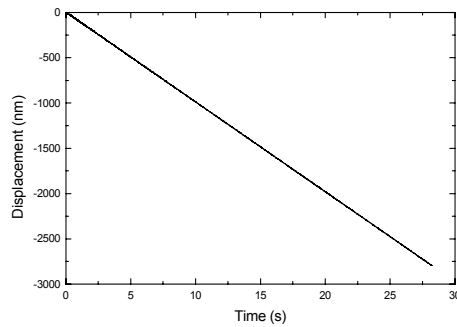


Fig. 4: Speed measurement

## 5. Conclusion

We have proposed a novel simple method to control step by step the displacement of a translation stage. Our method is based on the use of a heterodyne laser interferometer and a high frequency phase-jump electronic circuit. It allows to control the displacement of a target mirror with nanometric steps and sub-nanometric accuracy. This system of nanodisplacement is dedicated to the watt balance project of the Bureau National de Metrologie (France).

## Acknowledgements

Due to its multidisciplinary, this project brings together many researchers of different laboratories listed below: G. Geneves, P. Gournay (BNM-LNE/LAMA) – F. Alves (Satie, Ecole Normale Supérieure de Cachan) – M. Besbes (LGEP, Ecole Supérieure d'Electricité) – A. Clairon, D. Holleville, A. Landragin, F. Pereira Dos Santos (BNM-Syrte, Observatoire de Paris) – J. David (ENSAM) – A. Gosset (BNM-LNE) – M. Lecollinet, P. Pinot, F. Villar (BNM-INM/CNAM)

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