# Tip Waviness Compensation in a Polar Profilometer

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In 2001, the North Carolina State University Precision Engineering Center unveiled the polar profilometer Polaris [1]. Polaris was based on a system of two stacked axes with a linear axis residing atop a rotary axis. Using this system, an LVDT probe could be positioned to allow measurements of high aspect ratio aspheric surfaces not possible with other instruments. Initial specifications were for an overall measurement accuracy of 500 nm over a 50 mm circular measurement field. To achieve this level of performance, geometric machine errors such as probe tip misalignment, linear axis pitch and probe radius had to be compensated. As *Polaris* has given way to its commercial sibling, Ultraform 2D, shown in Figure 1, accuracy requirements have risen and perpetuated the need for more error compensation. Foremost, and most challenging of these new errors to address is probe tip waviness compensation.

The three limiting criteria presented for probe tip waviness on Ultraform 2D are: a commercially available Grade 5 (±125nm) probe tip must be used, waviness errors must be measured expediently on the instrument, and compensation must be performed automatically. Thus, achieving overall accuracy better than 100 nm requires more than simply choosing a more accurate and expensive probe tip.

The process of waviness compensation requires that, first, the waviness of the probe tip be evaluated and, second, the result be applied to every subsequent measurement. The unique geometry associated with a polar measuring system allows a certain degree of self calibration. This feature of the profilometer was exploited in *Polaris* to align the instrument's measuring probe with the rotary axis, thus establishing a true polar coordinate system. Two orthogonal alignment errors were identified: radial offset ($\rho_0$), and tangential offset ($\tau_0$). Radial offset has a straightforward effect on a measurement in that it merely needs to be added to the measured radial

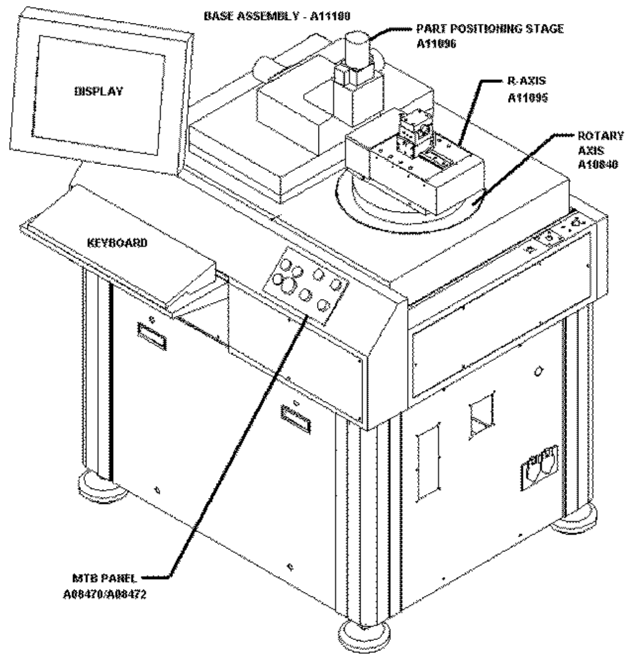


**Figure 1.** The Ultraform 2D polar profilometer.

position $R$ in order to obtain the corrected position $R'$.

$$R` = R + \rho_0 . \quad (1)$$

The impact on a measurement of the tangential offset is a bit more involved, since its influence is expressed as a function of both $R$ and $\theta$. Hence, the corrected radial position $R``$ is

$$R`` = (R`^2 + \tau_0^2)^{1/2} . \quad (2)$$

While the corrected angular position, $\theta`$, is

$$\theta = \theta` + \tan^{-1}\left(\frac{\tau_0}{R`}\right). \quad (3)$$

Waviness ($w$), on the other hand, is a function of the probe contact angle ($\gamma$), which is itself a function of $R$ and $\theta$.

As shown in Figure 2, radial offset error ($\rho_0$), tangential offset error ($\tau_0$) and waviness error ($w(\gamma)$) are all present in any measurement.

**Waviness Evaluation**

Evaluating the waviness of a given probe tip is inexorably linked to evaluation of probe misalignment. Probe misalignment is, in fact, just a special case of waviness with a high degree of symmetry. Unfortunately, this does not alleviate the need to find the relative position of the probe tip with respect to the rotary stage axis of rotation. The method for evaluating probe tip alignment that has been examined uses a flat as a reference standard.

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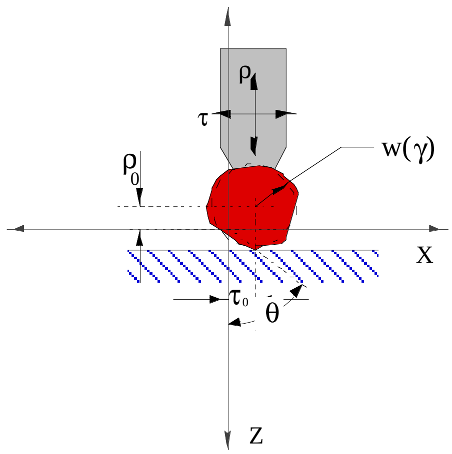


**Figure 2.** Probe error is a combination of radial offset error ($\rho_0$), tangential offset error ($\tau_0$) and waviness error ($w(\gamma)$).

For a probe tip with a radius of r, the probe position will be –r for all positions of $\theta$ if the center of the tip is positioned with its center at $\rho = 0$. One means of achieving this is by placing a flat plate ($\lambda/20$) at –r as shown in Figure 3. If the probe is truly centered, the probe output will give only the probe waviness as the rotary axis is traversed through the angular extent of the probe tip. If, however, there is an offset in either the $\rho$ or $\tau$- directions, the probe output will change in a characteristic way for each direction that is a linear combination of all these errors.

As shown in figure 4,or an offset of $\rho_0$ in the $\rho$-direction, the output of the LVDT will change as a function of the rotary axis position $\theta$:

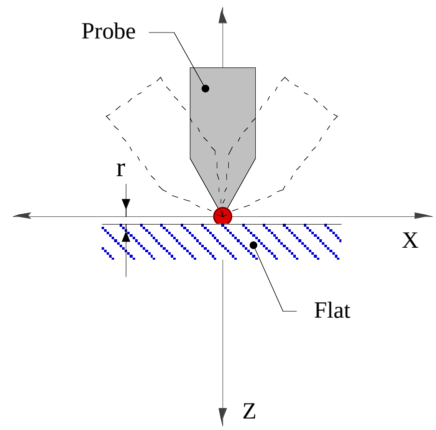


**Figure 3.** When the probe tip's center is aligned with the machine origin, rotation of the $\theta$-axis will yield only waviness data.

$$\rho = \rho_0\left(\frac{1}{\cos\theta} - 1\right). \qquad (4)$$

When an origin measurement is made and the $\rho$-offset is measured, the *R*-axis zero can be reset to eliminate the offset. In addition to $\rho$-offsets, there can be an offset in the $\tau$-direction. This offset produces a different response when measured using the flat method. The response of the LVDT to a $\tau$-offset is a function of $\theta$ as shown in Figure 5 can be written:

$$\rho = \tau_0(\tan\theta). \qquad (5)$$

Again, this characteristic output can be used to detect and correct the offset – this time in the $\tau$-direction.

In the actual alignment procedure, the

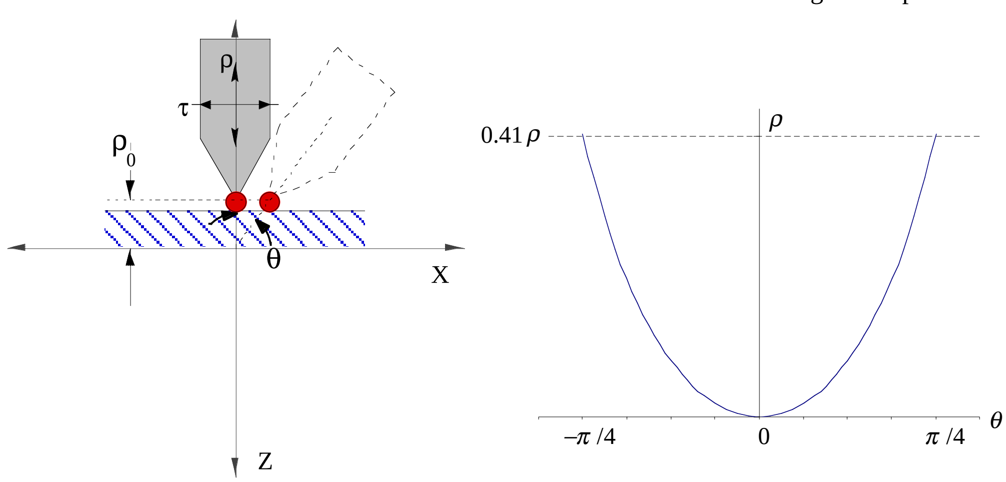


**Figure 4**. Schematic and plot of LVDT deflection with an offset of $\rho_0$ in the $\rho$-direction. The response as a function of $\theta$ can be used to determine the offset and adjust the machine.

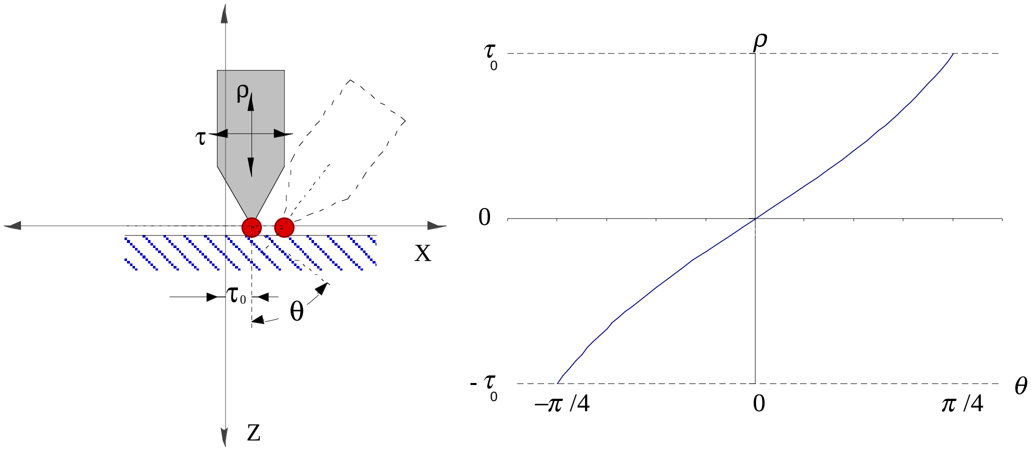


**Figure 5**. Schematic and plot of LVDT deflection with an offset of $\tau_0$ in the $\tau$-direction.

situation is slightly more complex. A combination of offsets will produce an output of the probe according to the sum of Equations (4) and (5),

$$\rho = \rho_0\left(\frac{1}{\cos\theta} - 1\right) + \tau_0(\tan\theta). \qquad (6)$$

Armed with this knowledge, one can now collect data and perform a curve fit of Equation (6) to the collected data and simultaneously solve for $\rho_0$ and $\tau_0$. The residual error is the waviness of the probe since, for a flat, $\theta = \gamma$.

The disadvantage of this method is that, for large amounts of waviness, the determination of $\rho_0$ and $\tau_0$ via a curve fit can be affected, increasing uncertainty in the location of the rotary axis center. Fortunately, by limiting magnitude of the waviness to ±125 nm, the influence on the fit is not appreciable.

**Error Correction**

Once the tip waviness and probe alignment errors are known, the data from subsequent measurements can be corrected in software. None of the errors are of sufficient magnitude to require real-time alteration of probe position during a measurement. Gage misalignment and nonlinearity are easily corrected by applying their compensation formulas sequentially.

Once the misalignment errors have been compensated using Equations 1-3, the procedure for probe radius and waviness correction is similar to that of modifying a tool path prior to a machining process. An important difference for a metrology instrument is that for each point in the

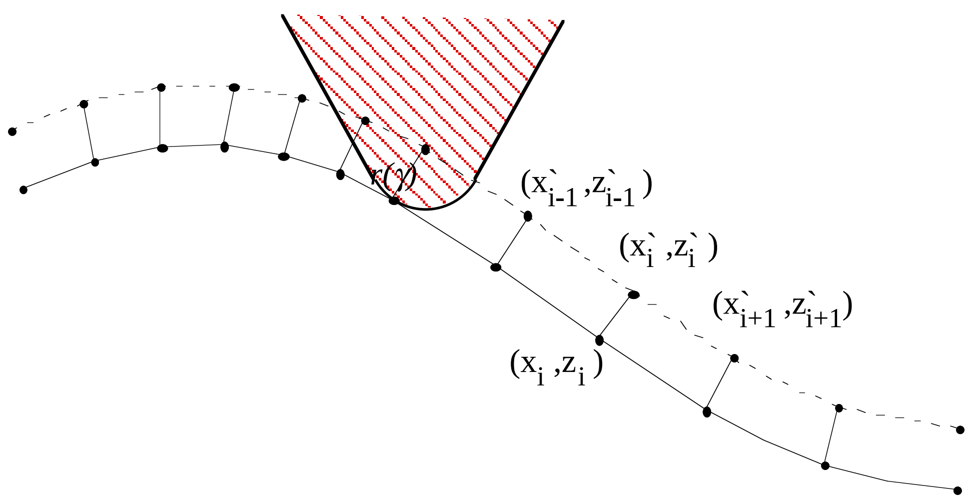


**Figure 6.** As the probe tip traverses the surface, the contact angle changes. This contact angle can be determined from the slopes of the interpolated line segments between data points.

data file the direction cosines of the correction vector must be estimated from the data itself.

In its most basic form, the waviness compensation is numerical and requires a minimum of three data points. The first step is the calculation of the slope of the surface at a data point ($x_i$`, $z_i$`). Since the slope of a point cannot be calculated, two neighboring points assumed to be on the surface are used. The slope *m* between any two points is given by

$$m_i = \frac{x`_i - x`_{i+1}}{z`_i - z`_{i+1}}. \quad (7)$$

To desensitize the data somewhat to local fluctuations, the slopes of the neighboring points are averaged

$$\overline{m_i} = \frac{m_i + m_{i-1}}{2} = \frac{1}{2}\left(\frac{x`_i - x`_{i+1}}{z`_i - z`_{i+1}} + \frac{x`_{i-1} - x`_i}{z`_{i-1} - z`_i}\right) \quad (8)$$

The inverse of this point slope is the normal slope, which, when inserted into the point-slope form of the normal line

$$z_i - z`_i = \overline{m_i}\left(x_i - x`_i\right) \quad (9)$$

can be used to solve for each point at a distance of the probe radius *r* along that line.

Considering waviness as a polar variation in probe radius, a combined correction for probe radius and waviness is found using Equations (10) and (11):

$$x_i = x_i` + r(\gamma)\cdot \overline{m} \quad \sqrt{\overline{m}^2 + 1} \quad (10)$$

Solving for $z_i$,

$$z_i = z_i` + r(\gamma)\cdot -1 \quad \sqrt{\overline{m}^2 + 1} \quad (11)$$

The function *r(γ)* is the radius as a function of contact angle or the sum of the nominal probe radius *r* and the waviness *w(γ).*

$$r(\gamma) = r + w(\gamma). \quad (12)$$

The contact angle $\gamma$ is a function of the local slope *m* and the probe angular position $\theta$:

$$\gamma = \theta - \tan^{-1}\left(-\frac{1}{m}\right). \quad (13)$$

The special cases of zero or infinite slope are easily detected by adding *r(γ)* to $x_i$` and $z_i$` respectively.

For smooth surfaces with slowly varying slopes this process is relatively straightforward and effective. However for high aspect surfaces, care must be taken to design a filtering process that removes the effects of noise without obscuring the presence and location of important surface features. The effect of probe waviness on measured data is a convolution of the probe and the actual surface. Although implemented iteratively as a filtering process, the extraction of corrected surface data from measured data is equivalent to spatial deconvolution. As such, the techniques developed for scanned probe microscopy are relevant to our implementation.

## Conclusion

Dealing with probe errors on a polar profilometer can be a challenging endeavor. The unique geometry of the system can, though, become an advantage in the process of evaluating these errors. Once found, misalignment errors and waviness can be applied post-process to correct raw measurement data.

## References


1. The Polar Profilometer *Polaris*, Proceedings of the ASPE 2001 Annual Meeting, **v.25**, pp. 28-31(2001)
2. The Polar Profilometer *Polaris,* 2001 PEC Annual Report, **v. 19**, pp.1-15 (2002)