

Time Series Modelling of Endmilling Process for Mode Estimation

M. C. Yoon¹, B. T. Kim¹, and D. S. Park²

1. School of Mechanical Engineering, Pukyong National University, Busan, South Korea; email: mcyoona@pknu.ac.kr, btkim@pknu.ac.kr
2. Department of Mechanical Engineering, Incheon University, Incheon, South Korea; email: dspark@lion.incheon.ac.kr

ABSTRACT

To get the natural mode of structural dynamics considering cutting process dynamics it is necessary to model the cutting system. Several time series modelling such as AR (burg, least square, yule walker, geometric lattice, instrument variable) algorithm, ARX (arx, iv4), ARMAX, ARMA, Box Jenkins, Output Error, recursive arx and recursive armax were analysed and compared with one another. As a result, arx, armax and iv4 were proved very reliable algorithms for the calculation of the modal parameters(natural frequency and damping ratio) in endmilling operation.

INSTRUCTION

In this study, we have modelled several time series algorithms with spectral analysis comparing with conventional FFT (fast fourier transform) method and calculated the natural mode with each algorithm in endmilling operation. The natural mode estimations with several time series algorithms may be practiced for latter study of chatter. Also these results may be compared with FFT spectrum and the advantage and limitation of each algorithm were discussed. For this purpose two experiments were practiced to get the experimental data and these data were used for the modelling of each algorithm and the results were compared and discussed together. In the first experiment, we have practiced impact test for the FRF of the time series modelling which needs two sensor signals in output/input FRF as accelerometer/impact hammer signals. Also, similar experiment for the output/input data of FRF was practiced using the accelerometer and the tool dynamometer sensor for the accelerometer/tool dynamometer FRF respectively.

ARMA (n,m) Model

The relationship between input and output in endmilling operation may be defined as a ARMA (Auto Regressive Moving Average) modelling as follows.

$$\begin{aligned} (1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}) y(t) \\ = (1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m}) e(t) \end{aligned} \quad (1)$$

where a_i ($i=1, 2, 3, \dots, n$) are auto regressive parameters and b_i ($i=1, 2, 3, \dots, m$) are moving average parameters. Also $e(t)$ is white noise

AR (n) Model

The AR model can be modelled by using the autoregressive parameters and it can be summarized as follows.

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}) y(t) = e(t) \quad (2)$$

where the detail several algorithm such as burg, least square, yule walker, geometric lattice and instrument variable ar(ivar) algorithms may be modelled like Eq.(2) in such a AR model⁽¹⁾⁻⁽⁴⁾

ARMAX (n,m,l,nk) Model

The ARMAX model is defined as follows and the A(z) in the equation is called autoregressive parameters and the B(z) and C(z) are moving average parameters.

$$\begin{aligned} (1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}) y(t) \\ = (b_1 + b_2 z^{-1} + b_3 z^{-2} + \dots + b_m z^{-m}) u(t - nk) \\ + (1 + c_1 z^{-1} + c_2 z^{-2} + \dots + c_l z^{-l}) e(t) \end{aligned} \quad (3)$$

ARX (n,m,nk) Model

The ARMAX model is modelled such as the moving average parameter part C(z) of ARMAX becomes unit by assuming that the past noise has no correlation to current one. So, the following model may be summarized between input and output.

$$\begin{aligned} (1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}) y(t) \\ = (b_1 + b_2 z^{-1} + b_3 z^{-2} + \dots + b_m z^{-m}) u(t - nk) + e(t) \end{aligned} \quad (4)$$

Box Jenkins (n,m,l,k,nk) Model

The algorithm may be modelled by assuming that there is different correlation between input and noise input with output. Such model may be represented and also may result in the Box Jenkins model as follows:

$$y(t) = \frac{(b_1 + b_2 z^{-1} + \dots + b_n z^{-n})}{(1 + f_1 z^{-1} + \dots + f_l z^{-l})} u(t - nk) + \frac{(1 + c_1 z^{-1} + \dots + c_m z^{-m})}{(1 + d_1 z^{-1} + \dots + d_k z^{-k})} e(t) \quad (5)$$

Output Error (n,l,nk) Model

By setting the part $C(z)/D(z)$ to be unit in Box Jenkins model by assuming that there is no correlation between past noise and current one, the output error model of time series may be defined as follows.

$$y(t) = \frac{(b_1 + b_2 z^{-1} + \dots + b_n z^{-n})}{(1 + f_1 z^{-1} + \dots + f_l z^{-l})} u(t - nk) + e(t) \quad (6)$$

From the time series models defined above the natural frequency and damping ratio from the denominator of transfer function in each model may be calculated as following.

$$\varpi_i = \text{abs}(\ln(\lambda_i)) / T_s \quad (7)$$

$$\xi_i = -\cos(\text{angle}(\ln(\lambda_i))) \quad (8)$$

Application to real data

In the first experiment an impulse hammer test was conducted under the condition of no rotation of spindle and the accelerometer signal was measured simultaneously. In the second, An experiment was also practiced to acquire the signal of cutting force and accelerometer in the process of cutting during the endmilling operation.

Frequency Response Function - Impact Hammer Test

Fig.1 (a) and (b) shows the impact hammer signal and accelerometer signal respectively. The right side was magnified for easier analysis.

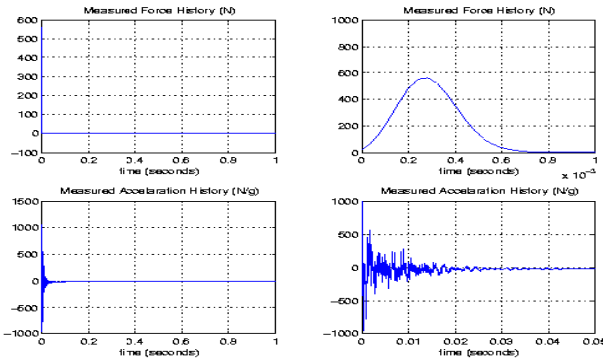
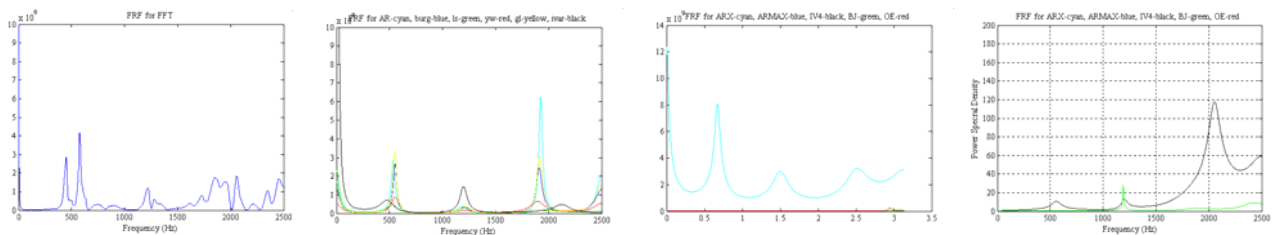


Fig.1 (a) Impulse signal (up - two figure), (b) Accelerometer signal (down) for the impact hammer test

The natural frequency and damping ratio were calculated by using Eq.(7)-(8) and they were summarized in Table 1. As a result, the four theoretical natural modes were calculated by Eq.(7)-(8) and the modes are consistent well with its spectrum modes. But the real mode can't be discriminated and just only shows the mode shifting. So it causes a wrong natural mode. If the sampling frequency is selected as 5000 Hz, it shows a natural mode that is consistent with its real mode a little bit well.

The transfer function $H(z) = A_c(z)/F_{imp}(z)$ of the time series models shows that the mode moves to its real one closer. Fig.2(a) ~ (f) are the results of the power spectrum of each algorithm with real modes about 470 and 600 Hz and they show the smoothing of the modes which are very close to one another. The result of Table 1 is the natural mode calculation with each algorithm. In the table, it was concluded that the calculation modes are consistent well with their real ones and the two nearby modes become into one because of the smoothing characteristics in different parameter selection.



a) FRF for FFT

(b) FRF for AR

(c) FRF for other algorithms in normalized frequency

(d) FRF for other algorithms ((c) is magnified)

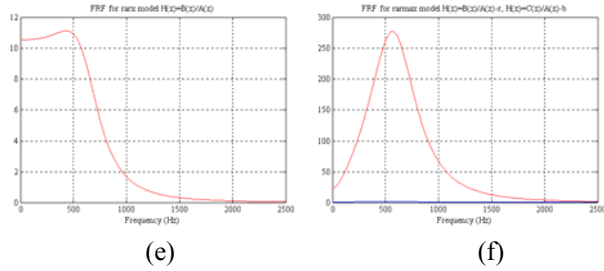


Fig.2 Power Spectrum for each method (a-f) with sampling frequency 5000Hz

The spectrum modes of each line show one of the algorithm and its mode also is consistent well with the mode in Table 1. Because of the different roots in the autoregressive parameters, which consist of the characteristic equation of the model, each algorithm show a little bit different value of its mode. But the shape also gives a good consistency with the real mode and it gives a little bit of mode shifting. Also, by selecting higher sampling frequency, the real frequency for the range 0~1200 Hz may be difficult to identify because of its poor resolution of the lower real range. So it is very important to select the suitable sampling frequency f_q . it was proved that in time series modelling it is enough to sample the data by selecting the sampling frequency f_q below 5000 Hz.

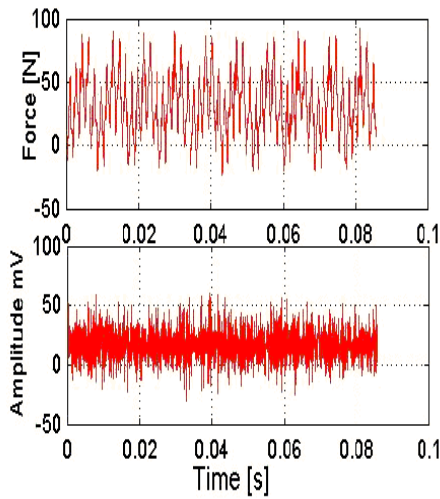
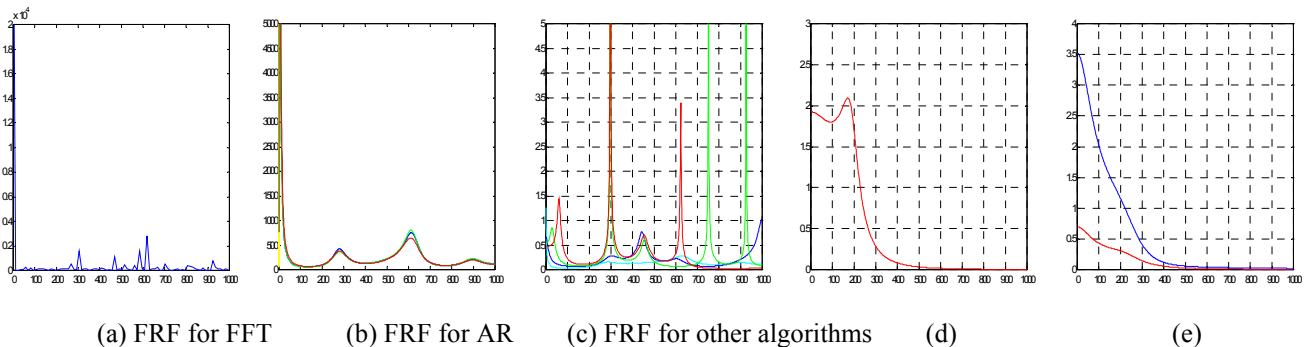


Fig.3 Experimental force and accelerometer signal

Table 1 Natural mode of structural dynamics

mode	1st	2nd	3rd	4th
ar(α_i) (ξ_i)	5.3617391e+002 3.4378699e-002	1.1971344e+003 5.5848027e-002	1.9248368e+003 1.6478783e-002	
burg	5.5286873e+002 3.3994177e-002	1.2140814e+003 6.2746282e-002	1.9088100e+003 2.7552828e-002	
ls	5.2972974e+002 9.0083952e-002	1.1963021e+003 6.2663993e-002	1.9473690e+003 1.1828698e-001	
yw	5.5495699e+002 6.0593751e-002	1.2092989e+003 1.2953545e-001	1.8977964e+003 6.8377535e-002	1.7736031e+003 4.5014260e-001
gl	5.5070053e+002 3.0100684e-002	1.2148282e+003 5.6566984e-002	1.9121992e+003 2.4355830e-002	1.7736031e+003 4.5014260e-001
ivar	4.9147926e+002 1.2164688e-001	1.1981895e+003 4.2452551e-002	2.1355418e+003 7.2070567e-002	1.8078015e+003 2.4311292e-001
arx	5.3575944e+002 4.4241368e-002	1.1973181e+003 7.9502739e-002	1.9927695e+003 9.0351934e-002	1.8245005e+003 5.8455895e-001
armax	1.9792561e+003 1.1544617e-001	5.1048995e+002 5.7283195e-002	1.2435657e+003 3.8357154e-002	1.0247127e+003 3.4731888e-001
iv4	5.6075439e+002 6.4704804e-002	1.1971848e+003 2.5695726e-002	2.0528913e+003 4.5152587e-002	1.8195641e+003 1.7867282e-001
bj(c/d)	5.4610182e+002 6.1545264e-002	1.2373347e+003 5.9670742e-002	2.0276169e+003 8.1668273e-002	1.0040063e+003 6.2406816e-001
bj(b/f)	5.5815957e+002 1.6086081e-001	1.1900335e+003 3.7050259e-003	1.8707502e+003 8.0266477e-002	2.3948874e+003 6.4016843e-002
oe	4.8063664e+002 1.4433280e-001	1.2004411e+003 3.1343098e-002	1.8833027e+003 7.6478752e-002	2.3464432e+003 8.3467112e-003
rax	8.3860471e+002 7.5941489e-002	1.5572194e+003 1.5021026e-001	1.8833027e+003 7.6478752e-002	2.3464432e+003 8.3467112e-003
rarmax	8.3482534e+002 1.5028969e-001	1.5773515e+003 1.6761302e-001	1.8833027e+003 7.6478752e-002	2.3464432e+003 8.3467112e-003

Frequency Response Function - Accelerometer/Tool Dynamometer Test



(a) FRF for FFT (b) FRF for AR (c) FRF for other algorithms (d) FRF for rarx ($H(z)=B(z)/A(z)$ -up line, $H(z)=1/A(z)$ -down line) (e) FRF for rarmax ($H(z)=B(z)/A(z)$ -down, $H(z)=C(z)/A(z)$ -up line)

Fig.4 Power Spectrum for each methods for sampling frequency 2000Hz with Accelerometer / force test.

Fig.3 shows the experimental results that were conducted in the full immersion slot endmilling with the diameter of 15 mm and four flutes endmill, 1500rpm, depth of cut in axial direction of 5mm and feedrate of 750 mm/min

respectively. In any different condition, the two natural frequencies around 470 Hz or 600 Hz may also be got with FFT. Fig.4 shows FRF for the sampling frequency of 4000 Hz. It gives us a good FRF than the case of the high sampling frequency (25000Hz) before. Also, the similar results may be got even if in same condition of impact test with less natural mode deviation. Also the natural mode (frequency, damping ratio) was calculated similarly by using the Eq.(7)-(8) in Table2. But because of the lower order of the model it is very difficult to discriminate the two close modes exactly in the real mode range. It results in the smoothing effect of the weak mode and they are merged into one natural mode. If the sampling frequency is decreased the discrimination of two close modes may be easy with the time series model in this range. The mode discrimination of two close modes was easier for the case of sampling 2000Hz than 4000 Hz by the lowering sampling frequency. This phenomenon may be seen well in Fig.4(c) with the ARX, ARMAX and Box Jenkins algorithm of two real modes about 470Hz and 600 Hz. So it is very important to select the sampling frequency and the time series algorithms appropriately. But using the conventional FFT algorithm it is very difficult to discriminate the close natural modes exactly because of the intrinsic aliasing effect. In general, the ARX and ARMAX are more desirable methods for the time series modelling in endmilling operation than the other algorithms. Also the natural modes were calculated exactly by using Eq.(7)-(8) and summarized in Table 2 for comparing with mode of the spectrum. It shows the exact consistency between two modes, the one by spectrum and the other by theoretical calculation. By lowering the sampling frequency the more precise analysis in the lower frequency range may be acquired in high resolution. Also, the similar mode may be matched for the characteristics of tool passing frequency, tool breakage and chatter, etc. Table 2 is the result of the natural mode that was acquired by setting the sampling frequency as 2000 Hz. In this case the frequency that can be analysed exist in the range of 0~1000 Hz.

Table 2 Natural mode of Cutting dynamics (sampling frequency = 2000Hz)

mode	1st	2nd	3rd	4th	5th
ar (ω_i)	2.8046498e+002	8.9801986e+002	6.1909418e+002	5.0351297e+002	
(ξ_i)	1.3798574e-001	1.1272540e-001	9.5336752e-002	2.7934171e-001	
burg	2.8072272e+002	8.9599909e+002	6.1951553e+002	5.0703165e+002	
	1.3387338e-001	1.1338032e-001	9.7001972e-002	2.8486687e-001	
ls	2.8073510e+002	8.9657807e+002	6.1814192e+002	5.0567556e+002	
	1.4441778e-001	1.0832206e-001	9.3836287e-002	2.7969961e-001	
yw	2.8299337e+002	8.9568149e+002	6.1970958e+002	5.2411474e+002	
	1.4748409e-001	1.1493127e-001	1.1056798e-001	2.8942964e-001	
gl	2.8073267e+002	8.9600304e+002	6.1951826e+002	5.0703459e+002	
	1.3384402e-001	1.1337181e-001	9.6985257e-002	2.8486672e-001	
ivar	8.3207628e+002	6.1219762e+002	2.8316786e+002	3.5512790e+002	
	3.0407962e-002	1.9393469e-002	2.5940489e-002	8.1214887e-002	
arx	8.9304017e+002	6.3212242e+002	2.7763453e+002	4.6061170e+002	
	8.3619450e-002	7.0921369e-002	1.6346990e-001	2.5758121e-001	
armax	2.9292011e+002	4.3275066e+002	6.1781229e+002	9.5479606e+002	
	1.1311945e-001	4.8553468e-002	7.5765545e-002	1.3941328e-001	
iv4	5.8082322e+002	9.0637634e+002	2.5596880e+002	4.5221593e+002	
	2.6150358e-001	1.6225629e-002	1.7410530e-003	5.0685203e-002	
bj(c/d)	2.8314758e+002	6.2777579e+002	7.9070803e+002	5.4764239e+002	
	1.1605418e-001	5.8963395e-002	9.9435217e-002	3.6389007e-001	
bj(b/f)	8.8260030e+002	7.0624556e+002	4.3062803e+002	1.2135657e+002	2.8965601e+002
	9.8813366e-003	5.3386788e-003	3.7443144e-002	9.9824969e-003	2.5726273e-001
oe	5.6371790e+001	4.2057702e+002	9.1880443e+002	6.6697031e+002	7.2775140e+002
	8.8624006e-002	9.1785345e-003	7.1884229e-003	7.7445028e-003	2.2196170e-001
rax	6.5316988e+002	3.5047675e+002	9.1880443e+002	6.6697031e+002	7.2775140e+002
	1.0976963e-001	2.1283801e-001	7.1884229e-003	7.7445028e-003	2.2196170e-001
rarmax	4.7144256e+002	7.0537816e+002	9.1880443e+002	6.6697031e+002	7.2775140e+002
	2.4839496e-001	1.3504902e-001	7.1884229e-003	7.7445028e-003	2.2196170e-001

Results

Among several algorithms, ARX and ARMAX models were proved to be suitable for the natural mode analysis in endmilling and these may be used in the more reliability for getting the natural mode with less error. These algorithms give us good calculated results of real natural mode ranging from 470 Hz to 600 Hz with less bias but these several time series methods also give us good natural modes in lower sampling frequency about 2000 Hz especially in ARX and ARMAX model. But it is impossible to get the natural mode in low sampling frequency with FFT method. In natural mode estimation the FRF $A_c(z)/F_{imp}(z)$ of impact hammer test and $A_c(z)/F_{dyn}(z)$ of dynamometer test give us the same natural mode. So the latter method can be used easily than former one with lower sampling frequency for natural mode estimation and other malfunction mode such as chatter and tool passing frequency.

5. References

- [1] L. Ljung, System Identification - Theory for the User, Prentice Hall (1999)
- [2] L. Ljung, T. Glad, Modeling of Dynamic Systems, Prentice Hall (1999)
- [3] M. C. Yoon et al[2], "A Study on Diagnostics of Machining System with ARMA Modeling and Spectrum Analysis, Journal of KSMTE, Vol.8 (1999), pp.42-51
- [4] M. C. Yoon et al[4], "Natural Mode Analysis for Chatter Lobe Estimation", Journal of KSMPE, June (2003)
- [5] Matlab Manual, Mathwork inc.(1997).