

Theoretical and Experimental Determination of the Stiffness Properties of a Capstan Drive

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Wire capstan drives are used as rotary transmission elements for their very low (nominally zero) backlash and high stiffness properties. To obtain high stiffness, the cable is typically wrapped around the input and output drum in a figure-eight pattern multiple times. This stiffness can be determined by analyzing the amount of deformation between the cable and the drums and the friction caused by this contact. In product literature and various textbooks [1][2][3], it was noticed that different methods of determining the stiffness of the transmission drive were presented depending how friction was considered. It is found the model which considers the extension from the external torque and preload force matches experimental data along with the other model that defined the area of traction to act in a certain section of the drum. As the coefficient of friction between the cable and transmission drum increases, the two models differ giving the method that defined the area of slip along a certain section of the drum, the conservative choice. This paper presents a comparison of two different stiffness models, along with data from experiments to study the model differences.

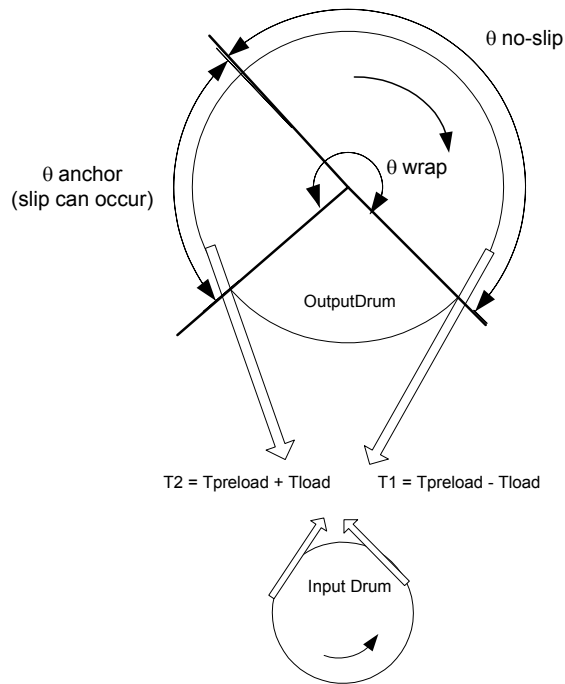


Figure 1: Definition of “angle-of-anchor” and “angle-of-wrap” for the output drum rotating counterclockwise.

The main difference discovered between the two stiffness models is due to how the traction region of the cable is defined. Traction can be defined as acting over the angle-of-anchor which is the section of the cable where slippage can occur while the cable is loaded. This angle is the angle one would derive from the classic capstan equation as being required to hold the varying load, given a cable preload. Alternatively, traction can be defined as acting over the angle-of-wrap which considers both sections of the cable where slip and no-slip occur. These two angles and the area of slip and no-slip are shown in Figure 1. If traction is defined by the angle-of-anchor, the section of slippage is due to the external torque applied. For traction defined by the angle-of-wrap, extension due to the preload force through the cable contacting the drum must also be considered in addition to the external torque that the angle-of-anchor considers. Therefore, half of the physical behavior of the cable is considered in the angle-of-anchor where the complete behavior of the cable is considered in the angle-of-wrap.

To determine which model was correct, experiments were conducted to measure rotary stiffness. The test setup consisted of a 3 mm steel cable wrapped in a figure-eight pattern around two transmission drums, with diameters 50 mm and 280 mm, refer to Figure 2. A load cell was spliced in the cable to determine the force around the drum and a rotary encoder attached to the output transmission drum determined the rotation of the output shaft, refer to Figure 3. A lever arm attached to the input transmission drum determined angular deflection of the input shaft. This value was subtracted to get the net deflection of the cable. A separate measurement which involved tensile testing with an Instron machine was performed to determine the effective modulus of the 3 mm 7X19 stranded steel cable.

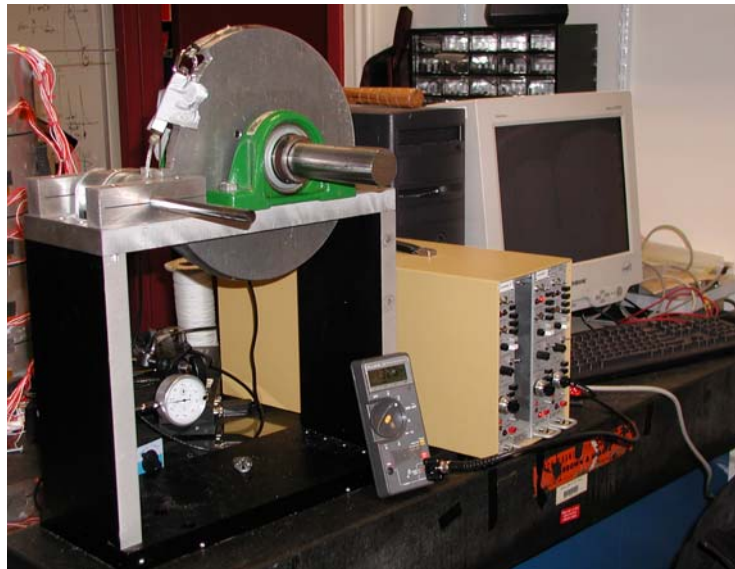


Figure 2: Configuration of the Capstan experimentation with load cell spliced between the cable.

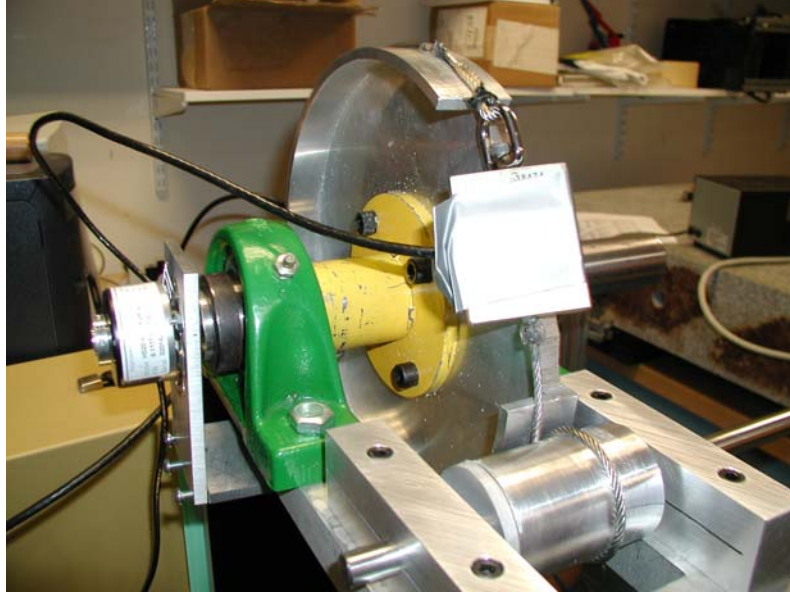


Figure 3: Encoder and load cell on output transmission drum. Input transmission drum shown in the bottom right.

Based on these experiments, it was determined that both models predict the correct torsional stiffness, varying 6% given an angle-of-anchor of 120° . When compared to experimental values, 60% of the results fell within both models, as shown in Figure 4. Likewise as the angle of anchor is increased, both models predict the same torsional stiffness within 6%. Therefore for low coefficient of friction the angle-of-anchor or the angle-of-wrap can be used where the cable extension is due to the external torque and preload in the cable but when the coefficient of friction increases and even though the angle-of-anchor is small, the angle-of-anchor is the conservative estimate. The recommended conservative torsional stiffness equations for a single free-set of cables are:

$$K_{torsional} = \frac{KD_o^2}{4} \tag{1}$$

$$K = \frac{2AE}{2\frac{D_o + D_l}{\mu} + L_{free}} \tag{2}$$

Given the maximum preload tension, $T_{preload}/2$, is used in the cable which makes the maximum torque:

$$\Gamma = \frac{T_{preload}D_l}{4} \tag{3}$$

where AE is the effective modulus of the cable area, D_O and D_I are the output and input drums respectively, L_{free} is the length of the cable not in contact with the drum, and the coefficient of friction between the drum and cable is μ .

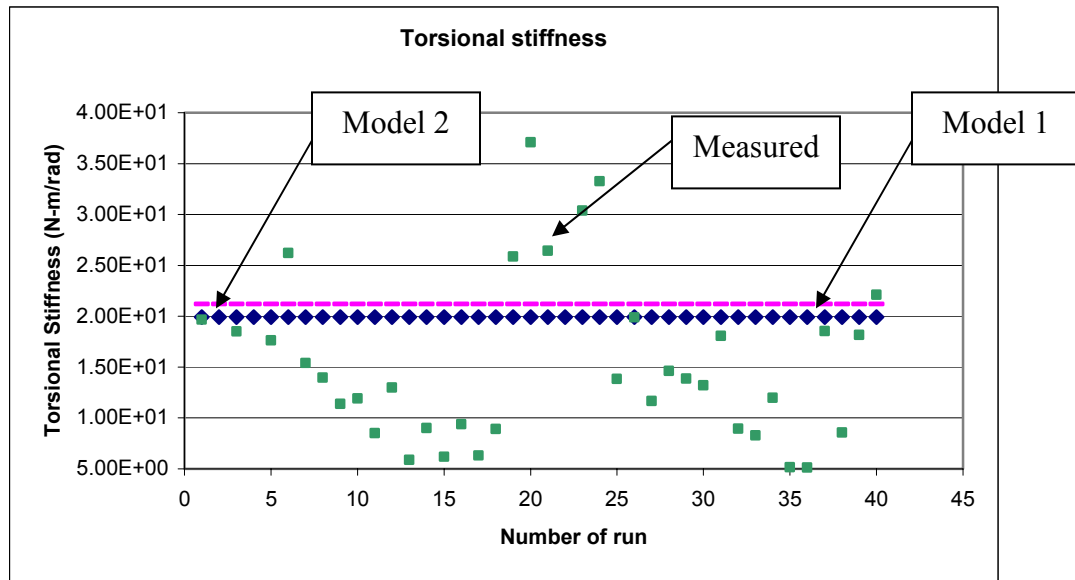


Figure 4: Torsional stiffness for measured and calculated values¹.

References:

- [1] Sagebrush technology, Inc “The Roto-Lok Drive Technology: A Revolutionary Innovation”, Sagebrush Technology, Inc. 2002.
<http://www.sagebrushtech.com/tech/technology.html>
- [2] Shigley, J., Mischke, C., *Standard Handbook of Machine Design*, 2nd ed. McGraw-Hill NY 1996, pp 31.5-31.9.
- [3] Alexander H. Slocum, *Precision Machine Design*, Society of Manufacturing Engineers 1992, pp. 691-693.

¹ Note two data points with values of 70 and 100 N-m/rad, are not shown on the graph due to randomness in value.