Identification of Nonlinear Characteristics for Precision Servomechanisms

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Abstract

A system identification method in the frequency-domain is proposed for a servomechanism with nonlinearities. A linear part of the servomechanism is identified exactly without distortion due to inherent nonlinearities through the proposed method. Parameters of the nonlinearities, including both Coulomb and viscous friction, are identified by the use of limit cycle analysis and describing function approximation. A model-based feedforward controller is applied to verify the availability of the proposed identification method.

Keywords: Coulomb and Viscous Friction, Describing Function, Friction Compensator, Limit Cycle, Nonlinearity, Servomechanism

1. Introduction

The need for high-density magnetic memory devices, processing of semiconductors, and optoelectronic elements has increased the demand for precision servomechanisms. Therefore, identification and compensation of the nonlinearities that seriously deteriorate system accuracy are indispensable to fabricate a high precision servomechanism [1].

In order to identify the nonlinear characteristics of servomechanism, an accurate linear element model should be obtained in advance. Two biased-square wave signals that have different magnitudes and the same frequency are used in identifying the linear element. This identification method does not suffer from the problem of nonlinear distortions and, therefore, is able to provide the accurate model of servomechanism dynamics.

We propose the application of relay feedback experiments to identify parameters of friction model, including both Coulomb and viscous characteristics, of servomechanisms. A relay with time delay is added to the system in order to induce various limit cycle conditions. Then an equivalent nonlinearity can be determined as a combination of the relay and the friction model in the form of describing functions. Using this method, the identification of nonlinearities becomes the identification of describing function parameters using the amplitude and frequency of limit cycle oscillations.

A two-degree-of-freedom controller including a PID feedback controller and a model-based feedforward friction compensator is applied to verify the effectiveness of the proposed identification method in this paper.

2. Servomechanism with nonlinearitiy

A. Decouple the servomechanism with nonlinearity

Consider a mechanical part of servomechanism that can be decomposed into a linear element $G_m(s)$ with a nonlinear feedback element $N(v)$ as shown in Fig. 1. The linear element describes the servomechanism dynamics and the nonlinear element describes the inherent friction. In order to identify the servomechanism, an input signal τ_c and an output signal v is used in conventional method with the assumption that the system to be identified is linear [2]. However the actual input signal τ_m to the linear element is quite different from the input signal τ_c owing to effects of nonlinearities.

Consider two square wave signals τ_{m1} and τ_{m2} having different magnitudes and the same frequency as input signals in the identification process. From the linear system characteristics (e.g. communicative law,

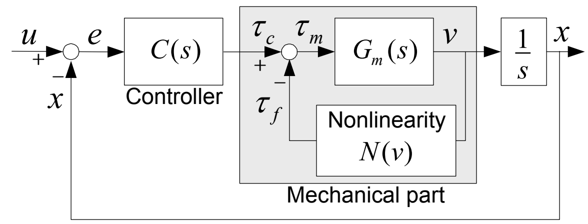

Fig. 1 Block diagram of servomechanism.

distributive law, and so on), the relationship between input and output signals to the linear element is given by

$$G_m(s) = \frac{v_1}{\tau_{m1}} = \frac{v_2}{\tau_{m2}} = \frac{v_1 - v_2}{\tau_{m1} - \tau_{m2}} \tag{1}$$

, where v_1 and v_2 are the output signal corresponding to the input signal τ_{m1} and τ_{m2}, respectively.

In general, the most significant friction components distorting the identification results are the stiction and the Coulomb friction:

$$\tau_m = \tau_c - \left[F_c \cdot \operatorname{sgn}(v) + F_s \cdot N_s(v)\right] \tag{2}$$

, where F_c is a Coulomb friction coefficient and $N_s(v)$ is a stiction nonlinearity.

If unidirectional signals which have a proper magnitude are used in the identification process, the stick phenomenon will be eliminated because there are no zero-velocity commands to the system at all times. Then, the difference between two square input signals τ_{m1} and τ_{m2} can be represented as;

$$\begin{aligned} \tau_{m1} - \tau_{m2} &= \tau_{c1} - \tau_{c2} - F_c\left[\operatorname{sgn}(v_2) - \operatorname{sgn}(v_2)\right] \\ &= \tau_{c1} - \tau_{c2} \end{aligned} \tag{3}$$

From Eq. (1) and (3), the transfer function of linear element can be described as;

$$G_m(s) = \frac{v_1 - v_2}{\tau_{m1} - \tau_{m2}} = \frac{v_1 - v_2}{\tau_{c1} - \tau_{c2}} \tag{4}$$

Consequently, the linear element shown in Fig. 1 can be exactly identified by taking $(\tau_{c1} - \tau_{c2})$ and $(v_{c1} - v_{c2})$ as the I/O pairs, which are equivalent to the I/O pairs $(\tau_{m1} - \tau_{m2})$ and $(v_{c1} - v_{c2})$.

B. Experimental Results

We apply the proposed identification method to a precision x-y positioning system. The positioning system is equipped with a ball–screw driving mechanism using AC-servo motors and amplifiers, encoders, digital I/O interfaces, and a PC-based controller.

The input signals τ_{c1} and τ_{c2} composed of Gaussian pseudo-random binary sequence are used as the torque command while motor speed v_1 and v_2 are synchronously recorded. The transfer function of linear element is obtained for the input-output sequence using the MATLAB System Identification Toolbox [3].

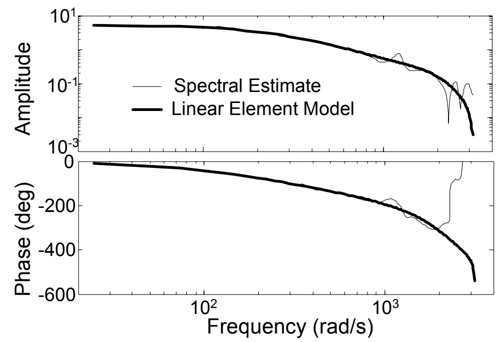

Fig. 2 Frequency response of identified linear element.

For the single axis, a 3rd-order model was obtained with coefficients f = [1, -1.058, -0.08528, 0.2177], b = [0.05741, 0.2937, 0.181]. The magnitude and phase response of the identified model are shown in Fig. 2 along with the power spectral estimates. Based on the identified linear element model, parameters of nonlinearities can be accurately determined.

3. Modeling and identification of friction

A. Modeling of friction with describing function

The friction model considered in this paper consists of Coulomb and viscous friction elements as shown in Fig.3 Conventional identification methods require accurate nonlinear models and the information of acceleration or velocity [1,4]. Therefore, relay feed-back experiments have been used for identification of friction parameters and frequency characteristics without using the acceleration data [5,6].

A system configuration containing linear element and two relays is devised to identify the friction parameters of the servomechanism. The first relay, RELAY Ⅰ, is considered as the friction model on the feedback path with the linear element. The second relay, RELAY II, with time delay is added to induce limit cycles as shown in Fig. 3.

The closed-loop configuration shown in Fig. 3 may be modified equivalently as in the configuration of Fig. 4, which consists of the equivalent nonlinearity acting on the linear element. A describing function approximation could be directly applicable toward the

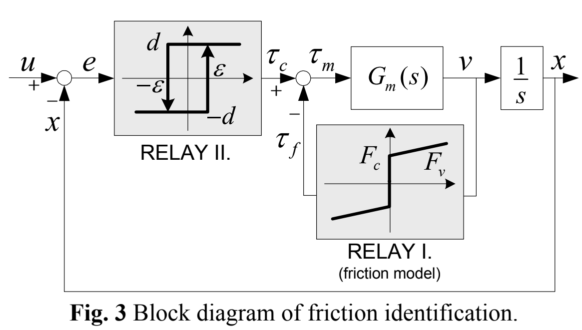

Fig. 3 Block diagram of friction identification.

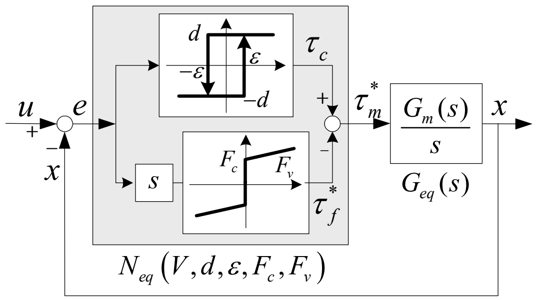

Fig. 4 Block diagram of equivalent nonlinearity.

analysis of the closed loop system. The describing functions of these two relays are given

$$NF = \frac{4F_c}{\pi V} + F_v$$

$$NR = \begin{cases} 0 & , V \le \varepsilon \\ \frac{4d}{\pi V}\sqrt{1-\left(\frac{\varepsilon}{V}\right)^2} - j\frac{4d\varepsilon}{\pi V^2} & , V > \varepsilon \end{cases} \quad (5)$$

, where NF is the describing function of the inherent friction model, NR is the describing function of the relay with time delay, V is the magnitude of input signal to nonlinear element, and F_c , F_v is the Coulomb and viscous friction coefficients, respectively.

Then, the describing function of the equivalent nonlinearity $N_{eq}(V,d,\varepsilon,F_c,F_v)$ can be determined as the parallel combination (i.e. the sum) of two describing functions of the two relays with a differential element.

$$N_{eq}(V,d,\varepsilon,F_c,F_v) = NR + j\,NF$$

$$= \begin{cases} j\left[\frac{4F_c}{\pi V} + F_v\right] & , V \le \varepsilon \\ \frac{4d}{\pi V}\sqrt{1-\left(\frac{\varepsilon}{V}\right)^2} - j\left[\frac{4(d\varepsilon - F_c V)}{\pi V^2} - F_v\right] & , V > \varepsilon \end{cases} \quad (6)$$

Using this method, the identification of nonlinearities becomes the identification of describing function parameters using the amplitude and frequency of the induced limit cycle oscillations. Another advantage of this method is that existing control hardware is usually sufficient to identify the friction model and implement its compensation.

B. Limit cycle analysis

The friction parameters can be identified in the equivalent system (see Fig. 4) using the limit cycle analysis [7,8]. The first step of the limit cycle analysis is to find the describing function of the equivalent nonlinearity. The second step is the derivation of limit cycle conditions. The limit cycle conditions can be expressed as follows;

$$1 + N_{eq} \cdot G_{eq}(j\omega) = 0 \quad (7)$$

The amplitude V and oscillating frequency ω of the limit cycle can be approximately given by the intersection of $G_{eq}(j\omega)$ and the negative inverse describing function of the equivalent nonlinearities.

When the describing function $N_{eq}(V,d,\varepsilon,F_c,F_v)$ is unknown and the transfer function of the linear element $G_{eq}(j\omega)$ is known, it is possible to use condition (7) to measure points on the frequency characteristics of the equivalent nonlinearity.

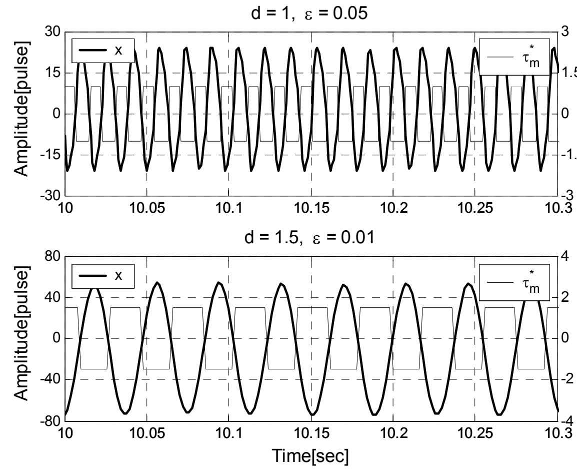

Fig. 5 Limit cycle oscillations by relay experiments.

C. Experimental Results

Two relay experiments are conducted in the precision x-y positioning system. By varying delay time and amplitude of the second relay, two limit cycle conditions can be generated. The limit cycle oscillations arising from the two experiments are shown in Fig. 5. Therefore two equations are derived from the Eq. (6), (7) to obtain the unknown parameters F_c and F_v . Clearly, two unknown parameters F_c and F_v can be obtained from the solution of Eq. (8).

$$\alpha_i X - Y = \beta_i,\ V > \varepsilon,\ i = 1,2 \quad (8)$$

where, $\begin{cases} X = F_c,\ Y = F_v,\ \alpha_i = -4/(\pi V_i) \\ \beta_i = \frac{4d_i}{\pi V_i}\sqrt{1-(\varepsilon_i/V_i)^2} \cdot \tan\left[\angle - G_{eq}(j\omega_i)\right] - \frac{4d_i\varepsilon_i}{\pi V_i^2} \end{cases}$

The friction parameters identified from two relay experiments are given by

$$\left.\begin{matrix} X = f(\alpha_1,\alpha_2,\beta_1,\beta_2) = 0.19801 \\ Y = f(\alpha_1,\alpha_2,\beta_1,\beta_2) = 0.87005 \end{matrix}\right\} \rightarrow \begin{cases} f_c = 0.19801 \\ f_v = 0.87005 \end{cases} \quad (9)$$

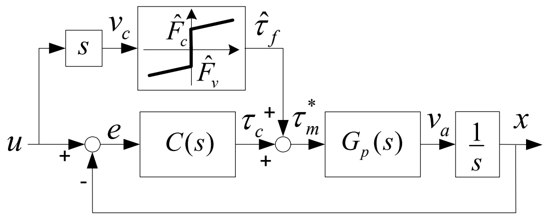

Fig. 6 Feedforward friction compensator.

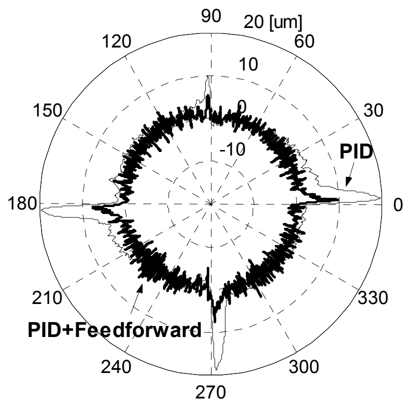

Fig. 7 Position error profile in circular motion. (Feedrate: 2000mm/min, Radius: 100mm)

4. Friction compensation

A two-degree-of-freedom controller including a PID feedback controller and a model-based feedforward friction compensator is applied to verify the identified friction model. Model-based friction compensators can be classified according to what estimate of velocity is used to evaluate the friction model and what parameters of the entire friction model are applied[1].

In this paper, the command velocity and the identified parameters of Coulomb and viscous friction are used in the friction compensator as shown in Fig 6, where $C(s)$ is the PID feedback controller.

Fig. 7 shows the position error in a circular motion with and without the feedforward friction compensator. Clearly, quadratic protrusion errors and the tracking error in the vicinity of zero-velocity are reduced with the friction compensator. The experimental results indicate that nonlinear characteristics, such as inherent frictions, can be identified accurately by using the propose method.

5. Conclusions

This paper presents a new application of relay feedback experiments in order to identify nonlinear characteristics in servomechanism.

Two biased-square wave signals that have different magnitudes and the same frequency are used in identification process to derive an accurate model of the system dynamics. This method does not suffer from the problem of nonlinear distortions and, therefore, is able to provide the accurate model of servomechanism dynamics.

A relatively simple identification method of the friction model is presented based on the limit cycle and the describing function analysis.

Experiments have been conducted with identified friction parameters in the precision x-y positioning system. Results from experiments have verified the availability of the proposed identification method

References

1. Armstrong-Hélouvry, B., Dupont, P. and Canudas de wit, C., 1994, "A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction," *Automatica*, Vol. 30, No. 7, pp. 1083~1138.
2. Ljung, L., 1999, *System Identification-Theory for the User*, Prentice Hall PTR, New Jersey, pp.79~246.
3. *MATLAB System Identification Toolbox User Guide*, Mathworks, Inc., 2000.
4. Hensen, R., H., A., van de Molengraft, M., J., G., and Steinbuch, M., 2002, "Frequency Domain Identification of Dynamic Friction Model Parameters," *IEEE Trans. On Control Systems Technology*, Vol. 10, No. 2, pp. 191~195.
5. Tan, K., K., Lee, T., H., Huang, S., N., and Jiang, X., 2001, "Friction Modeling and Adaptive Compensation Using a Relay Feedback Approach," *IEEE Trans. On Industrial Electronics*, Vol. 48, No. 1, pp. 169~175.
6. Besançon-Vode, A. and Besançon, G., 1999, "Analysis of a two-relay system configuration with application to Coulomb friction identification," *Automatica*, Vol. 35, pp. 1391~1399.
7. Slotine, J., J., E., and Li, W., 1991, *Applied Nonlinear Control*, Prentice Hall, Inc., New Jersey, pp.157~190.
8. Atherton, D., 1975, *Nonlinear Control Engineering*, Van Nostrand Reinhold Co., London, pp.75~166.